Why 4 Times 10 Is Not Always 40:
Using Simulation to Illustrate the Impact of Randomness on Training Heart and Lung Transplant Surgeons

In the world of heart and lung transplant surgery, four times ten does NOT always equal forty! Or, more precisely, if a hospital performs an average of **forty transplants** per year and has **four residents** (doctors-in-training) each of whom is responsible for surgeries every fourth day, it is **not** guaranteed that each of these residents will get to perform at least **ten transplants**. In fact, the odds of each resident getting ten or more transplants is very small – only about 5%! With this paper and the associated computer-based mathematical simulation tool, we hope to help you understand why.

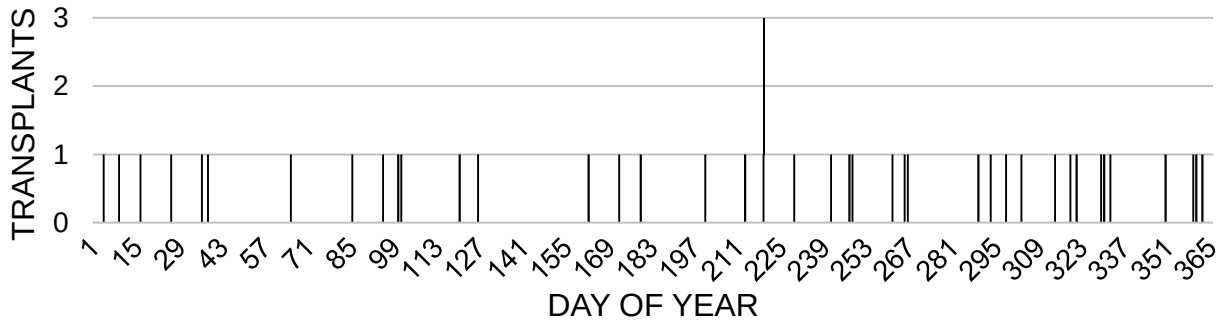
Surgical residents are medical school graduates who are training at a hospital under the supervision of more experienced doctors, called *attending physicians*, to learn surgical techniques and develop patient care skills necessary to treat patients on their own. Following residency, many complete a fellowship to refine their specialized surgical skills before independently practicing in their chosen field.

Before they are ready to practice medicine without supervision, residents and fellows must gain significant experience working under an attending physician, achieving more and more autonomy over time. In particular, prospective cardiothoracic (heart and lung) surgeons who hope to become certified in heart and/or lung transplant surgery must assist in a certain number of these transplants (to keep things simple, let's call it ten per year) during fellowship.

Fellows have many responsibilities during training, including clinical work, scheduled surgery, research, etc. In addition, they work a predetermined *call schedule* to cover unplanned and emergency operations including heart and lung transplants. At UMHS (and many other teaching hospitals), fellows are assigned to a rotating call schedule – if you have four fellows, then any given fellow will be on call every fourth day, as illustrated in the calendar below. If the organ transplant takes place on your day, then you participate in the procedure and accrue the experience toward certification.

July						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
	1 Chen	2 Jones	3 Smith	4 Reddy	5 Chen	6 Jones
7 Smith	8 Reddy	9 Chen	10 Jones	11 Smith	12 Reddy	13 Chen
14 Jones	15 Smith	16 Reddy	17 Chen	18 Jones	19 Smith	20 Reddy
21 Chen	22 Jones	23 Smith	24 Reddy	25 Chen	26 Jones	27 Smith
28 Reddy	29 Chen	30 Jones	31 Smith			

Unlike the fellows, who work on a planned and fixed call schedule, transplants arrive at random. Historical data indicate that the time between transplant opportunities at Michigan Medicine follows an *exponential distribution*. More plainly, transplant opportunities occur randomly – maybe one today, another in a few days, none for several weeks, and then two in a single day. The "timeline" below gives an example of a hypothetical year of transplant opportunities. The timeline was generated based on the assumption that the average time between transplants is approximately 9 days (roughly the same as at Michigan Medicine). For this hypothetical year, most days have zero, some have one, and occasionally two (or more!) occur on the same day. Notice there are both clusters of and droughts between transplants throughout the year.



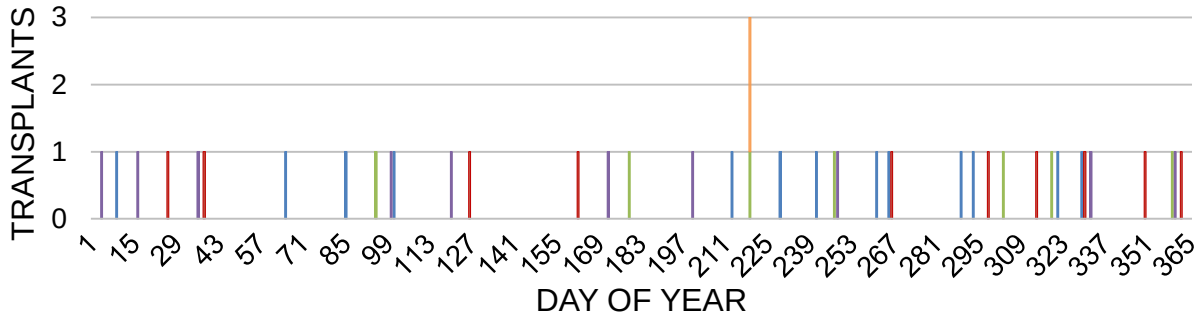
So why might not every fellow get 10 procedures and become certified in our earlier example? The first reason is simple. Even if you get 40 opportunities per year *on average*, there will be years with fewer (as well as years with more). If there are fewer than 40 cases, of course not everyone can get at least 10 cases.

But even when there are at least 40 cases, they may not be assigned evenly across the residents. The actual probability that all residents will get at least 10 transplants is only about 5.3%.

To put it another way, imagine a huge bag completely filled with equal numbers of green, purple, red, and blue marbles. Without looking, pull out 40 marbles. How likely is it that you will get

exactly 10 of each color? The odds are very low. This is the same basic idea as the fixed call schedule with randomly arriving transplant opportunities.

From the previous page, consider matching the rotating call schedule (extended over a year) with the timeline of transplant opportunities. As that hypothetical year played out, Dr. Chen would get thirteen opportunities, Drs. Jones and Reddy would each get ten transplants, and Dr. Smith would get seven transplants while two of the transplants would not go to any of the four fellows due to the arrival of three transplants in a single day. Thus, the residency program would only adequately train three of the four residents despite performing 42 transplants. We reproduce the timeline below, with the colors representing each of the surgeons from the call schedule with orange for the unassigned transplants. There clearly was potential to certify all four if there had been a schedule that allowed for three of the transplants performed by Dr. Chen to have gone to Dr. Smith instead.



Of course, this is just one "random realization" – in any given year, we do not know what the timeline of transplants will look like. But we *can* use *computer-based mathematical simulation tools* to generate many random possible outcomes and evaluate the performance *on average*. The University of Michigan Center for Healthcare Engineering and Patient Safety has developed a simulator to illustrate the likelihood with which programs achieve certain program requirements given the number of residents, residency duration, frequency of transplants, and call schedule design. You can download the simulator from:

https://cheps.engin.umich.edu/tools/stereo/