

Linear Programming Tools for Scheduling Trainees in Healthcare

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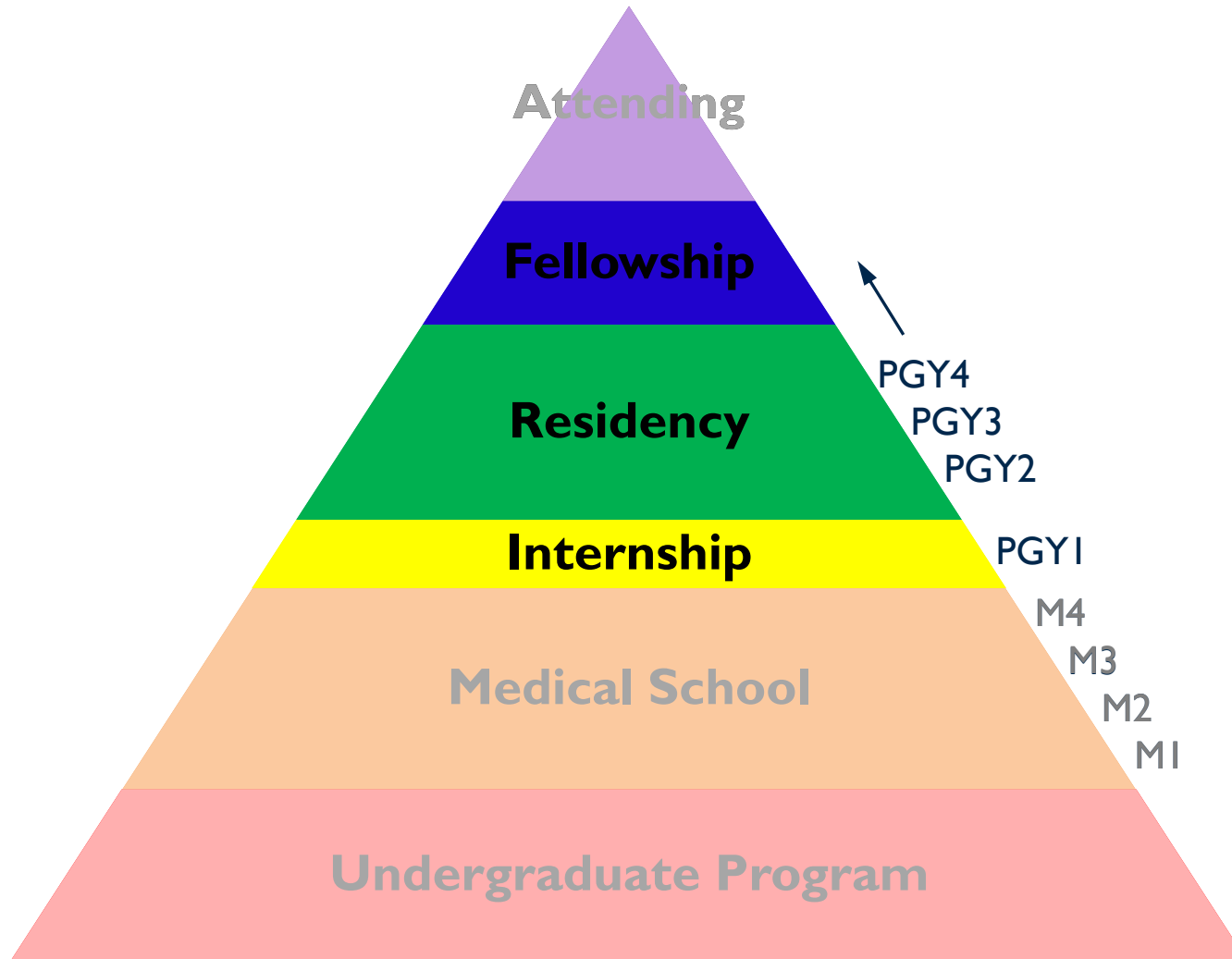
Presentation Outline

- Background
- Motivation
- Model Formulation
- Model Implementation
- Conclusions and Future Work

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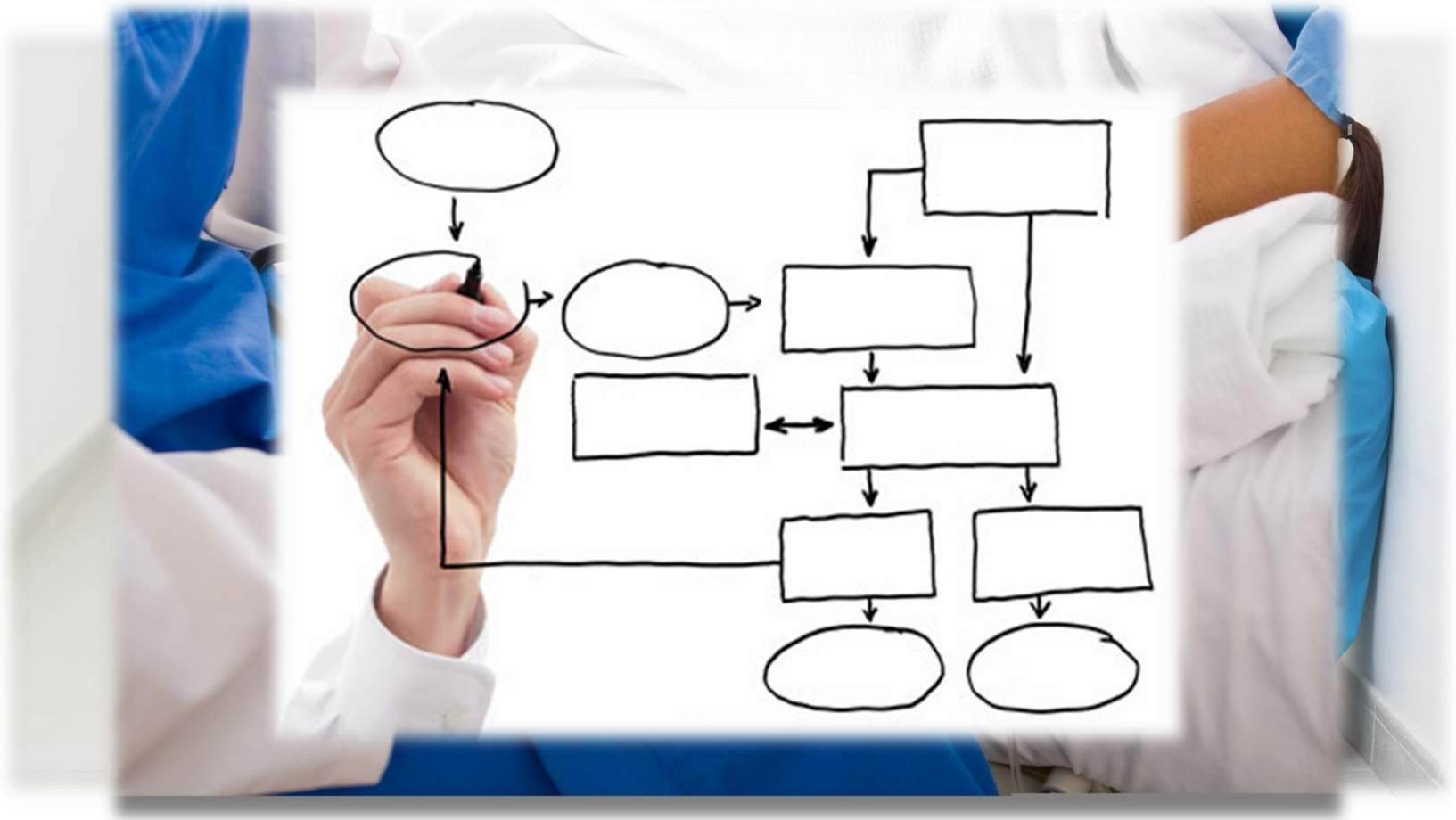
Healthcare Training Basics



Healthcare Training at Michigan



Importance of Scheduling



Who does the Scheduling?

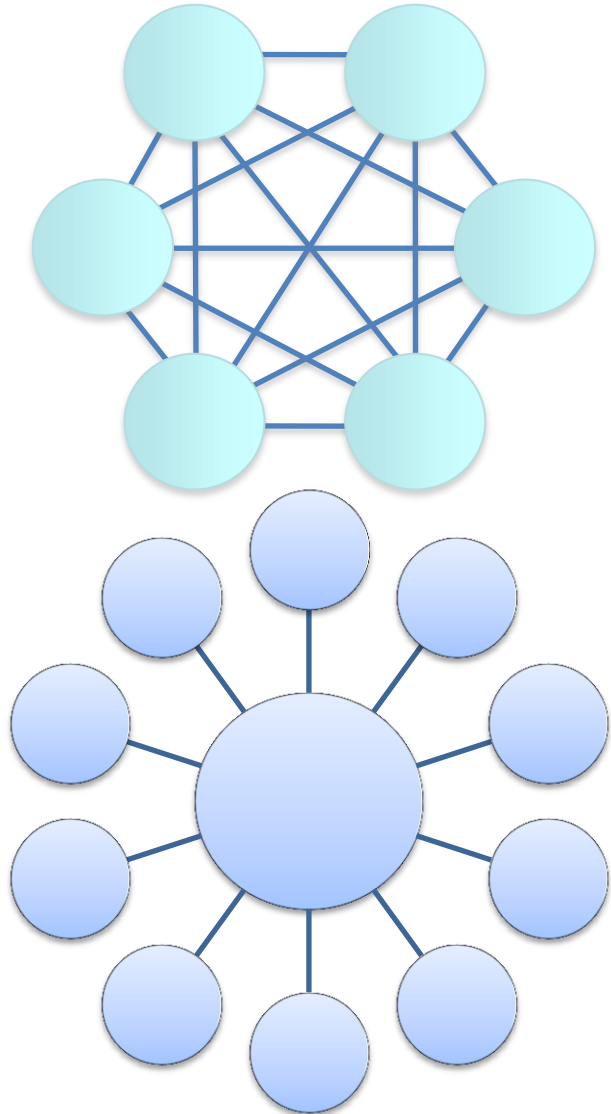
- Program dependent
 - Chief Resident
 - Faculty (Program Director)
 - Senior Administrative Staff

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Challenges in Scheduling

- Time-intensive process
- Numerous stakeholders
- Complex rules and legal requirements
- Conflicting goals
- Varying strategies and interdependencies
- “Good enough” mentality



Resident Education Requirements

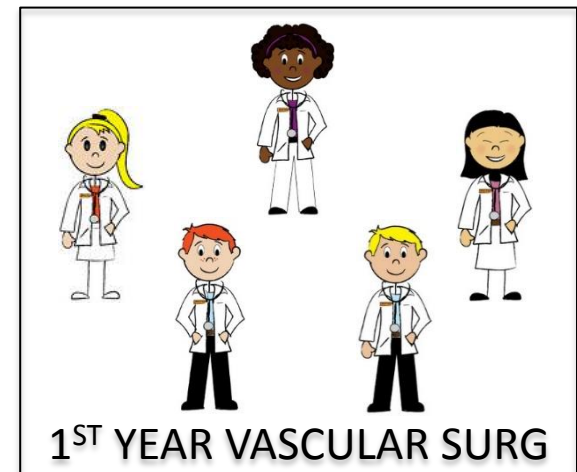
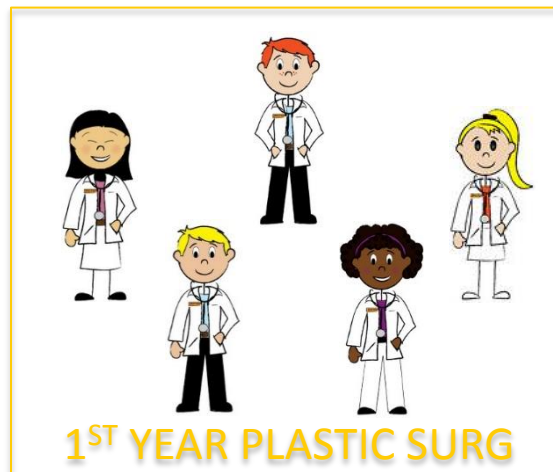
- Each program has unique educational requirements (operative and disease exposure)

	PGY1	PGY2	PGY3	PGY4	PGY5	PGY6	PGY7
1 1	MAIZE	MAIZE	MAIZE	MAIZE	MAIZE	MAIZE	MAIZE
2 2	BLUE	BLUE	BLUE/A	BLUE	BLUE	BLUE	MAIZE
3 3	WHITE	WHITE	WHITE	WHITE	WHITE	WHITE	RED
4 4	MAIZE/BLUE/WHITE	MAIZE/BLUE/WHITE	BLUE	BLUE	BLUE	BLUE	BLUE
5 5	ACS	ACS	ACS	ACS	ACS	ACS	WHITE
6 6	DSP	ACS	ACS	ACS	ACS	ACS	BLUE
7 7	TBEPLA	ANES	DSPOP	DSPOP	DSPOP	DSPOP	ACS
8 8	GI SUSICU	ANES	FLOAT	FLOAT	FLOAT	FLOAT	ACS
9 9	VA GS-STX	DSP	SICU	SICU	SICU	SICU	STX
10 10	VASVA	FLOAT	THS	THS	THS	THS	SVA
11 11	STVA CT	TBE	SICU	STX/SVA	STX/SVA	STX/SVA	VA GS
12 12	VA GS-VASC	VA GS-VASC	VA GS	VA GS	VA GS	VA GS	VA GS-VASC

General Surgery (6 yrs) (disease exposure per year)

Service Coverage Requirements

- Each service requires a resident complement comprised of varying skillsets and disciplines



WHITE

SICU

STX

Traditional Scheduling Approach

1. Build rotation templates
2. Adjust for coverage and educational needs
3. Renegotiate after reaching a dead-end

JULY	AUG	SEPT	OCT	NOV	DEC	JAN	FEB	MAR	APRIL	MAY	JUNE
BLUE	MAIZE	PLA	SVA	SICU	BLUE	WHITE	PLA	STX	VA G&V	VA CT	DSP
VA G&V	PLA	MAIZE	WHITE	ACS	BLUE	SICU	BLUE	PLA	STX	STX	VA CT
VA CT	PLA	BLUE	DSP	VA G&V	ACS	SICU	BLUE	MAIZE	WHITE	SVA	SVA
MAIZE	VA CT	VA G&V	BLUE	SVA	WHITE	ACS	SICU	BLUE	STX	PLA	DSP

Design a linear program which automates creation of a block schedule that satisfies the needs of the residents and services.

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Linear Programming Basics

- A technique to solve complicated story problems
- Four basic parts
 - Sets and parameters
 - Decision variables
 - Objective function
 - Constraints

$$\min \quad 2x_1 + x_2$$

$$\begin{aligned} \text{subject to} \quad & x_1 + x_2 \geq 5 \\ & 2x_1 + 3x_2 \leq 11 \\ & \underline{x_1}, x_2 \geq 0 \end{aligned}$$

Optimal Solution: (1, 4)
Objective Value = 6

R : residents

C : resident categories

S : services

M : months

$a_{rc} \in \{0, 1\}$: indicates if resident r fits category c

\mathcal{L}_{csm} : lower bound on number of residents fitting category c in service s during month m

\mathcal{U}_{csm} : upper bound on number of residents fitting category c in service s during month m

λ_{rs} : lower bound on number of months resident r must spend on service s

μ_{rs} : upper bound on number of months resident r must spend on service s

$x_{rsm} \in \{0, 1\}$: whether resident r is
assigned to service s in month m
 $\forall r \in R, s \in S, m \in M$

The base model does not have an objective function.

1. Every resident gets assigned to one service every month

$$\sum_{s \in S} x_{rsm} = 1, \quad \forall r \in R, m \in M$$

2. Every resident satisfies their educational requirements

$$\lambda_{rs} \leq \sum_{m \in M} x_{rsm} \leq \mu_{rs}, \quad \forall r \in R, s \in S$$

3. Every service satisfies their service coverage needs

$$\mathcal{L}_{csm} \leq \sum_{r \in R} a_{rc} x_{rsm} \leq \mathcal{U}_{csm}, \quad \forall c \in C, s \in S, m \in M$$

⋮

I. Every resident gets assigned to one service every month

$x_{\text{Smith},\text{Maize},\text{July}}$ Is Dr. Smith assigned to the Maize service in July?

If yes, $x_{\text{Smith},\text{Maize},\text{July}} = 1$. If no, $x_{\text{Smith},\text{Maize},\text{July}} = 0$.

$x_{\text{Smith},\text{Blue},\text{July}}$ Is Dr. Smith assigned to the Blue service in July?

$x_{\text{Smith},\text{White},\text{July}}$ Is Dr. Smith assigned to the White service in July?

$$x_{\text{Smith},\text{Maize},\text{July}} + x_{\text{Smith},\text{Blue},\text{July}} + x_{\text{Smith},\text{White},\text{July}} = 1$$

I. Every resident gets assigned to one service every month

$$x_{Smith, Maize, July} + x_{Smith, Blue, July} + x_{Smith, White, July} = 1$$

$$x_{Smith, Maize, Aug} + x_{Smith, Blue, Aug} + x_{Smith, White, Aug} = 1$$

\vdots

$$x_{Smith, Maize, June} + x_{Smith, Blue, June} + x_{Smith, White, June} = 1$$

$$x_{Jones, Maize, July} + x_{Jones, Blue, July} + x_{Jones, White, July} = 1$$

\vdots

$$x_{Jones, Maize, June} + x_{Jones, Blue, June} + x_{Jones, White, June} = 1$$

$$\sum_{s \in S} x_{rsm} = 1, \quad \forall r \in R, m \in M$$

2. Every resident satisfies their educational requirements

$x_{\text{Smith},\text{Maize},\text{July}}$ Is Dr. Smith assigned to the Maize service in July?

If yes, $x_{\text{Smith},\text{Maize},\text{July}} = 1$. If no, $x_{\text{Smith},\text{Maize},\text{July}} = 0$.

$x_{\text{Smith},\text{Maize},\text{Aug}}$ Is Dr. Smith assigned to the Maize service in August?

\vdots

$x_{\text{Smith},\text{Maize},\text{June}}$ Is Dr. Smith assigned to the Maize service in June?

$$1 \leq x_{\text{Smith},\text{Maize},\text{July}} + x_{\text{Smith},\text{Maize},\text{Aug}} + \dots + x_{\text{Smith},\text{Maize},\text{June}} \leq 2$$

2. Every resident satisfies their educational requirements

$$1 \leq x_{Smith, Maize, July} + x_{Smith, Maize, Aug} + \dots + x_{Smith, Maize, June} \leq 2$$

$$1 \leq x_{Smith, Blue, July} + x_{Smith, Blue, Aug} + \dots + x_{Smith, Blue, June} \leq 2$$

$$1 \leq x_{Smith, White, July} + x_{Smith, White, Aug} + \dots + x_{Smith, White, June} \leq 2$$

$$1 \leq x_{Jones, Maize, July} + x_{Jones, Maize, Aug} + \dots + x_{Jones, Maize, June} \leq 2$$

\vdots

$$1 \leq x_{Jones, Blue, July} + x_{Jones, Blue, Aug} + \dots + x_{Jones, Blue, June} \leq 2$$

$$\lambda_{rs} \leq \sum_{m \in M} x_{rsm} \leq \mu_{rs}, \quad \forall r \in R, s \in S$$

3. Every service satisfies their service coverage needs

$x_{\text{Smith},\text{Maize},\text{July}}$ Is Dr. Smith assigned to the Maize service in July?

If yes, $x_{\text{Smith},\text{Maize},\text{July}} = 1$. If no, $x_{\text{Smith},\text{Maize},\text{July}} = 0$.

$a_{\text{Smith},\text{GS}}$ Is Dr. Smith a General Surgery resident?

If yes, $a_{\text{Smith},\text{GS}} = 1$. If no, $a_{\text{Smith},\text{GS}} = 0$.

$a_{\text{Smith},\text{PGYI}}$ Is Dr. Smith a PGYI resident?

If yes, $a_{\text{Smith},\text{PGYI}} = 1$. If no, $a_{\text{Smith},\text{PGYI}} = 0$.

$a_{\text{Smith},\text{GS_PGYI}}$ Is Dr. Smith a General Surgery PGYI resident?

If yes, $a_{\text{Smith},\text{GS_PGYI}} = 1$. If no, $a_{\text{Smith},\text{GS_PGYI}} = 0$.

3. Every service satisfies their service coverage needs

$$\begin{aligned} 3 &\leq a_{Smith,GS}x_{Smith,Maize,July} + a_{Jones,GS}x_{Jones,Maize,July} \\ &\quad + a_{Chan,GS}x_{Chan,Maize,July} + \dots + a_{Gupta,GS}x_{Gupta,Maize,July} \\ &\leq 4 \end{aligned}$$

$$\begin{aligned} 1 &\leq a_{Smith,PGY1}x_{Smith,Maize,July} + a_{Jones,PGY1}x_{Jones,Maize,July} \\ &\quad + a_{Chan,PGY1}x_{Chan,Maize,July} + \dots + a_{Gupta,PGY1}x_{Gupta,Maize,July} \leq 2 \end{aligned}$$

$$\mathcal{L}_{csm} \leq \sum_{r \in R} a_{rc} x_{rsm} \leq \mathcal{U}_{csm}, \quad \forall c \in C, s \in S, m \in M$$

Expanded Model

- Distributed Educational Requirements
- Distributed Coverage Needs
- Extended Rotations
- Service Sequencing
- Service Spacing
- Resident Pairing

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Implementation Process

Sets

R : residents
 C : resident categories
 S : services
 M : months

Parameters

$a_{rc} \in \{0,1\}$: whether resident r fits category c
 ℓ_{csm}, u_{csm} : lower, upper bounds on staffing of residents fitting category c in service s during month m
 λ_{rs}, μ_{rs} : lower, upper bounds on months resident r must spend on service s

Decision Variables

$x_{rsm} \in \{0,1\}$: whether resident r is assigned to service s in month m
 $\forall r \in R, s \in S, m \in M$

Objective Function

```

1 build_model
2 build_model
3 build_model
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8 build_model
9 build_model
10 build_model
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100 build_model
    
```

Resident	Month	Service	Assignment
1	July	General	1
2	August	General	2
3	September	General	3
4	October	General	4
5	November	General	5
6	December	General	6

```

Nodes:
Node Left Objective Linf Best Integer Cuts/ Best Made ItCnt Gap
0 0 Infeasible 6.21

Root node processing (before bmc):
Real time = 0.20
Parallel bmc, 2 threads:
Real time = 0.00
Sync time Coverage = 0.00
Wait time Coverage = 0.00

Total (root+branch+mc) = 0.20 sec.
Variable Count = 37637
Constraint Count = 27678
Solution status = Infeasible
Time = 1
The schedule could not be generated!!!
Summary of each resident can be viewed
    
```

Resident	Month	Service	Assignment
1	July	General	1
2	August	General	2
3	September	General	3
4	October	General	4
5	November	General	5
6	December	General	6

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- Scheduling issues affect hospital workflow, training quality, and patient safety
- Scheduling residency programs at UMHS is highly interdependent, complex, and poorly executed
- We can address these scheduling needs using a linear programming formulation

- Define metrics for schedule optimality
 - Minimize deviation from desired resident complement by service
 - Maximize satisfied requests for educational customization
- Apply model to improve scheduling for other training programs

- Pediatric Medicine rotation schedule
- C.S. Mott Emergency Department shift schedule
- Chemotherapy infusion patient schedule
- Physician clinic/OR schedule
- Master surgical schedule problem
- Nurse staff scheduling

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- Center for Healthcare Engineering and Patient Safety
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Questions [?] and Comments [!]

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