A Data-driven Stochastic Optimization Approach to the Colonoscopy Scheduling Problem

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Colonoscopy Procedure

- The most common screening test for Colorectal Cancer, the third leading cause of cancer-related death in the US[1]
- Allows for direct visual examination of the entire colon and rectum
  - Spot existing cancer, prompting treatment
  - Prevent future cancer, by detecting precancerous growth
- Can help reduce CRC incidence by about 40% and mortality by about 50%[1]

[1] American Cancer Society, February 21, 2018

Challenges to Colonoscopy Scheduling at Michigan Medicine

- **Significant variability** in procedure duration due to the quality of the pre-procedure **bowel prep** that the patient must undergo

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- Patient **non-punctuality** (future talk)

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- **Significant variability** in procedure duration due to the quality of the pre-procedure **bowel prep** that the patient must undergo
- Patient **non-punctuality** (future talk)
- **Multiple** and **competing** performance criteria

Research Goal

- **Optimize** daily schedule **templates** and policies for filling these templates, to best account for patient characteristics and the associated **variability**

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- By building **higher-quality schedules**, it is possible to:
  - Reduce **costs**
  - Improve patient and provider **satisfaction**
  - Achieve **better clinical outcomes**, and more
- A valuable tool in creating such templates is the **ability to solve** the Stochastic Outpatient Procedure Scheduling Problem (**SOPSP**) as an embedded subproblem

Stochastic Outpatient Procedure Scheduling Problem(SOPSP)

A clinic manager must **schedule** the **start times** for a set of procedures, where

- Each procedure is of a **known type** and associated probability distribution of (non-negative) **random duration**
- Although patients may fail to show up to their appointments, we assume that those who do show up are **punctual**

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- The **provider is always available** at the start of the day, and immediately after each procedure
- There is **no opportunity to modify the schedule** on the day of service
- The performance metric is the **weighted sum** of total patient **waiting time** cost, total provider **idle time** cost, and clinic **overtime**

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A basic (yet still **challenging**) single-server stochastic appointment sequencing and scheduling (**SASS**) problem

So, We've Got Ourselves a Complex Appointment Scheduling Problem!!

What to do Next?

“Curse of Uncertainty"

Sample-Based Optimization and Mixed-Integer Programming (SMIP) to Rescue

SOPSP Complexity

Welch (1952)
Denton et al (2010)
Gupta and Denton (2008)
Mancilla and Storer (2012)
Gupta (2007)
Ahmadi-Javid et al (2017)
Berg et al (2014)
Weiss (1990)

Complex {combinatorial, stochastic, multi-criteria} Problem

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History of SMIP for SOPSP

- Denton et al., (2007)$\longrightarrow$ Mancilla and Storer., (2012)$\longrightarrow$ Berg et al., (2014)
- Not easy to solve, require specially-developed algorithms/heuristics,....

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What we need

- **Tractable SMIP**: can solve SOPSP instances of realistic sizes in an acceptable amount of time

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- **Implementable SMIP**: can be easily translated into standard optimization software packages, not requiring customized algorithmic development or tuning

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What we need

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Sorry, Superman, it's the #KnightsOfMIP business

Stochastic Mixed-Integer Program (SMIP) for SOPSP

Subject to: **Each procedure is assigned to one appointment**
Each appointment is assigned one procedure

Stochastic Mixed-Integer Program (SMIP) for SOPSP

$$\text{Subject to:} \quad \sum_{i=1}^{P} x_{i,p} = 1 \qquad \forall p \qquad (2)$$

$$\sum_{p=1}^{P} x_{i,p} = 1 \qquad \forall i \qquad (3)$$

$$\textbf{Actual start time}_{\mathbf{i}}^{\mathbf{n}} = \textbf{max}\,\{\textbf{scheduled time}_{\mathbf{i}}, \textbf{completion time}_{\boldsymbol{i-1}}^{\mathbf{n}}\}$$

Stochastic Mixed-Integer Program (SMIP) for SOPSP

Subject to:

$$\sum_{i=1}^{P} x_{i,p} = 1 \qquad \forall p \qquad (2)$$

$$\sum_{p=1}^{P} x_{i,p} = 1 \qquad \forall i \qquad (3)$$

$$s_i^n \geq t_i \qquad \forall (i,n) \qquad (4)$$

$$s_i^n \geq s_{i-1}^n + \sum_{p=1}^{P} d_p^n \cdot x_{i-1,p} \qquad \forall (i \geq 2, n) \qquad (5)$$

$$\text{Idle Time}_i^n = \max\{\text{actual start time}_{i+1}^n, \text{completion time}_i^n\}$$

$$\text{Overtime}^n = \max\{\text{completion time of last appt}_P^n - \text{scheduled closing time}\}$$

Stochastic Mixed-Integer Program (SMIP) for SOPSP

$$
\begin{aligned}
\text{Subject to:} \quad & \sum_{i=1}^{P} x_{i,p} = 1 && \forall p && (2)\\
& \sum_{p=1}^{P} x_{i,p} = 1 && \forall i && (3)\\
& s_i^n \geq t_i && \forall (i,n) && (4)\\
& s_i^n \geq s_{i-1}^n + \sum_{p=1}^{P} d_p^n \cdot x_{i-1,p} && \forall (i \geq 2, n) && (5)\\
& g_i^n = s_{i+1}^n - \left(s_i^n + \sum_{p=1}^{P} d_p^n \cdot x_{i,p} \right) && \forall (i < P, n) && (6)\\
& o^n \geq \left(s_P^n + \sum_{p=1}^{P} d_p^n \cdot x_{P,p} \right) - \mathcal{L} && \forall n && (7)\\
& (g_i^n, s_i^n, o^n) \geq 0 && \forall (i,n) && (8)\\
& x_{i,p} \in \{0,1\},\ t_i \geq 0 && \forall (i,p) && (9)
\end{aligned}
$$

Stochastic Mixed-Integer Program (SMIP) for SOPSP

$$\text{SOPSP}(\Omega) \qquad \text{minimize} \quad \mathbb{E}\Big[\lambda^w \cdot \texttt{Total Waiting} + \lambda^g \cdot \texttt{Total Idle} + \lambda^g \cdot \texttt{Overtime}\Big] \tag{1}$$

$$\text{Subject to:} \quad \sum_{i=1}^{P} x_{i,p} = 1 \qquad \forall p \tag{2}$$

$$\sum_{p=1}^{P} x_{i,p} = 1 \qquad \forall i \tag{3}$$

$$s_i^n \geq t_i \qquad \forall (i,n) \tag{4}$$

$$s_i^n \geq s_{i-1}^n + \sum_{p=1}^{P} d_p^n \cdot x_{i-1,p} \qquad \forall (i \geq 2, n) \tag{5}$$

$$g_i^n = s_{i+1}^n - \left(s_i^n + \sum_{p=1}^{P} d_p^n \cdot x_{i,p} \right) \qquad \forall (i < P, n) \tag{6}$$

$$o^n \geq \left(s_P^n + \sum_{p=1}^{P} d_p^n \cdot x_{P,p} \right) - \mathcal{L} \qquad \forall n \tag{7}$$

$$(g_i^n, s_i^n, o^n) \geq 0 \qquad \forall (i,n) \tag{8}$$

$$x_{i,p} \in \{0,1\}, \; t_i \geq 0 \qquad \forall (i,p) \tag{9}$$

Stochastic Mixed-Integer Program (SMIP) for SOPSP

$$\text{SOPSP}(N) \quad \text{minimize} \quad \frac{1}{N}\sum_{n=1}^{N}\left[\sum_{i=1}^{P}\lambda_i^w\cdot(s_i^n-t_i)+\sum_{i=1}^{P}\lambda_i^g\cdot g_i^n+\lambda^o\cdot o^n\right] \tag{1}$$

$$\text{Subject to:} \quad \sum_{i=1}^{P} x_{i,p}=1 \qquad \forall p \tag{2}$$

$$\sum_{p=1}^{P} x_{i,p}=1 \qquad \forall i \tag{3}$$

$$s_i^n \geq t_i \qquad \forall (i,n) \tag{4}$$

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Theoretical Analysis of the SMIP for SOPSP

(1) **Sizes** of SOPSP Formulations[2]

[2] Shehadeh et al. "*Analysis of Models for the Stochastic Outpatient Procedure Scheduling Problem.*"

Theoretical Analysis of the SMIP for SOPSP

(1) **Sizes** of SOPSP Formulations[2]

Table 1: Sizes of formulations of the SOPSP with P procedures and N scenarios

	Berg et al. (2014)*	Mancilla et al. (2012)	Shehadeh et al. (2018)
# Binary variables		P^2	P^2
# Continuous variables		$P + N(2P^2 + 2)$	$P + N(2P + 1)$
# First-stage constraints		$P^2 + 3P$	$P^2 + 3P$
# Second-stage constraints		$N(4P^2 + P + 2)$	$5NP$

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Theoretical Analysis of the SMIP for SOPSP

(1) **Sizes** of SOPSP Formulations[2]

$$\text{(Mancilla) minimize } \frac{1}{N}\sum_{n=1}^{N}\left[\sum_{i=1}^{P}\sum_{p=1}^{P}\lambda_p^w \cdot w_{i,p}^n + \sum_{i=1}^{P}\sum_{p=1}^{P}\lambda_p^g \cdot g_{i,p}^n + \lambda^o \cdot o^n\right] \tag{2a}$$

$$\text{subject to } \sum_{i=1}^{P} x_{i,p} = 1 \qquad \forall p \tag{2b}$$

$$\sum_{p=1}^{P} x_{i,p} = 1 \qquad \forall i \tag{2c}$$

$$t_i - t_{i+1} - \sum_{p=1}^{P} w_{i+1,p}^n + \sum_{p=1}^{P} g_{i,p}^n + \sum_{p=1}^{P} w_{i,p}^n = -\sum_{p=1}^{P} d_p^n \cdot x_{i,p} \qquad \forall (i < P, n) \tag{2d}$$

$$t_P + \sum_{p=1}^{P} w_{P,p}^n - o^n + e^n = -\sum_{p=1}^{P} d_p^n \cdot x_{P,p} + \mathcal{L} \qquad \forall n \tag{2e}$$

$$w_{i,p}^n \le M_1^i \cdot x_{i,p} \qquad \forall (i,p,n) \tag{2f}$$

$$g_{i,p}^n \le M_2 \cdot x_{i,p} \qquad \forall (i,p,n) \tag{2g}$$

$$(w_{i,p}^n, g_{i,p}^n, o^n, e^n) \ge 0 \qquad \forall (i,p,n) \tag{2h}$$

$$t_i \ge 0 \qquad \forall i \tag{2i}$$

$$x_{i,p} \in \{0,1\} \qquad \forall (i,p) \tag{2j}$$

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Mancilla et al. (2012)	**Shehadeh et al. (201**
P^2	P^2
$P + N(2P^2 + 2)$	$P + N(2P + 1)$
$P^2 + 3P$	$P^2 + 3P$
$N(4P^2 + P + 2)$	$5NP$

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	Berg et al. (2014)*	**Mancilla et al. (2012)**	**Shehadeh et al. (2018)**
# Binary variables	$2P^2 + 4P + 2$	P^2	P^2
# Continuous variables	$P + 1 + N(2P^2 + 4P + 4)$	$P + N(2P^2 + 2)$	$P + N(2P + 1)$
# First-stage constraints	$P^3 + 5P^2 + 11P + 10$	$P^2 + 3P$	$P^2 + 3P$
# Second-stage constraints	$N(4P^2 + 9P + 5)$	$N(4P^2 + P + 2)$	$5NP$

[*] **An enhancement of Denton et al. (2007)**

[2]

Theoretical Analysis of the SMIP for SOPSP

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	Berg et al. (2014)*	Mancilla et al. (2012)	Shehadeh et al. (2018)
(Berg) minimize $\frac{1}{N}\sum_{n=1}^{N}\left[\sum_{p=1}^{P+1}\sum_{p'=1}^{P}\lambda^{w}_{p,p'}\cdot A^{n}_{p'}w^{n}_{p,p'}+\sum_{p=1}^{P+1}\sum_{p'=1}^{P+1}\lambda^{g}_{p,p'}\cdot g^{n}_{p,p'}+\lambda^{o}\cdot o^{n}\right]$	(3a)		P^2
subject to $\sum_{p'=1}^{P+1} r_{p,p'} \leq 1$	$\forall p$ (3b)		$P+N(2P+1)$
$\sum_{p=1}^{P+1}\sum_{p'=1}^{P+1} r_{p,p'} = P$	(3c)		P^2+3P
$x_{i,p}+x_{i+1,p'}-1 \leq r_{p,p'}$	$\forall(p,p',i\leq P)$ (3d)		$5NP$
$\sum_{i=1}^{P+1} x_{i,p} = 1$	$\forall p$ (3e)		
$\sum_{p=1}^{P+1} x_{i,p} = 1$	$\forall i$ (3f)		
$\sum_{p=1}^{P+1} r_{p,P+1} = 1$	(3g)		
$\sum_{p=1}^{P+1} r_{P+1,p} = 0$	(3h)		
$x_{P+1,P+1} = 1$	(3i)		
$w^{n}_{p,p'} \leq M_1 r_{p,p'}$	$\forall(p,p',n)$ (3j)		
$g^{n}_{p,p'} \leq M_2 r_{p,p'}$	$\forall(p,p',n)$ (3k)		
$-\sum_{p'=1}^{P+1} w^{n}_{p',p}+\sum_{p'=1}^{P+1} w^{n}_{p,p'}-\sum_{p'=1}^{P+1} g^{n}_{p,p'} = A^{n}_{p}d^{n}_{p}-y_p$	$\forall(p:p\leq P,n)$ (3l)		
$\sum_{p=1}^{P+1}\sum_{p'=1}^{P} g^{n}_{p,p'}-o^{n}+e^{n} = \mathcal{L}-\sum_{p=1}^{P+1} A^{n}_{p}d^{n}_{p}$	$\forall n$ (3m)		
$r_{p,p'}, x_{i,p} \in \{0,1\}\ , y_i \geq 0$	$\forall(p,p',i)$ (3n)		
$(w^{n}_{p,p'}, g^{n}_{p,p'}, o^{n}, e^{n} \geq 0)$	$\forall(p,p',n)$ (3o)		

Theoretical Analysis of the SMIP for SOPSP

(1) Our SMIP formulation is **smaller** ✓

(2) The **tightness** of the SOPSP Formulations

Theorem 1. *Suppose* $\lambda^w > 0$, *and* $\lambda^g > 0$ *and/or* $\lambda^o > 0$.

- *The linear programming relaxation (LPR) of Shehadeh et al. and Mancilla et al. are* ***equivalent.***

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- *The linear programming relaxation (LPR) of Shehadeh et al. and Mancilla et al. are* ***equivalent.***
- *Furthermore, Shehadeh et al. is a* ***tighter*** *formulation than Berg et al.* ✓

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Do We Really Have a Good Model?
Is it Tractable?
Can We Implement it in Practice?

Description of Experiments

- 14 different SOPSP instances

Table 2: Characteristics of SOPSP instances.

Instance	# of Procedures	# of Types	Procedures to be scheduled (by type)
1[a]	4 procedures	3 types	(2A, 1C, 1J)
2[a]	5 procedures	4 types	(2A, 1G, 1H,1J)
3[a]	5 procedures	4 types	(1A, 1D,2G, 1J)
4[a]	6 procedures	5 types	(1A, 1B, 1F, 2G, 1H)
5[a]	7 procedures	5 types	(1C, 1D, 1F, 1H, 3J)
6[a]	7 procedures	6 types	(1A, 1B, 1D, 1E, 2G, 1J)
7[a]	10 procedures	6 types	(3A, 1C, 1D, 1G, 1I,3J)
8[a]	10 procedures	6 types	(2A, 1B, 1D, 2G, 2I, 2J)
9[b]	**10 procedures**	**2 types**	**(6CP, 4CPU)**
10[a]	11 procedures	8 types	(2A, 1C, 2E, 1F, 1G, 1H, 2I, 1J)
11[a]	11 procedures	6 types	(2A, 2F, 1G, 2H, 2I, 2J)
12[c]	**12 procedures**	**2 types**	**(9R, 3N)**
13[c]	**16 procedures**	**2 types**	**(12R, 4N)**
14[c]	**20 procedures**	**2 types**	**(15R, 5N)**

[a] From the AIMMS-MOPTA 5th Optimization Modeling Competition
[b] **From Berg et al. (2014)**
[c] **From Deceuninck et al. (2018)**

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Table 3: Distribution information for procedure duration, by type.

Procedure type	Mean	Variance	Distribution
A	9.83	12.08	Lognormal
B	81.46	804.56	Normal
C	59.75	652.69	Lognormal
D	34.53	303.94	Lognormal
E	120.84	2.38e+3	Lognormal
F	47.76	232.06	Lognormal
G	43.94	469.86	Gamma
H	39.90	129.28	Lognormal
I	95.13	2.430e+3	Lognormal
J	19.51	99.36	Lognormal
CPU	12.05	188.57	Weibull
CP	30.96	58.75	Weibull
R	20.00	256.00	Lognormal
N	30.00	576.00	Lognormal

Description of Experiments

- 14 different SOPSP instances
- Three different sets of weights for the multi-criteria objective function
  (1) $\lambda^w = \lambda^g = \lambda^o$
  (2) $\lambda^w = 1, \lambda^g = 0, \lambda^o = 10$ (Berg et al., 2014)
  (3) $\lambda^w = 1, \lambda^g = 5, \lambda^o = 7.5$ (Deceuninck et al., 2018)

[3] Ostrowski et al. 2011, Berg et al. 2014

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  (3) $\lambda^w = 1, \lambda^g = 5, \lambda^o = 7.5$ (Deceuninck et al., 2018)
- For each of the 14 SOPSP instances and 3 sets of weights, we generated 10 sample average approximations, each with $N = 1,000$ scenarios

[3] Ostrowski et al. 2011, Berg et al. 2014

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- 420 sample average approximations (SAA), each with N =1,000 scenarios

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- Symmetry-breaking constraints[3]

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- Symmetry-breaking constraints[3]

$$x_{i,p} - \sum_{j>i}^{P} x_{j,p+1} \leq 0, \quad \forall i = 1, ..., P, \ \forall p : p, p+1 \in P_q, \ \forall q = 1, \ \ldots, Q,$$

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e.g., procedure mix= (2 Type A, 1 Type B)

$$\boldsymbol{A_1 \rightarrow B_1 \rightarrow A_2}$$

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e.g., procedure mix= (2 Type A, 1 Type B)

$$\boldsymbol{A_1 \rightarrow B_1 \rightarrow A_2 \Longleftrightarrow A_2 \rightarrow B_1 \rightarrow A_1}$$

Description of Experiments

- 14 different SOPSP instances
- Three different sets of weights for the multi-criteria objective function
  (1) $\lambda^w = \lambda^g = \lambda^o$
  (2) $\lambda^w = 1, \lambda^g = 0, \lambda^o = 10$ (Berg et al., 2014)
  (3) $\lambda^w = 1, \lambda^g = 5, \lambda^o = 7.5$ (Deceuninck et al., 2018)
- 420 sample average approximations (SAA), each with N =1,000 scenarios
- Symmetry-breaking constraints[3]
- Time limit: 2 hours
- Using a **standard optimization modeling tool** (AMPL), and a **commercial MILP solver** (CPLEX), with **default settings**

[3] Ostrowski et al. 2011, Berg et al. 2014

Comparison with Berg et al. (2014)

- Using **our model**, we were **able to solve all of the 420 SAAs** in **less than 25 minutes**

Table 4: Solution times (in seconds) using Shehadeh et al. model

SOPS	$\lambda^w = \lambda^g = \lambda^o$			$\lambda^w = 1, \lambda^g = 0, \lambda^o = 10$			$\lambda^w = 1, \lambda^g = 5, \lambda^o = 7.5$		
Instance	Min	Avg±stdv	Max	Min	Avg±stdv	Max	Min	Avg±stdv	Max
1	2	3 ±0.34	3	3	3±1	7	3	3±0.2	7
2	10	13±2	17	8	11±3	17	4	5±0.9	7
3	8	9±0.9	11	5	5±0.4	6	5	6±0.6	7
4	33	41±6	55	21	23±2	26	23	25±2	28
5	53	65±9	77	44	51±6	60	41	49±5	57
6	99	111±7	122	52	58±8	80	57	70±8	79
7	215	276±46	334	153	176±36	276	168	197±28	248
8	237	284±24	310	140	170±29	242	205	226±18	269
9	57	70±8	85	44	55±6	61	46	53±4	58
10	588	769±105	937	178	226±37	293	233	270±33	342
11	254	357±61	460	660	770±37	987	251	326±43	375
12	83	107±12	123	70	78±5	86	100	116±11	130
13	363	466±59	551	242	297±35	349	455	512±55	602
14	862	1218±164	1464	930	1189±193	1500	461	549 ± 76	703

Comparison with Berg et al. (2014)

- Using **Berg et al. model**, we were **able to solve 160** SAAs to optimality within the 2 hrs time limit.

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Table 5: Ratios of solution times of Berg et al. and Shehadeh et al. on SAAs solved by both.

$\lambda^w = \lambda^g = \lambda^o$ (a)			$\lambda^w = 1, \lambda^g = 0, \lambda^o = 10$ (b)			$\lambda^w = 1, \lambda^g = 5, \lambda^o = 7.5$ (b)		
Min	Avg±stdv	Max	Min	Avg±stdv	Max	Min	Avg±stdv	Max
6	31±29	116	4	33±27	107	8	51±35	138

[a] SOPS Instances 1–6, 10 SAA instances each.
[b] SOPS Instances 1–5, 10 SAA instances each.

Comparison with Berg et al. (2014)

- Using **Berg et al. model**, we were **able to solve 160** SAAs to optimality within the 2 hrs time limit.
  - Berg takes from **6 to 138** times longer than our model to solve such instances
- The **remaining 260 SAAs** that were not solved terminated with **poor quality** solutions

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  - **180 SAAs** terminated with a relative MIP **gap** ($\frac{UB-LB}{UB} \times 100\%$) between **16%** and **70%**

Table 6: Relative MIP gap at termination for SAAs not solved by Berg et al. in two hours.

$\lambda^w = \lambda^g = \lambda^o$(a)			$\lambda^w = 1, \lambda^g = 0, \lambda^o = 10$(b)			$\lambda^w = 1, \lambda^g = 5, \lambda^o = 7.5$(b)		
Min	Avg±stdv	Max	Min	Avg±stdv	Max	Min	Avg±stdv	Max
41%	54±0.08%	70%	19%	34±0.09%	53%	16%	40±0.09%	52%

[a] SOPS Instances 7–12, 10 SAA instances each.
[b] SOPS Instances 6–11, 10 SAA instances each.

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- **So, WHY?**

Table 7: Ratios of optimal objective values of LP relaxations of Shehadeh et al. and Berg et al.

$\lambda^w = \lambda^g = \lambda^o$			$\lambda^w = 1, \lambda^g = 0, \lambda^o = 10$			$\lambda^w = 1, \lambda^g = 5, \lambda^o = 7.5$		
Min	Avg±stdv	Max	Min	Avg±stdv	Max	Min	Avg±stdv	Max
1.95	2.62±0.41	3.48	1.11	1.38±0.26	2.08	1.27	1.64±0.33	2.49

Comparison with Mancilla et al. (2012)

- Using **our model**, we were **able to solve all of the 420 SAAs** in **less than 25 minutes**
- Using **Mancilla et al.** model, we were **able to solve 340** of the 420 SAAs to optimality within the two hour time limit.

Comparison with Mancilla et al. (2012)

- Using **our model**, we were **able to solve all of the 420 SAAs**
- Using **Mancilla et al.** model, we were **able to solve 340** of the 420 SAAs to optimality within the two hour time limit.

Table 8: Comparison of performance of Mancilla et al. and Shehadeh et al. on SAAs solved by both

	$\lambda^w = \lambda^g = \lambda^o$			$\lambda^w = 1, \lambda^g = 0, \lambda^o = 10$			$\lambda^w = 1, \lambda^g = 5, \lambda^o = 7.5$		
Ratio	Min	Avg±stdv	Max	Min	Avg±stdv	Max	Min	Avg±stdv	Max
$\frac{\text{(M) sol. time}}{\text{(S) sol. time}}$	1.2	7±4	21	2	13±9	43	1.1	7±5	27

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- The **remaining 80 SAAs** that were not solved by Mancilla et al. terminated with **relative MIP gaps between 15% and 25%**.

Do We Really Have a Good Model? YES ✓
Is it Tractable? YES ✓
Can We Implement it in Practice? YES ✓

- ⋆ Scheduling of surgeries in an operating room
- ⋆ Scheduling of ships in a port
- ⋆ Scheduling of exams in an examination facility, and more....

Conclusion

- ⋆ Outpatient colonoscopy scheduling is challenging primarily due to uncertainty in procedure time
- ⋆ We developed a new SMIP model for the Stochastic Outpatient Procedure Scheduling Problem
  - a basic (yet still challenging) offline SASS problem

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.... And we SOLVED **SASS**

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Absolutely Not! The Rest is Still Coming

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  - Proposition 1. Appointment order policy provides an upper bound
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- ⋆ All in all, we developed tractable operations research frameworks for stochastic outpatient appointment scheduling

And for Colonoscopy Scheduling at Michigan Medicine

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Well,

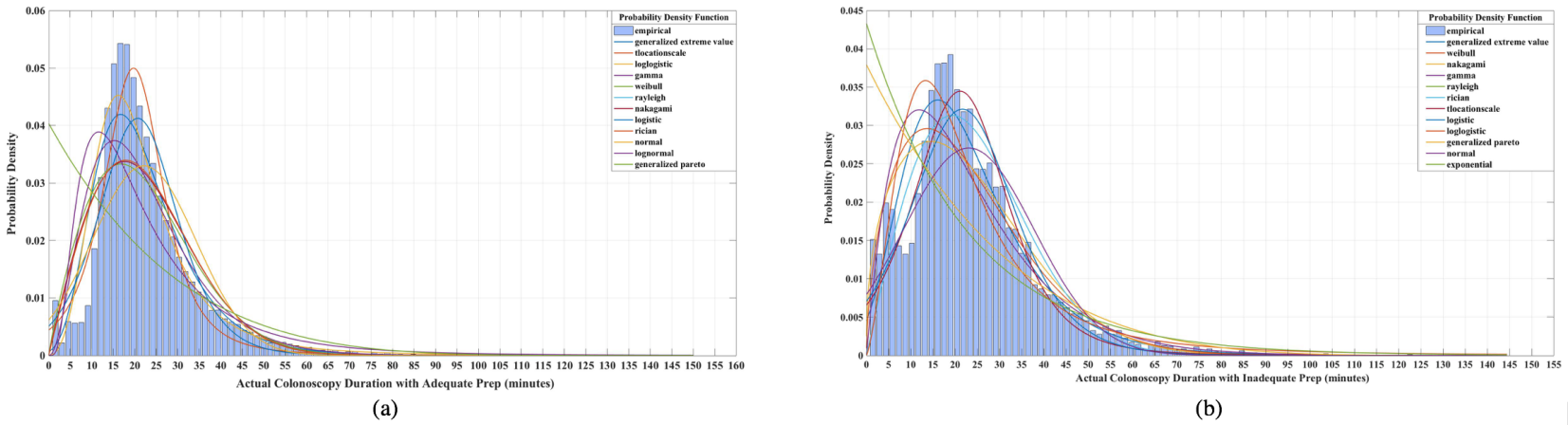

Figure 1: Variability of colonoscopy duration with (a) adequately prep, and (b) inadequate prep(2013-2017)

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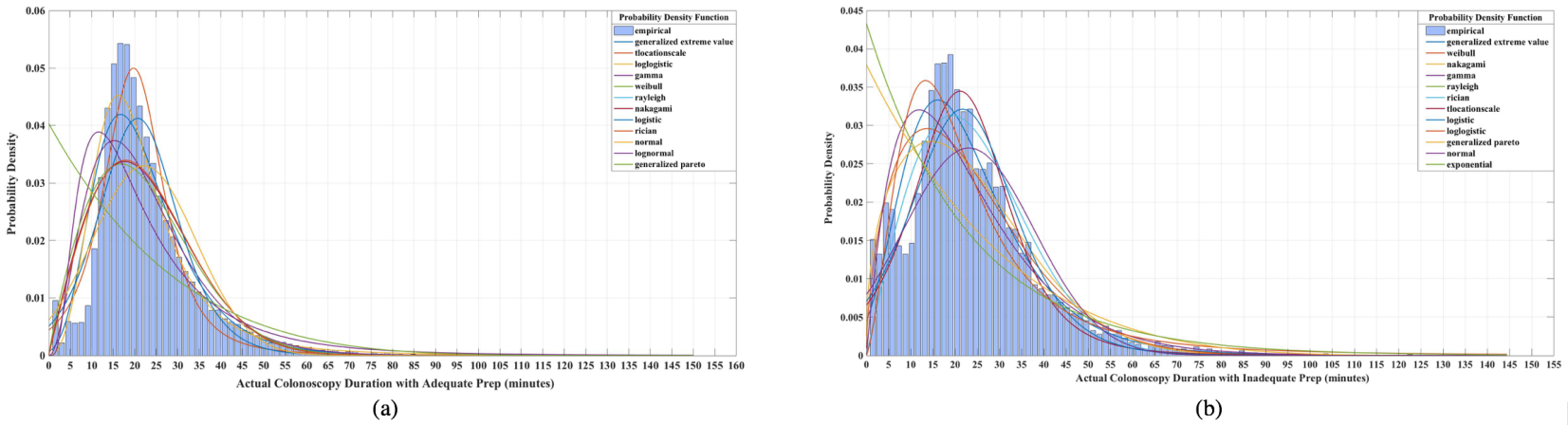

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Always Remember....

"*The theory of sequencing and scheduling, more than any other area in operations research, is characterized by a virtually unlimited number of problem types.*"

–Lawler et al. (1993). Sequencing and scheduling: Algorithms and complexity

Thank You!

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Professor. Amy Cohn
amycohn@umich.edu

A Data-Driven DRO Approach for Colonoscopy Scheduling

- We interpret the randomness in bowel prep-adequacy by a 0-1 Bernoulli random variable, q_p

$$d_p = q_p \cdot d^A + (1 - q_p) \cdot d^I$$

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$$\mathcal{S}^A := \{d^A \geq 0 : d_i^{AL} \leq d_i^A \leq d_i^{AU},\ \forall i \in [P],\ d_{P+1}^A = 0\},\quad \mathcal{S}^I := \{d^I \geq 0 : d_i^{IL} \leq d_i^I \leq d_i^{IU},\ \forall i \in [P]\}$$
$$\mathcal{S}^u := \{u : u_i^L \leq u \leq u_i^U,\ i \in [P],\ d_{P+1}^I = 0, u_{P+1} = 0\},\quad \mathcal{S}^q : \{q : q \in \{0,1\}^P\}$$

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- With expectation as a risk measure

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- OR Journey: NMIP $\Rightarrow$ IP $\Rightarrow$ LP