A Delayed Column Generation Approach for Solving a Cargo Crew Scheduling Problem

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Introduction

Given a set of flights in the planning horizon for the goods delivery, the crew scheduling problem here is to generate a set of ***crew pairings*** and assign each of them to a crew to operate in practice, such that the flights scheduled in the planning horizon are covered as many as possible while no flight is covered by more than one crew.

A crew pairing is a sequence of flights, where rules categorized into the following three types must be respected:

- Basic "lows" of physics
- Regulatory policies
- Corporate policies

To achieve higher flight coverage, we consider allowing a "break" to take place in the "middle" of the crew pairing, where the crew will fly commercially home to have a short vacation. However, this relaxation to the basic "laws" of physics prevents us from solving our problem in a direct manner, as tons of valid crew pairing will exist (Table 1).

1. Enumerating all valid crew pairings takes a great amount of time.
2. The math programming becomes too large to be solved explicitly.

Table 1: General information of three real-world datasets

Dataset	#Flights	#Arcs	#Valid Pairings	Enum. Time
No.1	606	123,612	142,777,637	3day 02hr
No.2	541	97,716	79,648,029	1day 21hr
No.3	644	134,907	133,208,846	3day 02hr

$$\begin{aligned} \min \quad & \sum_{p \in P} -n_p \cdot x_p \\ \text{s.t.} \quad & \sum_{p \in P} a_{f,p} \cdot x_p \leq 1 \qquad \forall f \in F \\ & x_p \in \{0,1\} \qquad \forall p \in P \end{aligned}$$

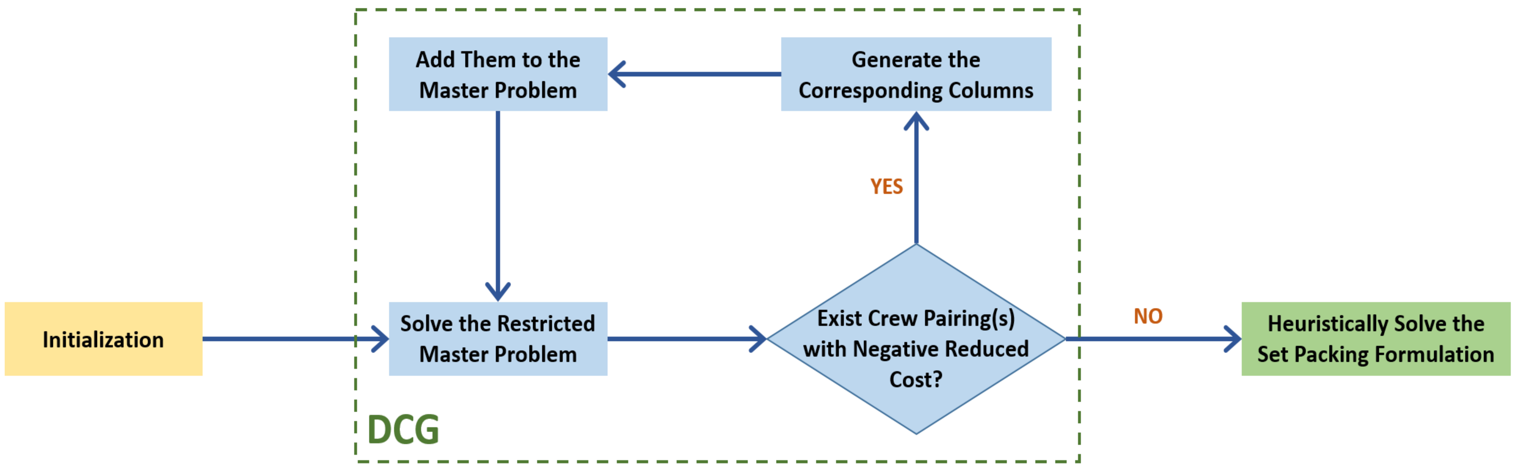
Figure 1: The math programming, a set packing formulation, for our problem

Objectives

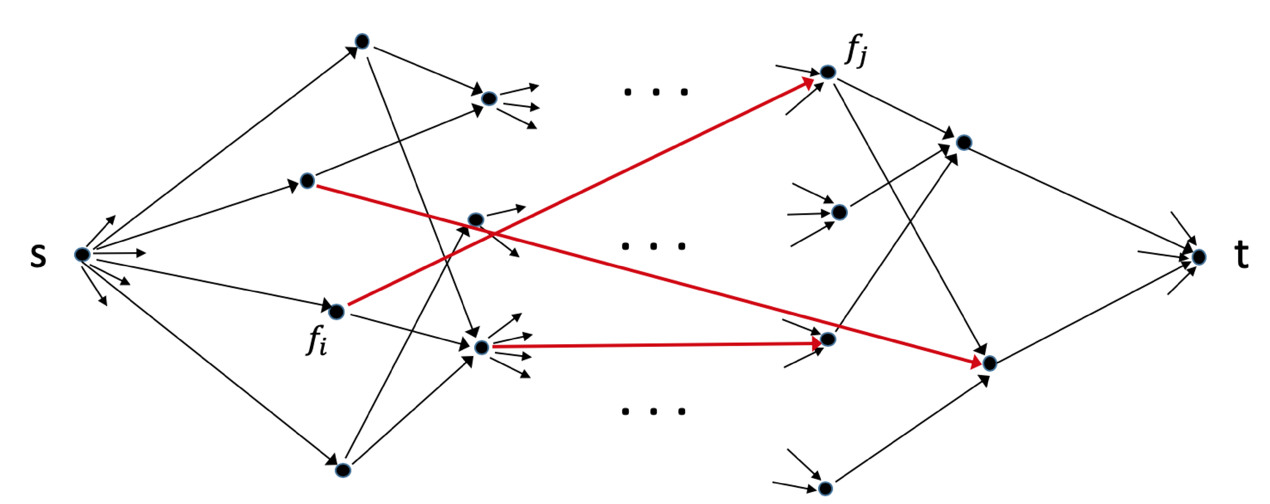
1. Evaluate whether incorporating the break feature into the pairing generation can improve the flight coverage to a desirable rate.
2. Develop efficient frameworks and approaches for solving the relaxed problem with the break feature incorporated.

Solution Framework

1. Solve the LP-relaxation of the set packing formation (Figure 1) to optimality via a delayed column generation (DCG) approach.
2. Heuristically solve the original integrality-constrained formulation with pairings limited to those generated during the previous step.

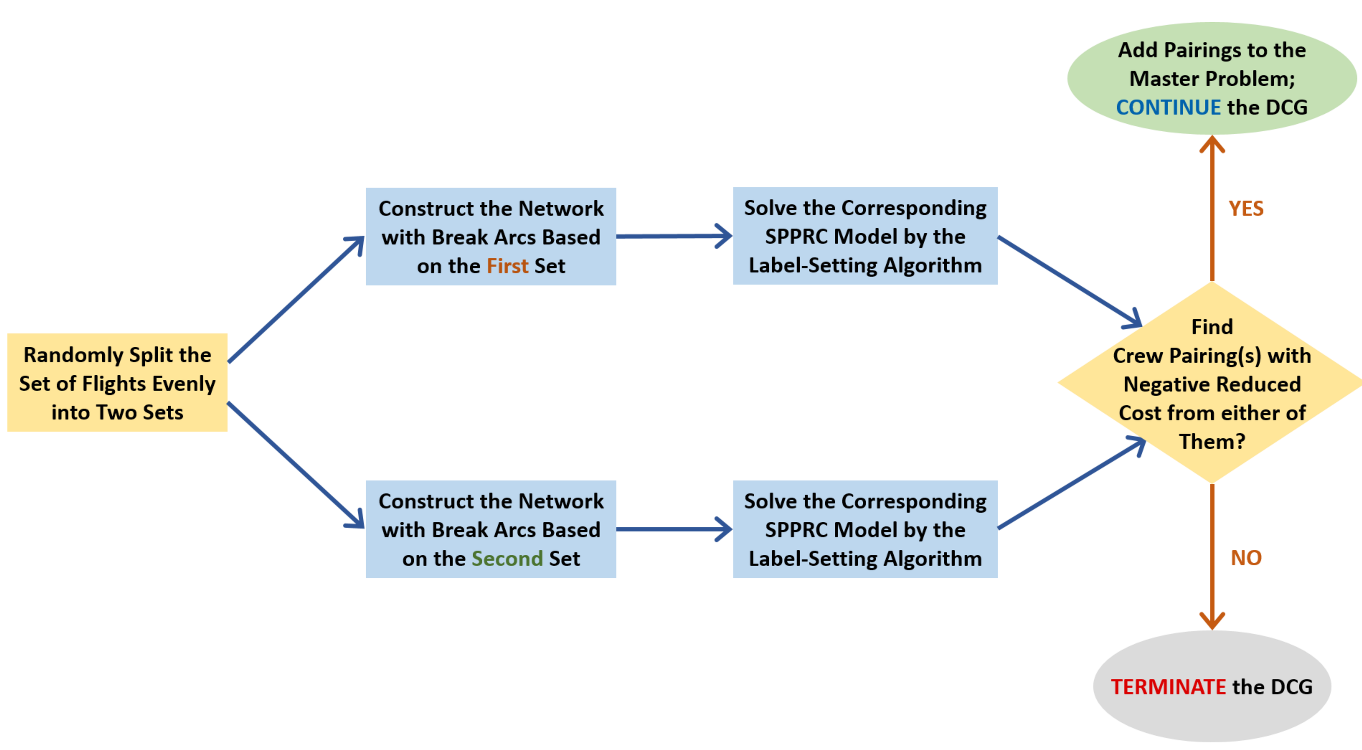


The pricing problem of the DCG is modeled as a shortest path problem with resource constraints (SPPRC) on a flight-based network.

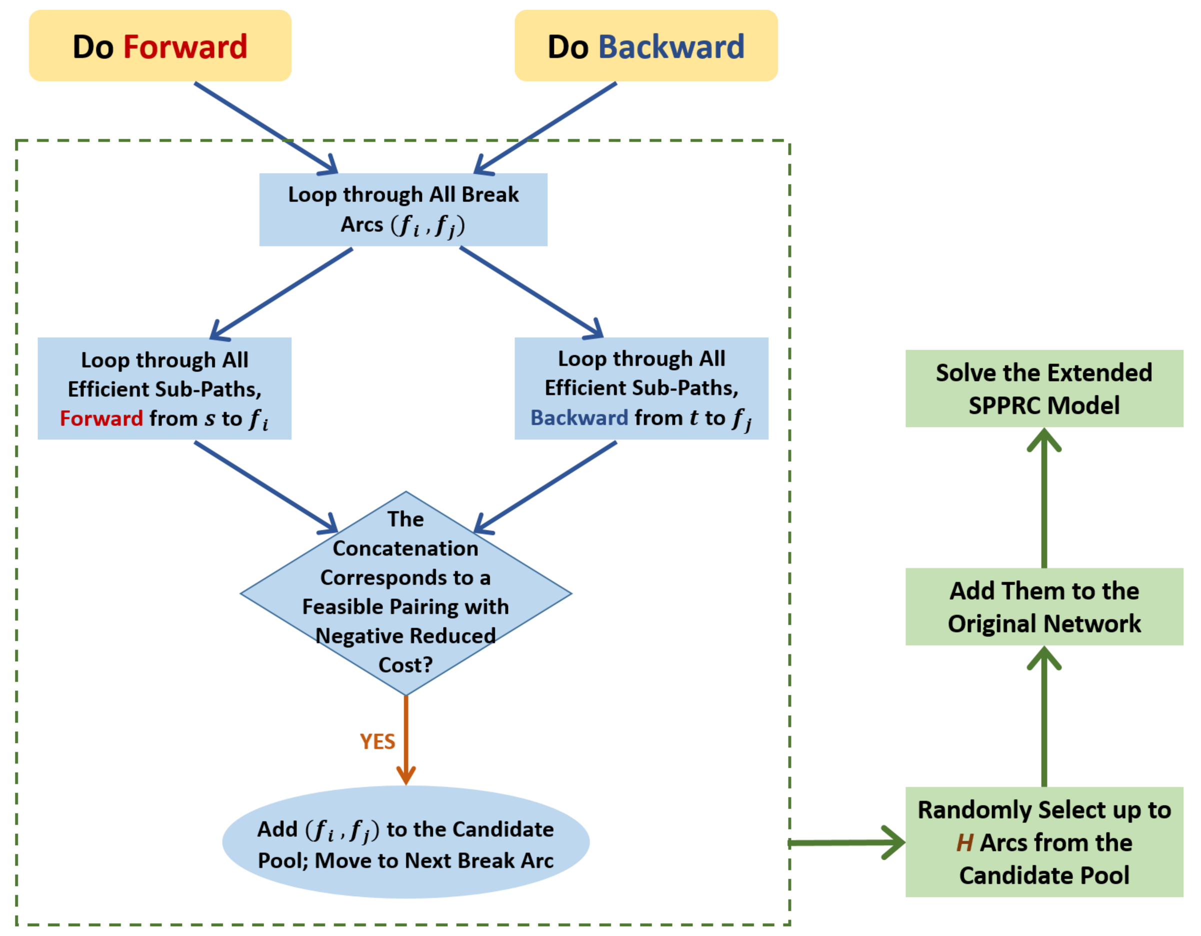


Solving the SPPRC model completely, by a label-setting algorithm even with a handful speed-up improvements, will take a long time.

A Flight Partitioning Heuristic



An Arc Selection Exact Approach



Results and Conclusions

The Flight Partitioning Heuristic

Dataset	LP-obj	#Itr.	LP Time	#Pairings Gen.
No.1	551.53 (0.70)	129.5 (4.5)	4.0hr (0.00)	61,600 (4,809)
No.2	475.28 (4.13)	133.2 (29.5)	1.7hr (0.37)	46,363 (4,393)
No.3	569.37 (2.71)	132.4 (22.1)	3.6hr (0.65)	58,888 (3,425)

Dataset	IP-obj	IP Time	B&C Gap (%)	Coverage (%)
No.1	494.20 (4.64)	2hr (0)	10.89 (1.07)	81.55 (0.77)
No.2	426.90 (5.96)	2hr (0)	9.93 (1.21)	78.91 (1.10)
No.3	510.30 (6.83)	2hr (0)	10.85 (1.12)	79.24 (1.06)

The Arc Selection Exact Approach

Dataset	LP-obj	#Itr.	LP Time	#Pairings Gen.
No.1	563.29	41	2hr 09min	79,730
No.2	492.10	35	1hr 14min	58,448
No.3	584.42	37	1hr 57min	85,050

Dataset	IP-obj	IP Time	B&C Gap (%)	Coverage (%)
No.1	521	2hr	7.81	85.97
No.2	454	2hr	7.89	83.92
No.3	551	2hr	5.72	85.56

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