This document formulates the basic notation and constraints for the Shift Scheduling Game.

1 Inputs: Sets and other Parameters

1.1 Sets

R	set of residents
$I \subseteq R$	set of interns
S	set of shifts {1,2,...,7}
$N \subseteq S$	set of night shifts
D	set of days
$D_r \subseteq D \forall r \in R$	set of days that intern r cannot work
$L_{sd} \subseteq (S \times D)$	the set of shifts incompatible with (s, d)

1.2 Parameters

$ut = 16$	upper bound for number of total shifts
$lt = 12$	lower bound for number of total shifts
$un = 5$	upper bound for number of night shifts
$ln = 2$	lower bound for number of night shifts

2 Decision Variables

x_{rsd}	do we assign resident r to shift s on day d?

3 Objective Function

This simplified version of the shift scheduling problem does not have an objective function as the goal is simply to create a $feasible$ schedule rather than one that is optimal based on certain metrics.

4 Variable Restrictions

x_{rsd} {0,1} $\quad \forall r \in R, s \in S, d \in D$

5 Constraints

1. **Every shift needs a resident**
   For every shift s on every day d, exactly one resident r must be assigned to work.

$$\sum_{r \in R} x_{rsd} = 1, \quad \forall s \in S, d \in D \tag{1}$$

2. **Every resident needs between 12 and 16 shifts**
   For every resident r, we must assign between 12 and 16 shifts across all shifts s over all days d.

$$\sum_{s \in S, d \in D} x_{rsd} \geq lt \quad \forall r \in R \tag{2}$$

$$\sum_{s \in S, d \in D} x_{rsd} \leq ut \quad \forall r \in R \tag{3}$$

3. **Every resident needs between 2 and 5 night shifts**
   For every resident r, we must assign between 2 and 5 shifts across all **night** shifts s in N over all days d.

$$\sum_{s \in N, d \in D} x_{rsd} \geq ln, \quad \forall r \in R \tag{4}$$

$$\sum_{s \in N, d \in D} x_{rsd} \leq un, \quad \forall r \in R \tag{5}$$

4. **Every resident needs adequate rest between shifts**
   For every resident r, day d, and shift s the sum of x_{rsd} and its set of incompatible shifts in (s', d') must be equal to at most 1.

$$x_{rsd} + \sum_{(s',d') \in L_{sd}} x_{rs'd'} \leq 1 \quad \forall r \in R, s \in S, d \in D \tag{6}$$

5. **Interns cannot work the first or last shift of the day**
   Interns r in I cannot work the first (shift 1) or the last shift (shift 7) s of the day d.

$$\sum_{s \in \{1\}} \sum_{d \in D} x_{rsd} = 0 \quad \forall r \in I \tag{7}$$

$$\sum_{s \in \{7\}} \sum_{d \in D} x_{rsd} = 0 \quad \forall r \in I \tag{8}$$

6. **Each resident has a specific day of week he or she cannot work**
   Resident r has a specific day of week that he or she cannot work. D_r is the set of corresponding days.

$$x_{rsd} = 0 \quad \forall r \in R, s \in S, d \in D_r \tag{9}$$