Stochastic Optimization to Reduce Wait Times in an Outpatient Infusion Center

Jeremy Castaing

Informs 2015 - Philadelphia

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Our Collaborators

- **Research Team (CHEPS):**
  - Pr. Amy Cohn
  - Pamela Martinez
  - Donald Richardson
  - Matthew Rouhana
  - Stephanie See
  - Gabriel Zayas-Caban

- **Contacts at the UM Cancer Center:**
  - Alon Weizer, Medical Director
  - Louise Salamin, Nurse Manager
  - Carolina Typaldos, Project Manager

Motivations

- **Current state:**

  Average waiting time from arrival to infusion area to beginning of treatment is over 40 minutes

- **Challenge:**

  Scheduling appointment times under uncertainty of treatment duration

- **Goal:**

  Generate appointment schedules that reduce patient waiting times and total length of day of operations

Outline of the presentation

- Description of the problem
- Stochastic Optimization Model
- Decomposition Algorithm
- Results
- Future Research

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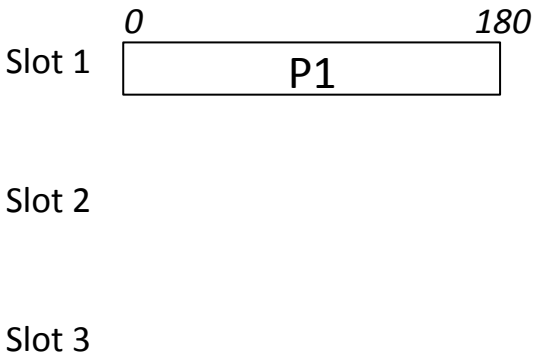
The Scheduling Process

- **Phase 1: Online Scheduling (Day-15 to Day-2)**
  - Patient/Physician calls to schedule an appointment
  - Scheduler assigns patient to a day and a slot
  - Scheduler gives approximate appointment time



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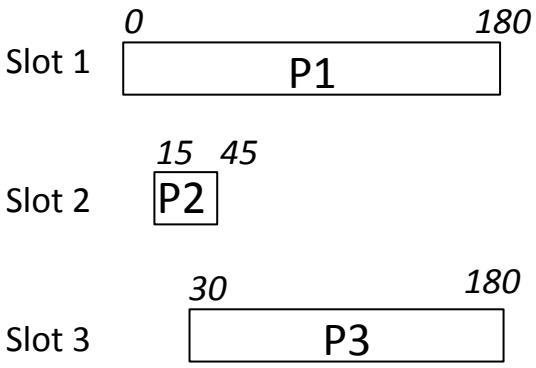

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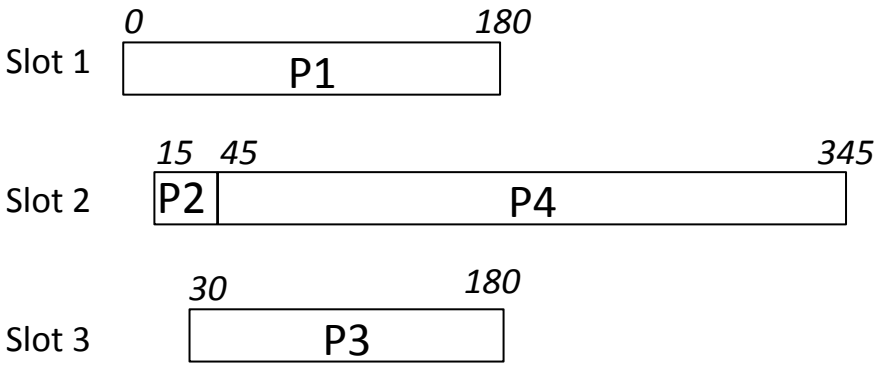

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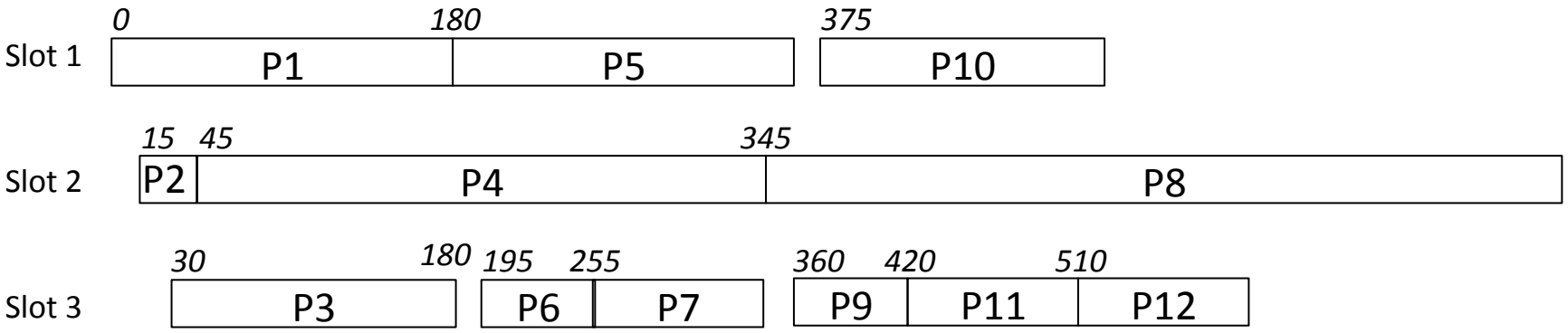

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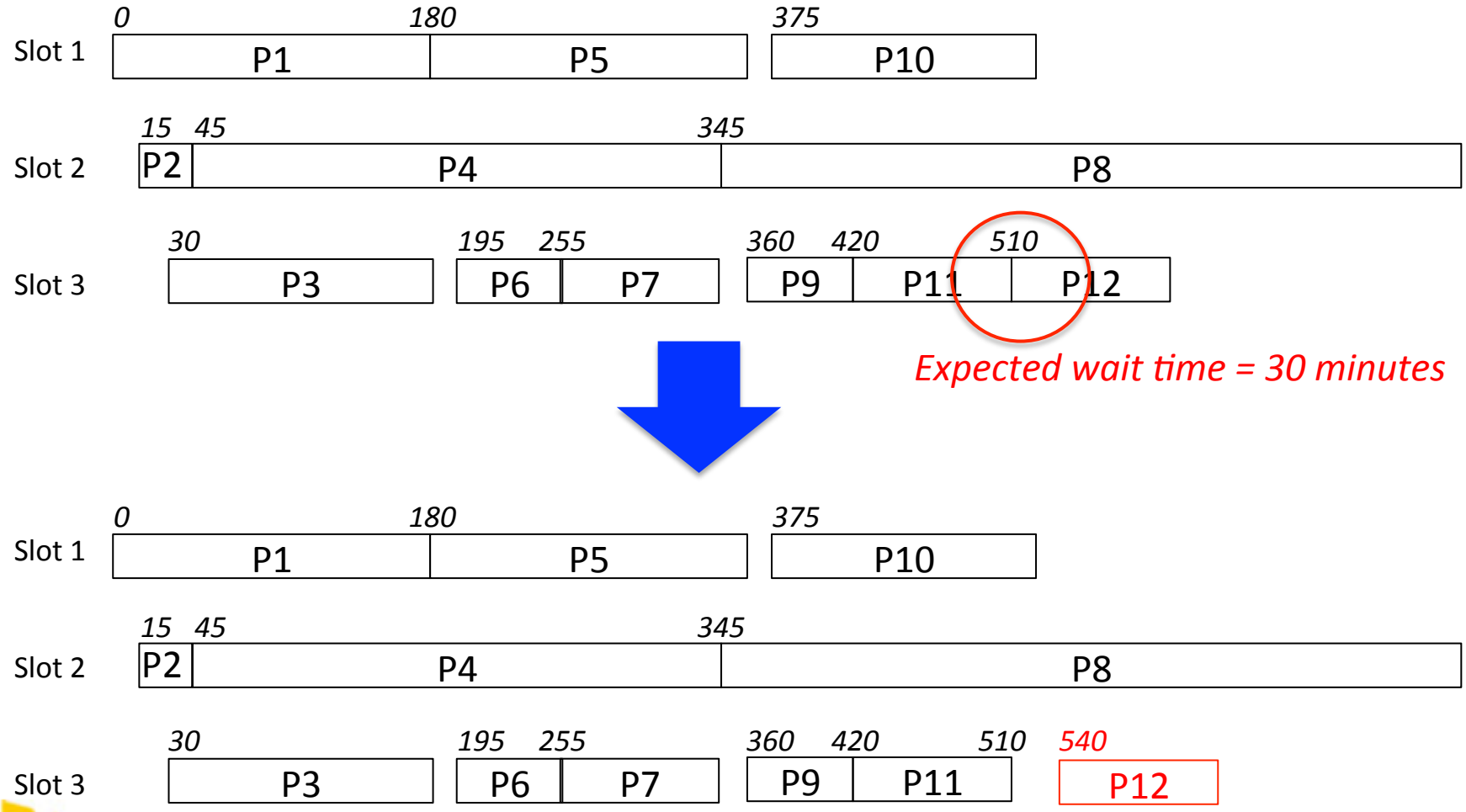
  - Scheduler gives approximate appointment time

- **Phase 2: Fine-Tuning Optimization (Day-2)**
  - Once the list for a day is full, we set final appointment times
  - We preserve patients sequence so that final times are close to original estimates
  - We optimize appointment times to minimize patient waiting and staff overtime

The Scheduling Process

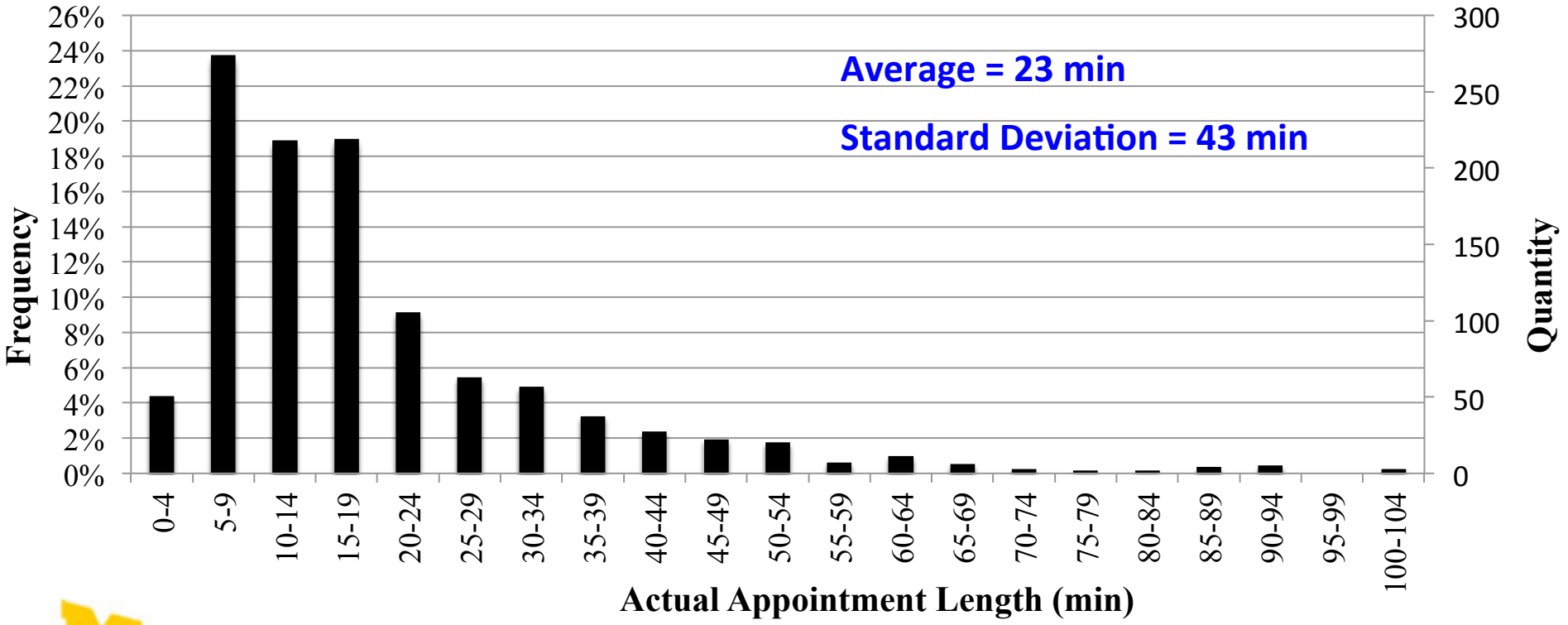


Analysis of Historical Data

- Data set collected via *MiChart*
- 10,000 patient visits from August to November 2014
- Comparison of **scheduled** appointment lengths VS **actual** appointment lengths

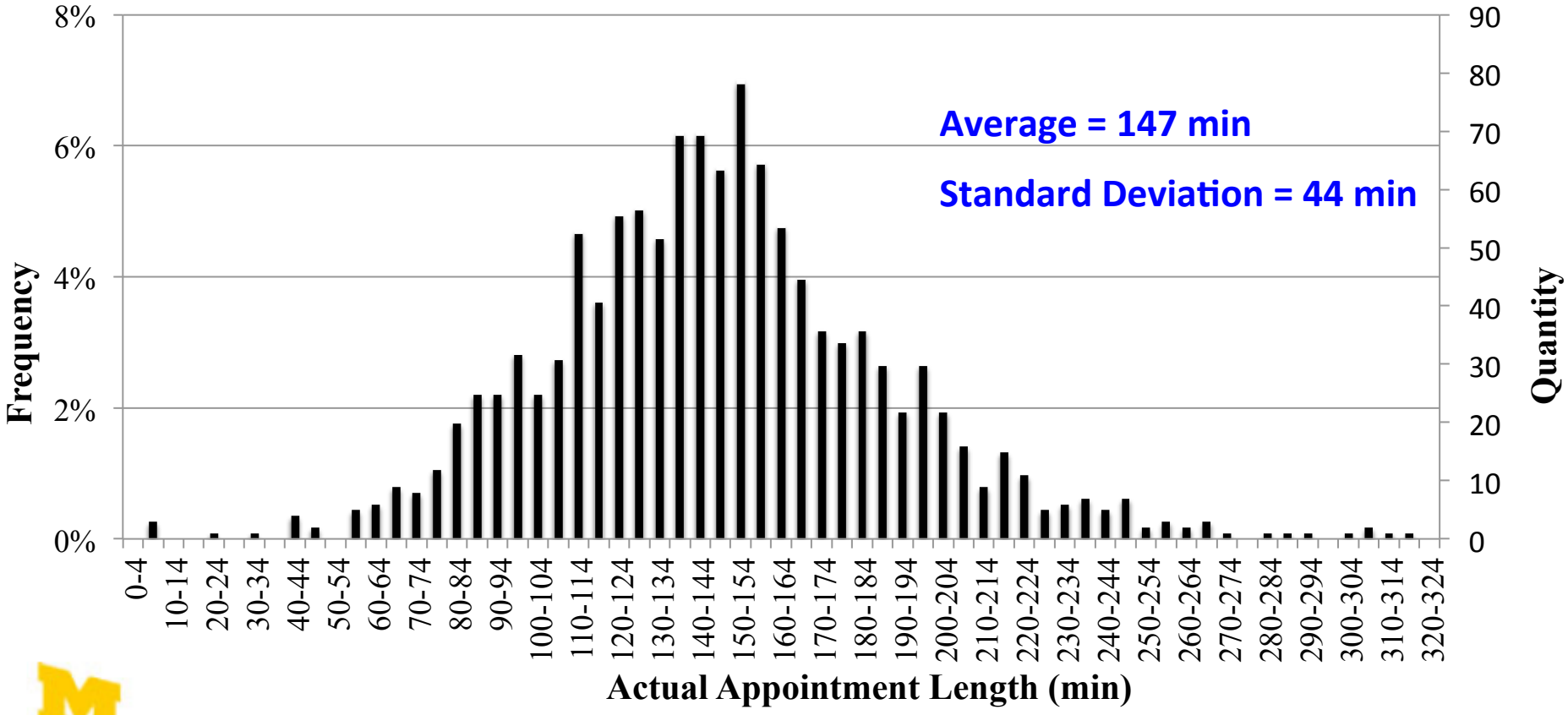
Analysis of Historical Data

- 30 minute scheduled appointment lengths



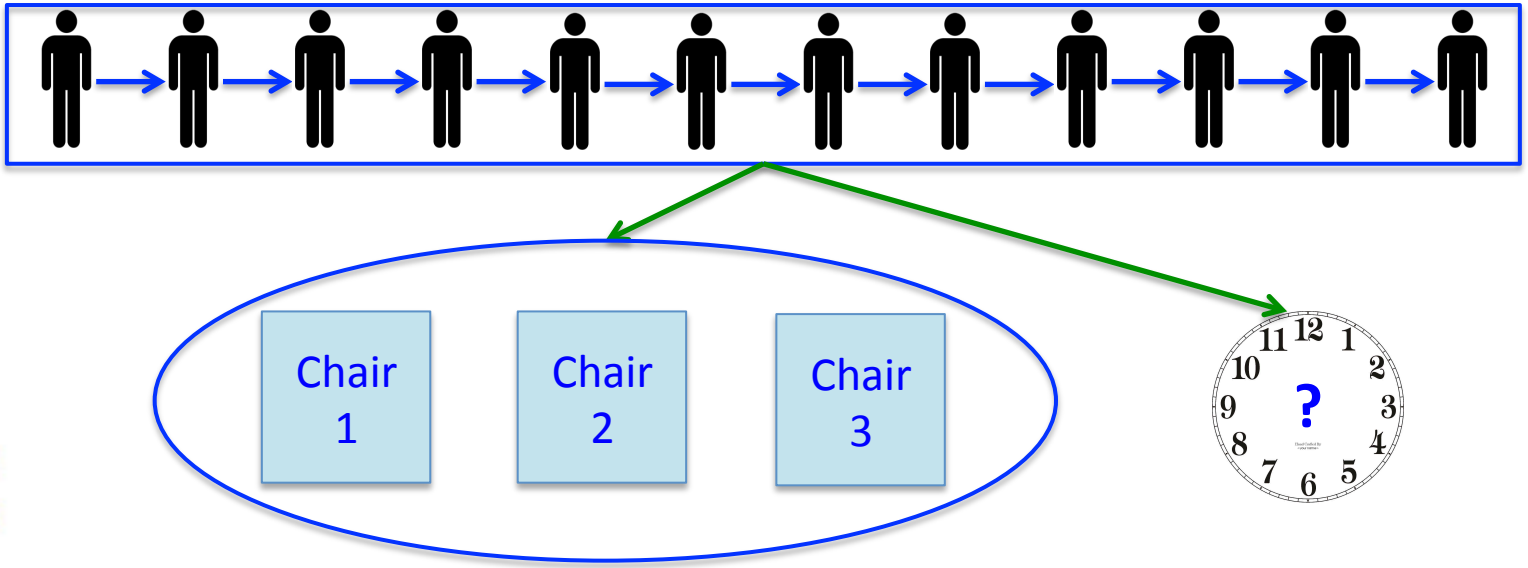
Analysis of Historical Data

- 150 minute scheduled appointment lengths

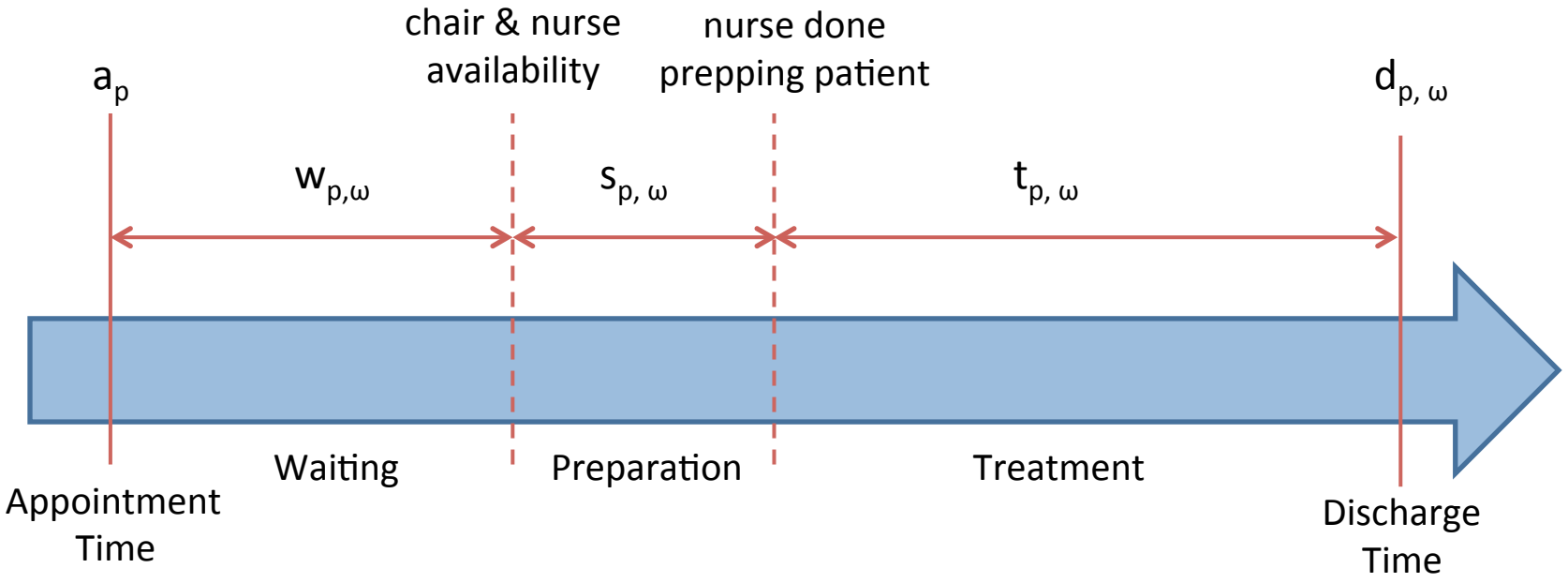


Assumptions

- 12 patients have to be scheduled (according to a given sequence)
  - Appointment time
  - Chair assignment
- 3 chairs are available (infusion pod)
- 1 nurse is responsible for the patients assigned to those 3 chairs
- We assume that patients arrive on time



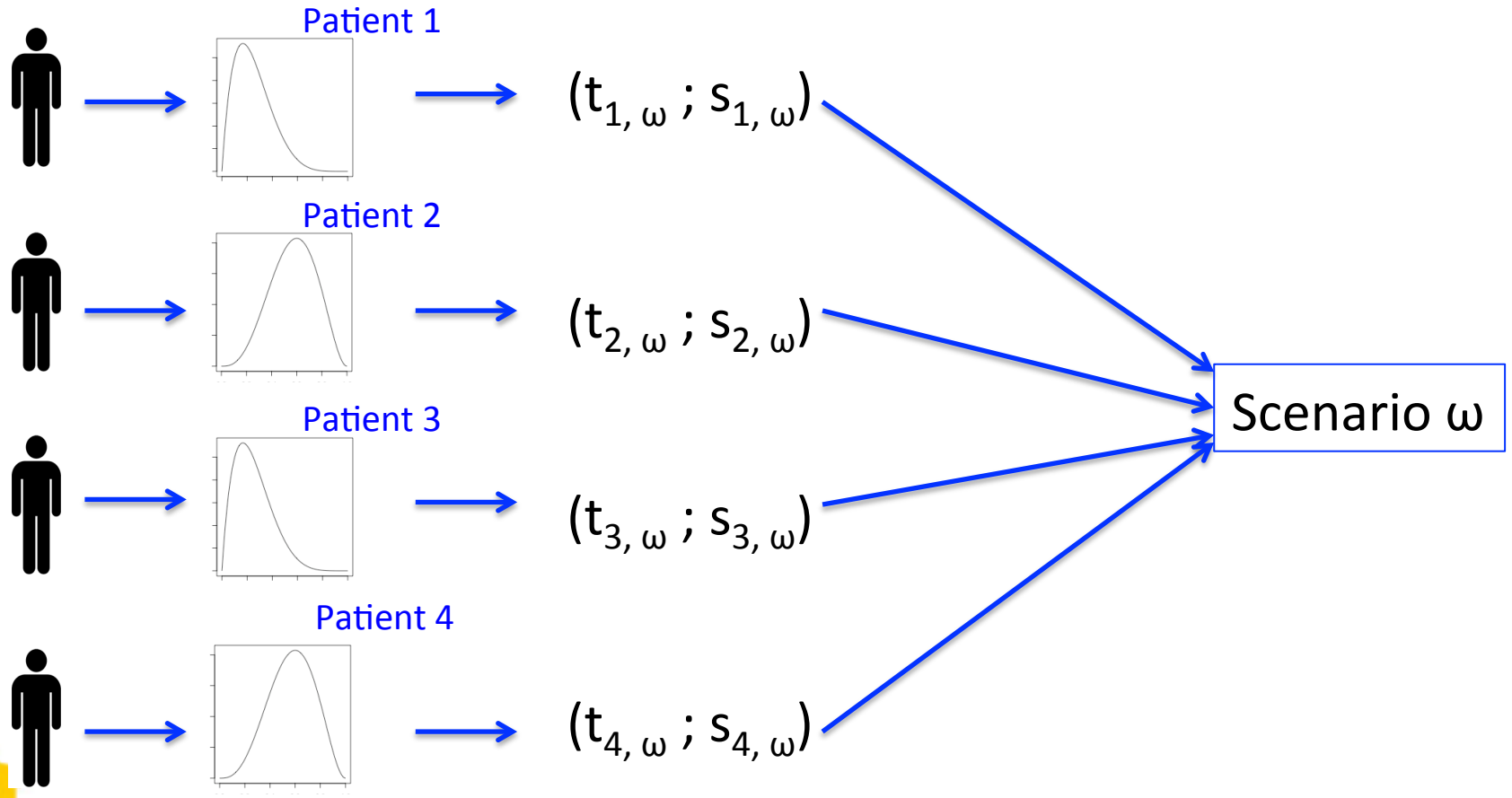
Patient timeline



- Treatment $\boldsymbol{t_{p,\omega}}$ and preparation times $\boldsymbol{s_{p,\omega}}$ are random parameters
- We use a Sample Average Approximation (SAA) approach
- Each scenario contains a realization of $\boldsymbol{t}$ and $\boldsymbol{s}$ for each patient

Definition of Scenarios

- How to construct a scenario ω:



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Overview of the model

First Stage Decision:

Appointment Times: a_p	Continuous	≥0

Second Stage Decision:

Chair Assignment: x^{ω}_{pc}	**Binary**	1 iff patient p is assigned to chair c in scenario ω

Other Variables:

Waiting Time: w^{ω}_{p}	Continuous	≥0
End of Day: E^{ω}	Continuous	≥0
Discharge Time: d^{ω}_{p}	Continuous	≥0

$$\min_{a,x^\omega,d^\omega,w^\omega,E^\omega} \quad \lambda \sum_{p\in P}\sum_{\omega\in\Omega} w_p^\omega + (1-\lambda)\sum_{\omega\in\Omega} E^\omega \tag{0}$$

Objective: Trade-off between the total expected waiting time and the expected end of the day.

Variables :

a_p : appointment time of patient p
d_p^ω : discharge time of patient p in scenario ω
w_p^ω : waiting time of patient p in scenario ω
E^ω : end of the day in scenario ω

Parameters :

s_p^ω : treatment time of patient p in scenario ω
t_p^ω : treatment time of patient p in scenario ω
λ : weight in objective
M : big constant

$$\min_{a,x^\omega,d^\omega,w^\omega,E^\omega} \quad \lambda \sum_{p\in P}\sum_{\omega\in\Omega} w_p^\omega + (1-\lambda)\sum_{\omega\in\Omega} E^\omega \tag{0}$$

$$\text{subject to} \quad \sum_{c\in C} x_{pc}^\omega = 1 \qquad \forall p \in P,\ \forall \omega \in \Omega \tag{1}$$

Each patient is assigned to exactly one chair in each scenario.

Variables :

a_p : appointment time of patient p
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$$a_p + w_p^\omega + s_p^\omega + t_p^\omega = d_p^\omega \qquad \forall p \in P,\ \forall \omega \in \Omega \tag{2}$$

Definition of the discharge time of each patient in each scenario.

Variables :

a_p : appointment time of patient p
d_p^ω : discharge time of patient p in scenario ω
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E^ω : end of the day in scenario ω

Parameters :

s_p^ω : treatment time of patient p in scenario ω
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$$a_p + w_p^\omega + s_p^\omega + t_p^\omega = d_p^\omega \qquad \forall p \in P,\ \forall \omega \in \Omega \tag{2}$$

$$a_{p_1} + w_{p_1}^\omega + M(2 - x_{p_1 c}^\omega - x_{p_2 c}^\omega) \geq d_{p_2}^\omega \qquad \forall c \in C,\ \forall p_1 > p_2 \in P,\ \forall \omega \in \Omega \tag{3}$$

A patient can sit on his chair only if all the previous patients assigned to his chair have been discharged.

Variables :

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d_p^ω : discharge time of patient p in scenario ω
w_p^ω : waiting time of patient p in scenario ω
E^ω : end of the day in scenario ω

Parameters :

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A patient can sit on his chair if the nurse has finished to prepare all previous patients on his pod.

Variables :

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$$E^\omega \geq d_p^\omega \qquad \forall p\in P,\ \forall \omega\in\Omega \tag{5}$$

Definition of the end of the day in each scenario.

Variables :

a_p : appointment time of patient p
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$$E^\omega \geq d_p^\omega \qquad \forall p\in P,\ \forall \omega\in\Omega \tag{5}$$

$$x_{pc}^\omega \in \{0,1\} \qquad \forall c\in C,\ \forall p\in P,\ \forall \omega\in\Omega \tag{6}$$

$$a_p \geq 0 \qquad \forall p\in P \tag{7}$$

$$w_p^\omega, d_p^\omega \geq 0 \qquad \forall p\in P,\ \forall \omega\in\Omega \tag{8}$$

Binary and non-negativity constraints.

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Intractability of this model

- Large scale MIP (large number of scenarios)
- Weak relaxation bound

Number of Scenarios	Solve Time
2	1s
4	4s
6	30s
8	150s
10	500s

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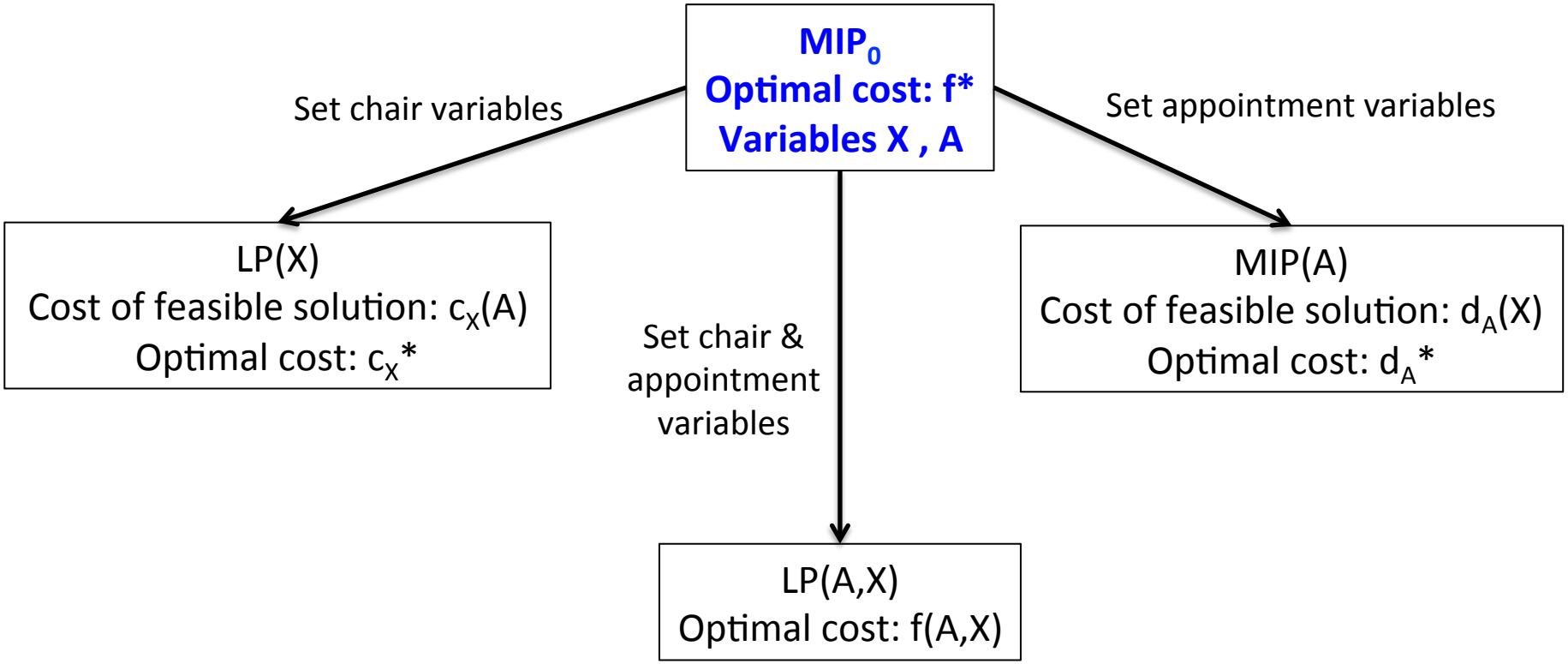
Decomposition

Appointment Times: a_p (continuous)

Chair Assignment: x^{ω}_{cp} (binary)

Decomposition



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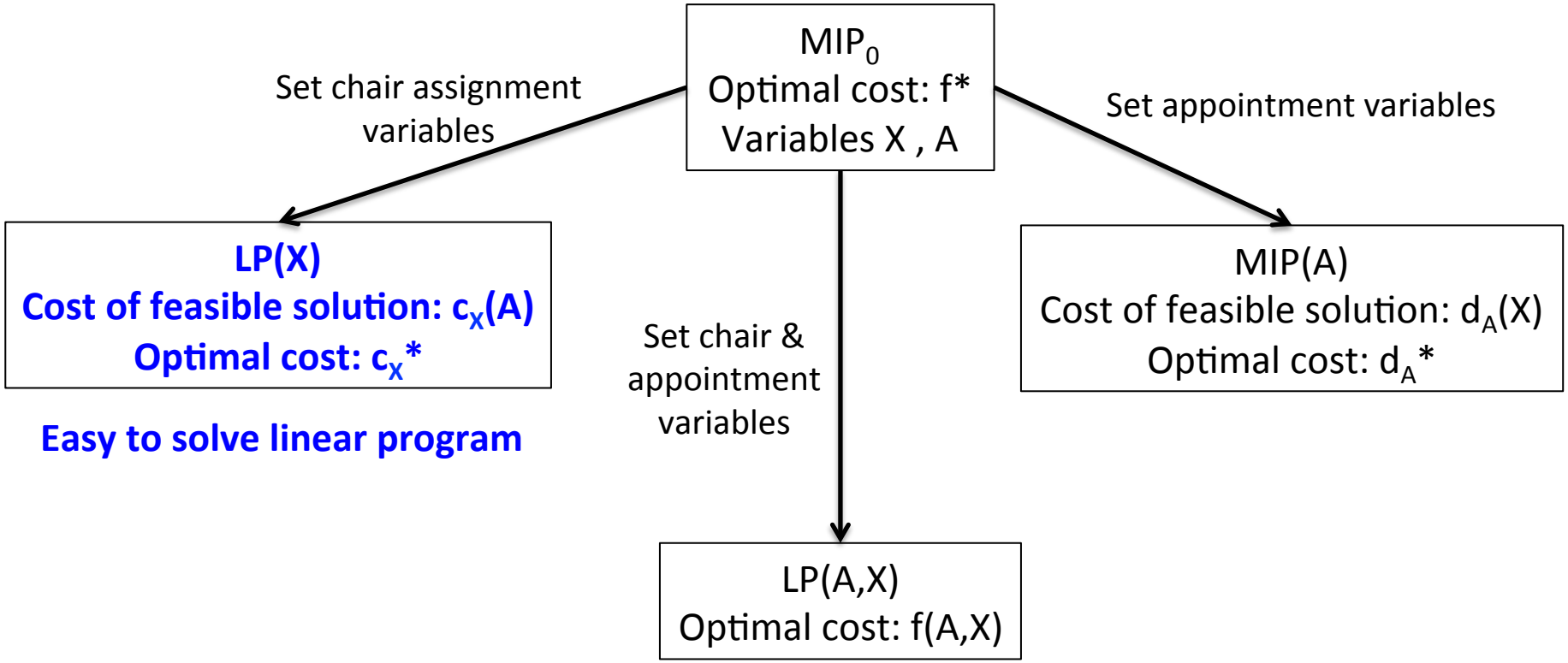
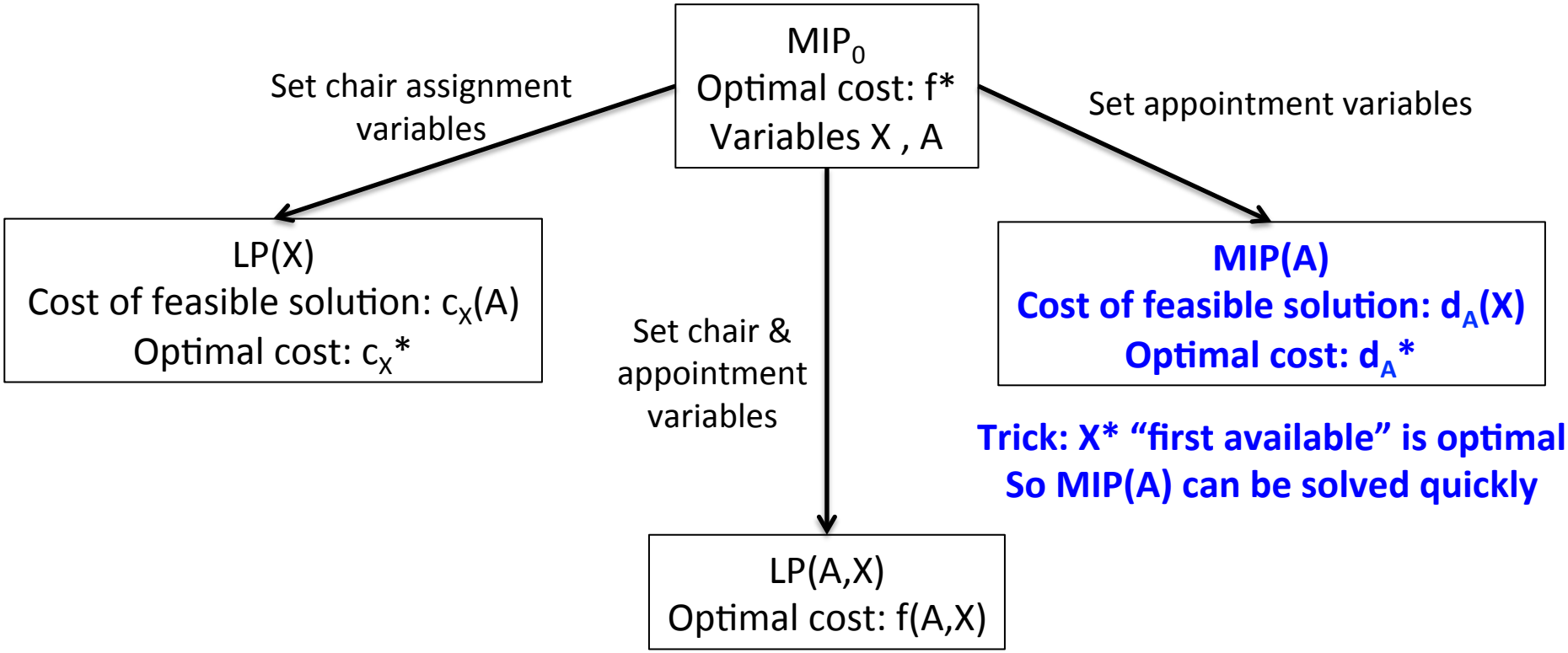


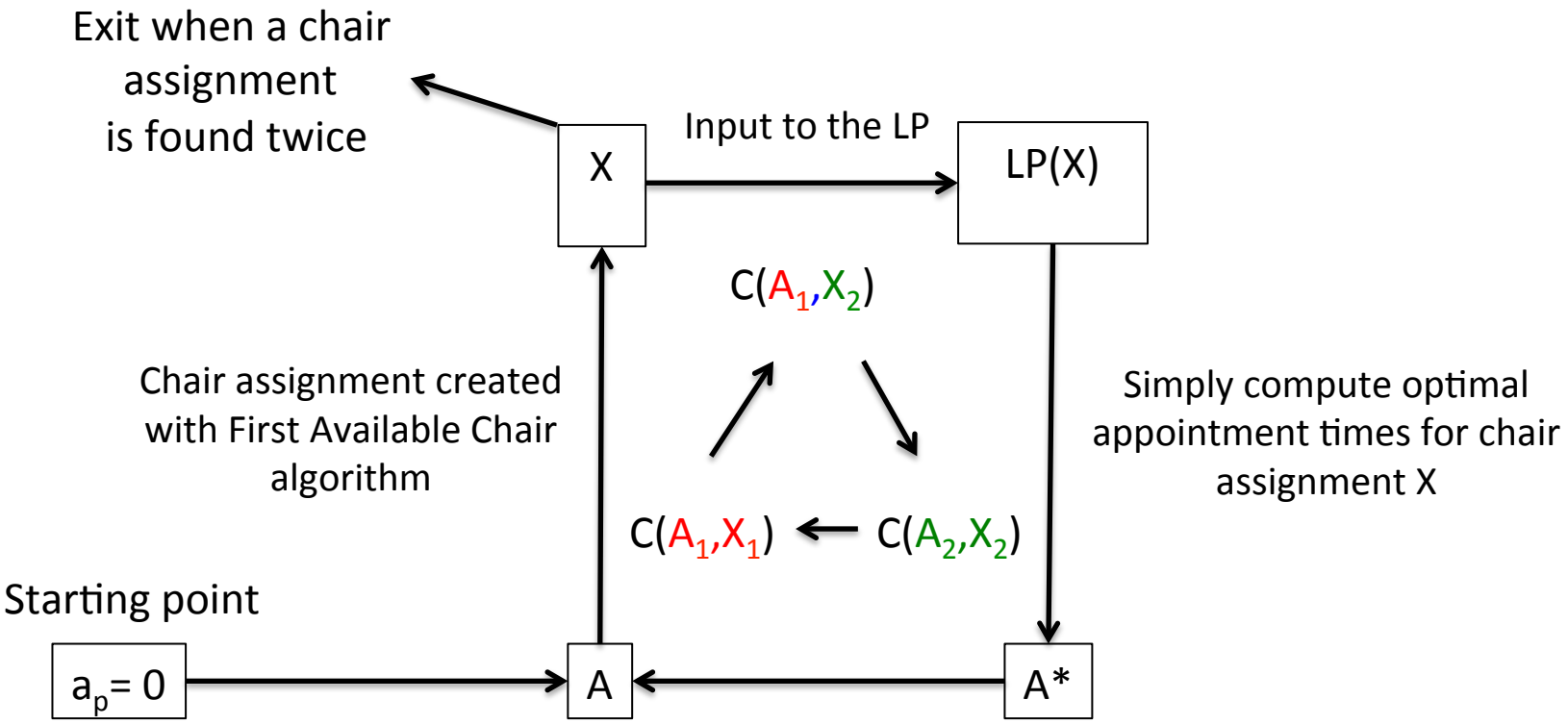
Easy to solve linear program

Decomposition



Trick: X* "first available" is optimal
So MIP(A) can be solved quickly

The Fix-Unfix Algorithm



Improvement: $c(A_1,X_1) \geq c(A_1,X_2) \geq c(A_2,X_2)$

Termination: We iterate until we find the same chair assignment twice

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Questions to answer

1. Can the algorithm solve large instances of the problem?
2. How far from optimal are the solutions?
3. Are optimized schedules better than initial schedules?

1) Tractability

- Solve instance with 100 scenarios
- Runtime < 1 sec
- Termination after 3 iterations
- Solve instances with 1000 scenarios in less than 10 sec

Iteration Count	Optimized Variables	Objective Value	Total Wait Time (min)	Length of Operations (min)
0	Chairs	1203	2453	667
1	Appointments	535	116	715
1	Chairs	525.9	95	710
2	Appointments	525.2	98	708
2	Chairs	524.4	97	707
3	Appointments	524.3	95	708
3	Chairs	524.3	95	708

2) Solution Quality

- **Small instances** - Comparison to MIP (100 instances with 5 scenarios):
  - Heuristic optimal in 32 cases
  - Average optimality gap in the other instances: 0.41%

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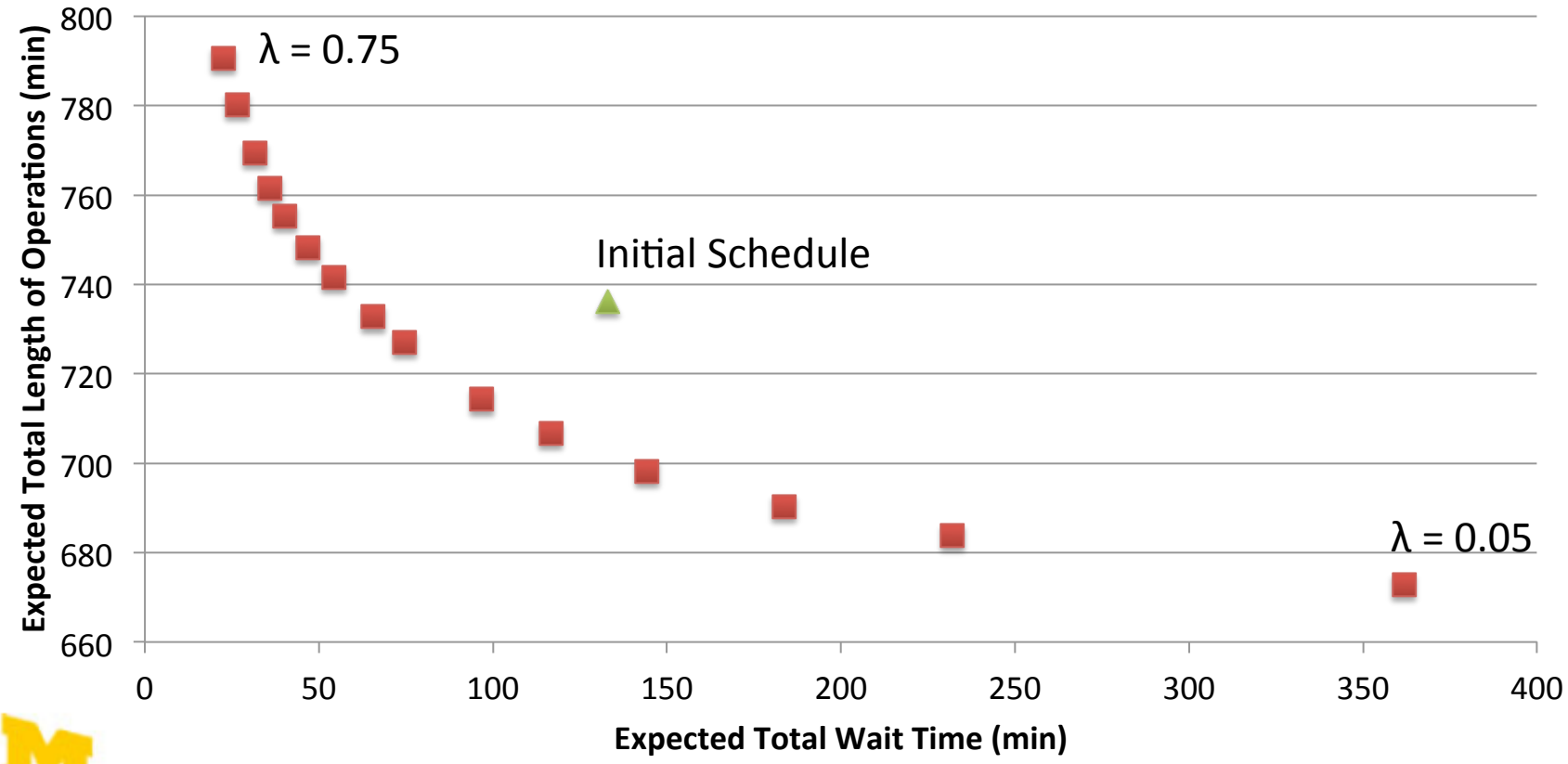
- **Small instances** - Comparison to MIP (100 instances with 5 scenarios):
  - Heuristic optimal in 32 cases
  - Average optimality gap in the other instances: 0.41%
- **Large instances** - Computation of lower bounds via scenarios decomposition (10 instances with 100 scenarios)
  - Gap: Average = 2% / Max = 3.5%

3) Application to real data

- Schedules obtained for different values of λ



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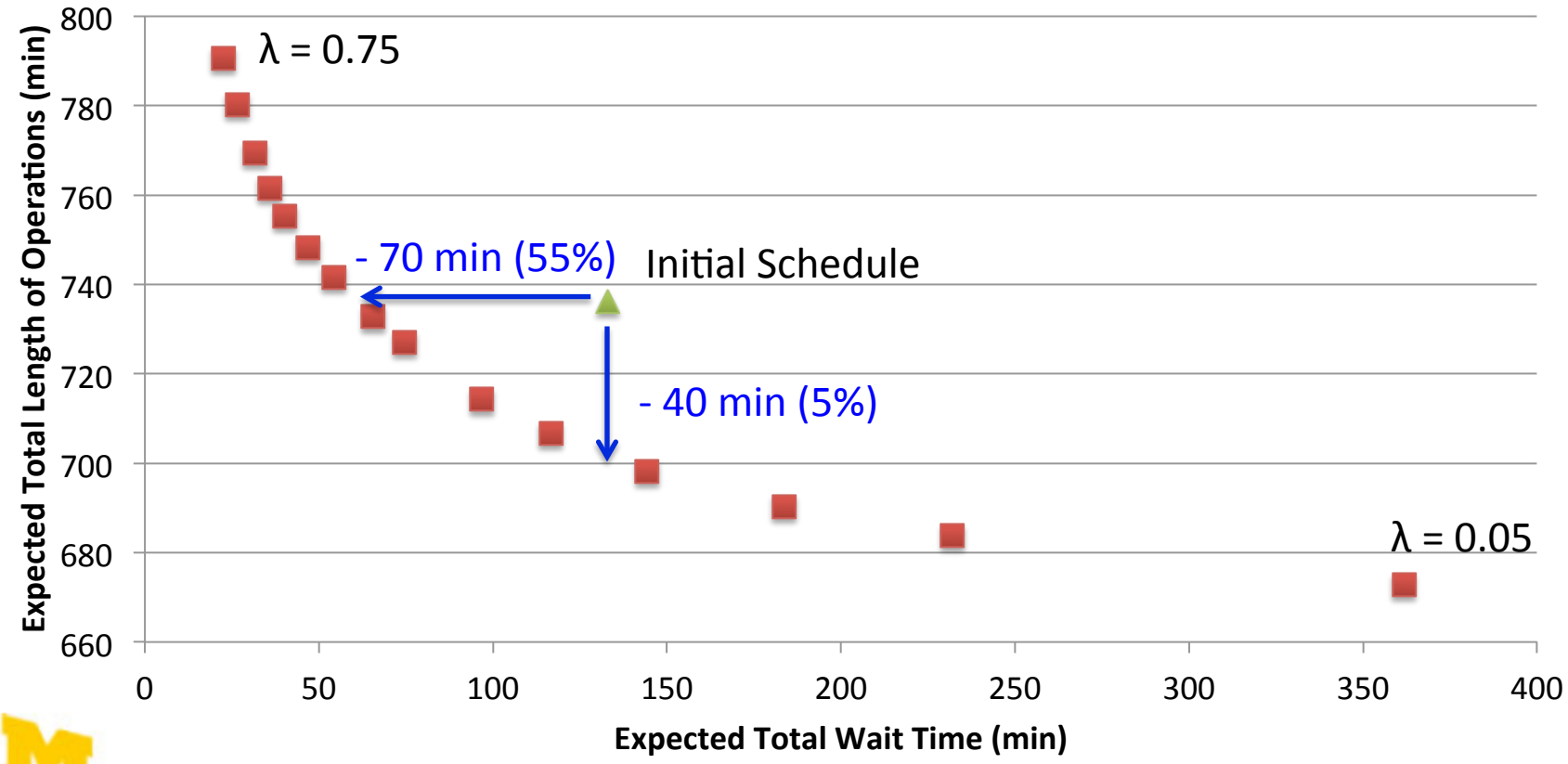

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Future Work

- **Theoretical Work:**
  - Understand why the *Fix-Unfix* algorithm works so well
  - Can this algorithm be successfully applied other optimization problems?
  - Merge scheduling phases 1 and 2 in a single optimization model
- **Towards an implementation at the Cancer Center:**
  - Define general rules/guidelines to help the phase 1 scheduling process
  - Find easy-to-implement good sequences: Longest (Shortest) Processing Time First, Shortest Variance First...

Thank you

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University of Michigan

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