A Delayed Column Generation Approach for Solving a Cargo Crew Scheduling Problem

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Outline

1. Introduction
2. Model and Solution Framework
3. Basic Case: Crew Pairings with No Breaks
4. Advanced Case: Crew Pairings That Allow Breaks
5. Generalization
6. Conclusion and Future Work

Cargo Aviation Industry

- Global air cargo traffic is forecast to grow a **robust 4.2%** per year over the next 20 years (Boeing 2017)
  - The revenue ton-kilometers (RKT) will more than double from **256 billion** in 2017 to **584 billion** in 2037
  - The number of freighter airplanes will grow by more than **70 percent** in total

Introduction

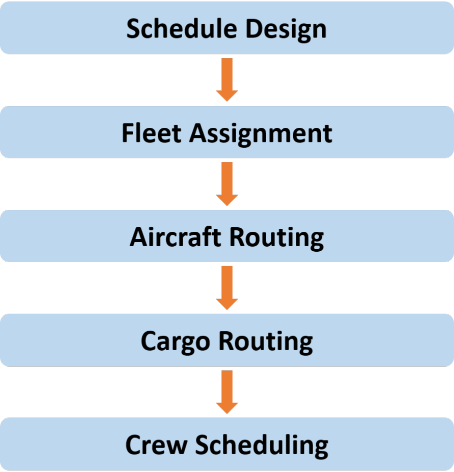
Background

- The cargo airline that we partnered with accepts requests for goods delivery, from one location to another, from customers including logistic companies, manufacturers, the military, and so on
- Requests are gathered and further partitioned into different planning horizons, typically a calendar month
- The cargo airline has a fleet of airplanes and a group of crews it can contract, and needs to determine and schedule all necessary tasks and activities accordingly so that the requests in each planning horizon will be delivered as planned

Introduction

Background

Cargo Aviation Planning Procedure



Schedule Design

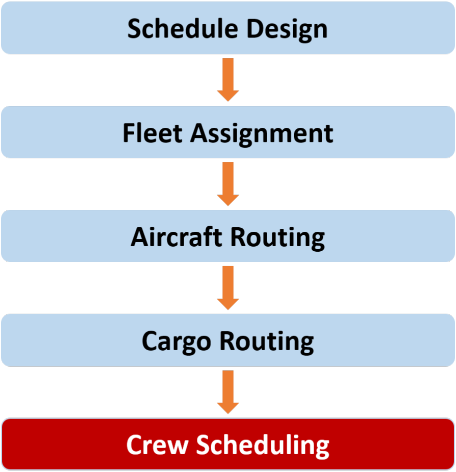
Fleet Assignment

Aircraft Routing

Cargo Routing

Crew Scheduling

Cargo Aviation Planning Procedure



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Crew Pairing

A sequence of flights that will be assigned to a single crew to carry out

- Specific requirements like labor regulations must be satisfied

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Crew Scheduling

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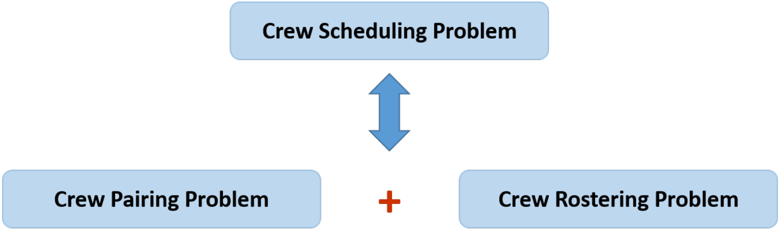
Traditional Passenger Aviation:



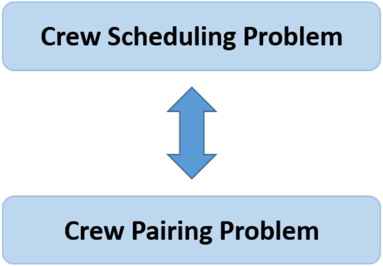
Crew Scheduling Problem

Crew Pairing Problem + Crew Rostering Problem

Introduction

Crew Scheduling

Our Cargo Aviation Problem:



Crew Scheduling Problem

Crew Pairing Problem

- △ The majority of flights are long-haul, international flights
- △ Lack repeating daily pattern

Introduction

Crew Scheduling

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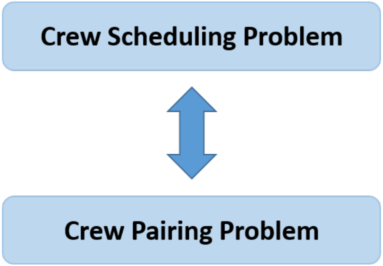

Crew Scheduling Problem

Crew Pairing Problem

△ The majority of flights are long-haul, international flights

△ Lack repeating daily pattern

- Each crew pairing spans a much longer time, e.g. a half month
- The crew pairing will correspond to a complete crew schedule
- The crew are possibly away from home for the whole duration of the assigned pairing
- The crew often fly commercially from/to their base when starting/finishing the pairing

Introduction

Problem Statement

Requirements

- Basic "laws" of physics
- Regulatory policies

  E.g. Each *duty period* cannot exceed 17 hours, and the crew must have a minimum 10-hour *layover* for rest before starting next duty period
- Corporate policies

  E.g. The time span of the crew pairing must be at least 12 days

Introduction

Problem Statement

We have a **different objective**, unlike the traditional ones

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- UNABLE to cover ALL flights in the planning horizon
    - The structure of the network - lacks opportunities for quick turns; includes many airports with a small number of associated flights
    - The requirements - avoid assigning short pairings to crews

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Objective

Cover as many flights in the planning horizon as possible with valid crew pairings that satisfy all of the requirements

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Remedy

Allow a "**break**" to take place in the "middle" of the crew pairing

- Two extra requirements must be satisfied to have a break:
  - The duration of the break must be at least 6 days
  - The break cannot take place before the crew completes the fourth flight, or after the seventh flight in the pairing

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Basic Case: **No Breaks**

Advanced Case: **Allow Breaks**

Model and Solution Framework

A Set Packing Problem

Formulation

$$\begin{aligned} \min \quad & \sum_{p \in P} -n_p \cdot x_p \\ \text{s.t.} \quad & \sum_{p \in P} a_{f,p} \cdot x_p \leq 1 && \forall f \in F \\ & x_p \in \{0,1\} && \forall p \in P \end{aligned}$$

Sets and Parameters

F	the set of flights
P	the set of valid crew pairings
n_p	the number of flights covered by pairing p, for $\forall p \in P$
$a_{f,p}$	1 if flight f is covered by pairing p; 0 otherwise, for $\forall f \in F, \forall p \in P$

Decision Variables

x_p	Binary for $\forall p \in P$. 1 if pairing p is assigned to a crew; 0 otherwise

Model and Solution Framework

Delayed Column Generation

- First, solve the LP-relaxation to optimality, with the crew pairings iteratively incoporated on demand, driven by the dual values via a delayed column generation (DCG) framework

  (Lavoie et al. (1988), Anbil et al. (1998), Wei and Vaze (2018), ...)

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Delayed Column Generation

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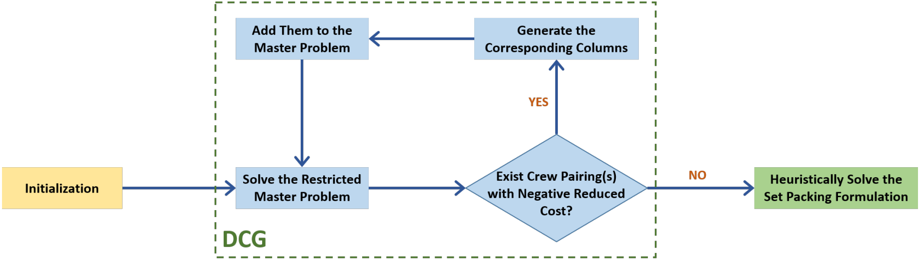
  (Lavoie et al. (1988), Anbil et al. (1998), Wei and Vaze (2018), ...)

- Heuristically solve the original integrality-constrained set packing formulation with pairings limited to those generated during the DCG

  (Barnhart et al. (1994), Dunbar et al. (2012), ...)

Model and Solution Framework

Delayed Column Generation



Initialization

Solve the Restricted Master Problem

Exist Crew Pairing(s) with Negative Reduced Cost?

YES

Generate the Corresponding Columns

Add Them to the Master Problem

NO

Heuristically Solve the Set Packing Formulation

DCG

Generating Pairings with No Breaks via SPPRC

Previous Work

- Shortest Path Problem with Resource Constraints (SPPRC) was first introduced for solving a routing problem with time windows for bus transportation (Desrosiers et al. (1984)).
- It has since been generalized, and several variants have been proposed, to address a wide range of problems in transportation
  - **Routing:** Desrochers and Soumis (1989), Dumas et al. (1991), Ioachim et al. (1998), Feillet et al. (2004), ...
  - **Crew Scheduling:** Vance et al. (1997), Gamache et al. (1999), Dunbar et al. (2012), Shao et al. (2015), ...
  - Etc.
- Irnich and Desaulniers (2005) provides a comprehensive instruction and survey on SPPRC.

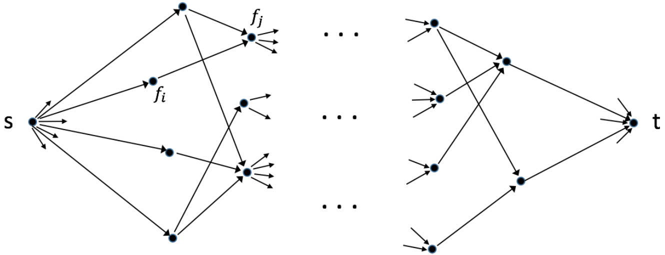
Generating Pairings with No Breaks via SPPRC

Flight-based Network

Directed graph $G(V, E)$

V: Nodes in the network, consist of $F \cup \{s, t\}$

E: Arcs in the network. For $\forall f_1, f_2 \in F$, arc (f_1, f_2) exists *iff* the basic "laws" of physics (i.e. Requirement 1) hold on this follow-up. In addition, for $\forall f \in F$, there are arcs (s, f) and (f, t)



An $s - t$ path in the network corresponds to a potential crew pairing

Generating Pairings with No Breaks via SPPRC

Model

Every path p in the network G is tracked with a resource vector $T^p \in \mathbb{R}^k$:

- Each of the first $k-1$ resources is defined to prevent the violation of one of the remaining requirements

  E.g. r_a: The amount of time the current duty period has spanned so far

  (Ensure no duty period exceeds the maximum 17-hour duty length)

  E.g. r_b: The remaining amount of total time required by the current crew pairing to fulfill the minimum requirement on the total time span

  (Ensure the length of the finalized crew pairing at the end is not shorter than 12 days)

- The last resource r_k is defined for the calculation of the reduced cost of the current crew pairing

Generating Pairings with No Breaks via SPPRC

Model

- Let $\mathcal{P}$ be the set of feasible $s - t$ paths respecting all resource constraints in the network G.

The original pricing problem is then equivalent to solving the following formulation based on our SPPRC model:

$$\min_{p \in \mathcal{P}} \; T_k^p$$

Generating Pairings with No Breaks via SPPRC

Model

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The original pricing problem is then equivalent to solving the following formulation based on our SPPRC model:

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- A **label-setting algorithm** is used to solve this formulation
  - Similar like Dijkstra's algorithm, but in a higher dimension
  - Infeasible and inferior sub-paths are discarded during the path extension

Generating Pairings with No Breaks via SPPRC

Computational Experiments

Dataset	#Flights	#Valid Pairings	Enum. Time	#Flights Cov.
No.1	606	440,641	30min 34sec	332
No.2	541	329,145	26min 40sec	281
No.3	644	462,395	35min 52sec	334

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Dataset	LP-obj	#Itr.	LP Time	#Pairings Gen.
No.1	336.382	11	6min 13sec	22,052
No.2	284.447	9	3min 36sec	16,642
No.3	340.327	10	6min 24sec	23,736

Dataset	IP-obj	IP Time	Coverage
No.1	332	28sec	54.79%
No.2	281	24sec	51.94%
No.3	333	5min 20sec	51.71%

Generating Pairings That Allow Breaks

A Straight Extension

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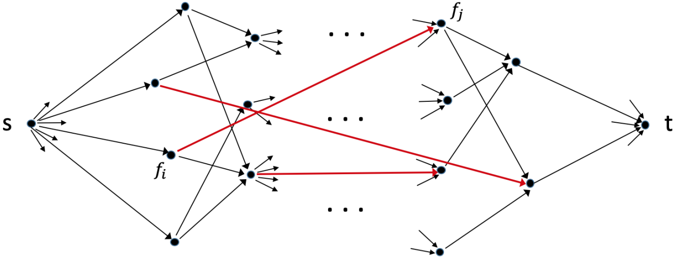
- An additional set of "*break arcs*" B are introduced into the flight-based network $G(V, \bar{E})$, where $\bar{E} = E \bigcup B$



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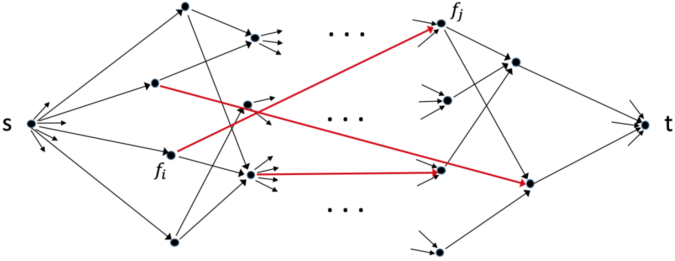

- The SPPRC model is updated accordingly,
  i.e. necessary resources are additionally incorporated to ensure the requirements introduced by the break feature are satisfied

Generating Pairings That Allow Breaks

A Straight Extension

Computational Challenge

For the first iteration:

- Runtime: > **10 hours**
- #pairings found: > **2.6 million**

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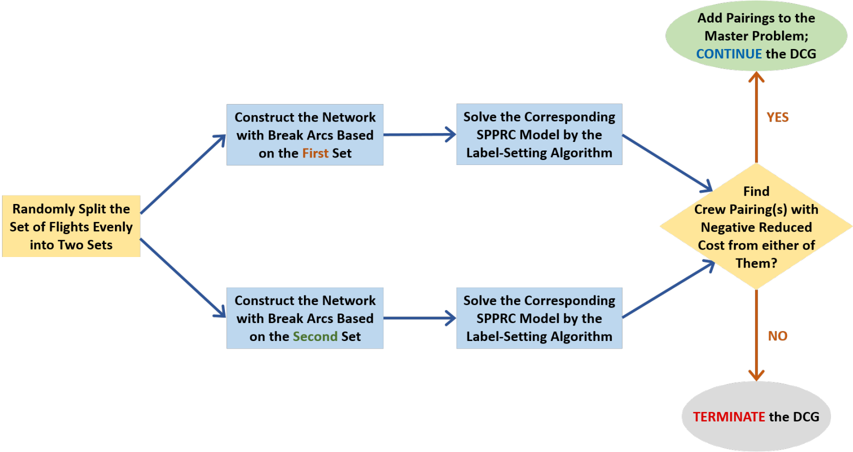
#Nodes	#Arcs (no breaks)	#Arcs (allow breaks)
608	12,539	**123,612**

Generating Pairings That Allow Breaks

A Heuristic - Flight Partitioning Approach



Randomly Split the Set of Flights Evenly into Two Sets

Construct the Network with Break Arcs Based on the First Set

Solve the Corresponding SPPRC Model by the Label-Setting Algorithm

Construct the Network with Break Arcs Based on the Second Set

Solve the Corresponding SPPRC Model by the Label-Setting Algorithm

Find Crew Pairing(s) with Negative Reduced Cost from either of Them?

YES

Add Pairings to the Master Problem; CONTINUE the DCG

NO

TERMINATE the DCG

Generating Pairings That Allow Breaks

An Exact Algorithm - Arc Selection Approach

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- Rather than dumping ALL break arcs into the original network, we only introduce a SUBSET of them each time (Barnhart et al. (1995))

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- Each selected arc should be beneficial. That is, crew pairing(s) containing the corresponding break which has a negative reduced cost should then be introduced

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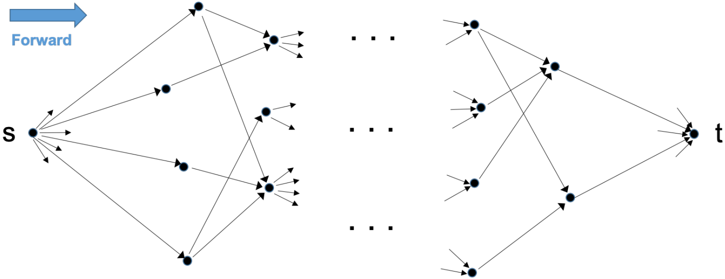

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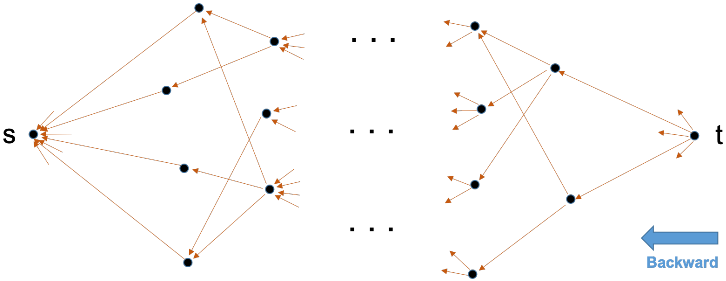

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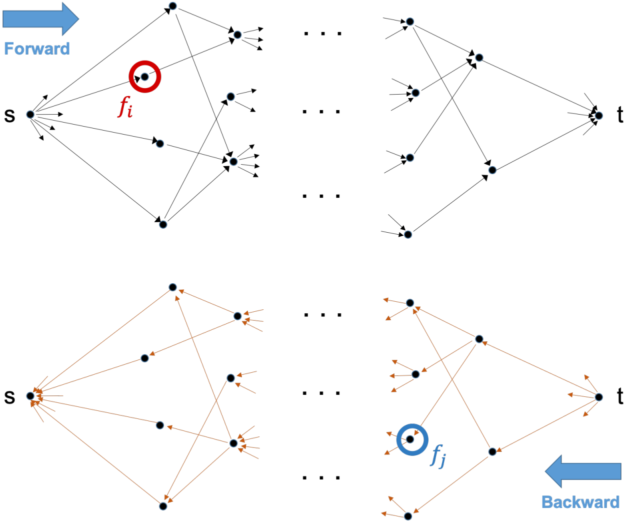

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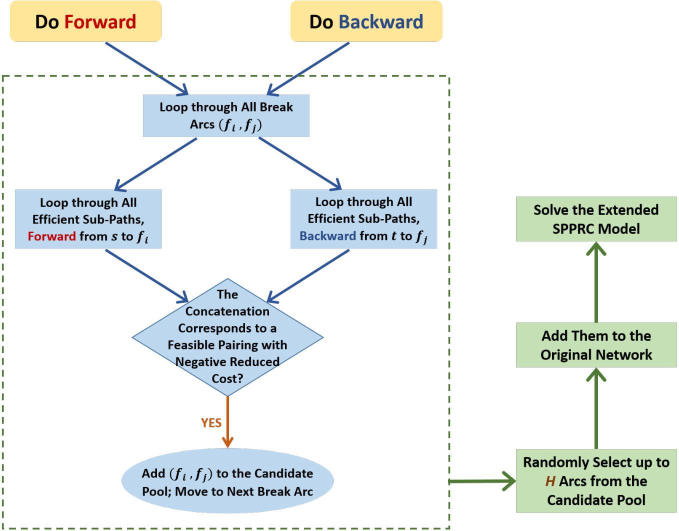

Do Forward

Do Backward

Loop through All Break Arcs (f_i, f_j)

Loop through All Efficient Sub-Paths, Forward from s to f_i

Loop through All Efficient Sub-Paths, Backward from t to f_j

The Concatenation Corresponds to a Feasible Pairing with Negative Reduced Cost?

YES

Add (f_i, f_j) to the Candidate Pool; Move to Next Break Arc

Randomly Select up to H Arcs from the Candidate Pool

Add Them to the Original Network

Solve the Extended SPPRC Model

Generating Pairings That Allow Breaks

An Exact Algorithm - Arc Selection Approach

- The accumulation of resource consumption is **independent augmentation**, *or* **independent augmentation with reset** for all resources

Proposition

A break arc is pruned out during the proposed arc selection procedure *iff* there does NOT exist any negative reduced cost crew pairing which contains a break corresponding to this arc

Generating Pairings That Allow Breaks

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Proposition

A break arc is pruned out during the proposed arc selection procedure *iff* there does NOT exist any negative reduced cost crew pairing which contains a break corresponding to this arc

- If NO pairing is found by solving the extended SPPRC model, then:
  - NO break arc is selected and added to the network
    - NO negative reduced cost pairing which contains a break exists
  - NO negative reduced cost pairing which does not contain a break exists

Generating Pairings That Allow Breaks

Computational Experiments

Basic Case: No Breaks

Dataset	#Flights	#Arcs	#Valid Pairings	Enum. Time
No.1	606	12,539	440,641	30min 34sec
No.2	541	10,113	329,145	26min 40sec
No.3	644	12,201	462,395	35min 52sec

Advanced Case: Allow Breaks

Dataset	#Flights	#Arcs	#Valid Pairings	Enum. Time
No.1	606	123,612	142,777,637	3day 02hr
No.2	541	97,716	79,648,029	1day 21hr
No.3	644	134,907	133,208,846	3day 02hr

Generating Pairings That Allow Breaks

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Flight Partitioning Heuristic Approach (Time Limit on DCG is 4hr, on IP is 2hr)

Dataset	LP-obj	#Itr.	LP Time	#Pairings Gen.
No.1	551.53 (0.70)	129.5 (4.5)	4.0hr (0.00)	61,600 (4,809)
No.2	475.28 (4.13)	133.2 (29.5)	1.7hr (0.37)	46,363 (4,393)
No.3	569.37 (2.71)	132.4 (22.1)	3.6hr (0.65)	58,888 (3,425)

Dataset	IP-obj	IP Time	B&C Gap (%)	Coverage (%)
No.1	494.20 (4.64)	2hr (0)	10.89 (1.07)	81.55 (0.77)
No.2	426.90 (5.96)	2hr (0)	9.93 (1.21)	78.91 (1.10)
No.3	510.30 (6.83)	2hr (0)	10.85 (1.12)	79.24 (1.06)

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Arc Selection Exact Approach ($H = 250$; Time Limit on IP is 2hr; No Limit on DCG)

Dataset	LP-obj	#Itr.	LP Time	#Pairings Gen.
No.1	563.29	41	2hr 09min	79,730
No.2	492.10	35	1hr 14min	58,448
No.3	584.42	37	1hr 57min	85,050

Dataset	IP-obj	IP Time	B&C Gap (%)	Coverage (%)
No.1	521	2hr	7.81	85.97
No.2	454	2hr	7.89	83.92
No.3	551	2hr	5.72	85.56

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- The **arc selection approach** would be **effective** if the density of the underlying network prevents the tractability of the SPPRC model (particularly for scheduling and routing problems)
    - Guaranteed to be exact, if the resource consumption is accumulated in an independently augmenting (possibly with reset) manner.

Generalization

- The **arc selection approach** would be **effective** if the density of the underlying network prevents the tractability of the SPPRC model (particularly for scheduling and routing problems)
  - Guaranteed to be exact, if the resource consumption is accumulated in an independently augmenting (possibly with reset) manner.
- The **flight partitioning approach** and the **arc selection approach** can be **integrated together** to work (especially when the number of nodes and arcs are both extremely huge)
  - Use the flight partitioning approach to rapidly improve the objective value (the arc selection approach can be applied to solve each instance)
  - Turn to the arc selection approach to help proving optimality

Conclusion and Future Work

Conclusion

- We consider the problem of **generating high-quality crew pairings** to **cover as many scheduled flights as possible** for a cargo airline
- Two variations are considered, where the advanced case additionally incorporates a **“break”** feature to boost the flight coverage
- We model the problem as a **set packing problem**, and solve it using a **delayed column generation** framework
- The pricing problem is formulated as a **SPPRC**, and solved by a **label-setting algorithm** integrated with speed-up improvements

Conclusion and Future Work

Conclusion

- **A heuristic approach** and **an exact approach** are proposed to address the tractability issue when solving the advanced case
- Computational experiments on real-world datasets demonstrate the **benefits** of incorporating breaks into the pairing generation, and the **effectiveness** of our proposed approach
- The proposed heuristic and exact approaches can be used together to potentially solve instances of larger size, and can be generalized and applied to solve a wide range of other scheduling and routing problems

Conclusion and Future Work

Future Work

- Reduce the B&C gap when solving the IP
    - SDP (i.e. Theta Bag) to provide a tighter upper bound
    - Generate maximal clique cuts

- Apply other heuristics to deal with the integrality constraints (e.g. price-and-dive), or implement the exact B&P framework

- Incorporate deadhead into the pairing generation to further improve flight coverage

- Consider more complicated cost structure of crew pairings

Thank You for Your Attention

Q & A

Reference I

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