Collaborators

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Motivation

C.S. Mott Emergency Room shift scheduling

Residency rotation scheduling

Conclusions and potential opportunities
Scheduling needs in healthcare

Physician scheduling

Nurse scheduling

Operating room scheduling

Appointment scheduling

Many more…
Scheduling affects...

...patient anxiety, patient quality of life, and satisfaction...
Traditional approach

Schedules hand-built by program director, chief resident(s), or administrator

**Benefits**

1) Intimate problem knowledge
2) Administrative consolidation
3) Streamlined approval process

**Drawbacks**

1) Time-consuming process
2) High cognitive demand
3) Limited consideration of tradeoffs
Medical training at UMHS

1,199 trainees

105 training programs

25 residencies

80 fellowships
Motivation

C.S. Mott Emergency Room shift scheduling

Residency rotation scheduling

Conclusions and potential opportunities
C.S. Mott Pediatric Emergency Room

Level I Pediatric Trauma Center

About 25,000 visits per year

Staffed by 5 residency programs

- Pediatrics
- Medicine-Pediatrics
- Family Medicine
- Emergency Medicine
- Psychology
Resident scheduling challenges

Resource-intensive process
  – Chief resident spends 20 – 25 hours per month
  – Numerous revisions

Complicated requirements
  – Legal, regulatory, and administrative rules
  – Resident education
  – Service coverage
Decision variables

Whether to assign resident \( r \) to shift \( s \) on day \( d \)

\[ x_{r,s,d} \in \{0, 1\}, \quad \forall \ r \in R, \ s \in S, \ d \in D \]
Shift coverage

Must provide sufficient shift coverage for every day and shift

\[
\sum_{r \in R} x_{rsd} = 1, \quad \forall \ d \in D, \ s \in S \{\text{flex, EOM, EMSr}\}
\]

\[
0 \leq \sum_{r \in R} x_{rsd} \leq 1, \quad \forall \ d \in D, \ s \in \{\text{flex, EOM}\}
\]

\[
\sum_{r \in R} x_{rsd} = 0, \quad \forall \ d \in D, \ s \in \{\text{EMSr}\}
\]
Total shifts

Must provide adequate educational experience for every resident

\[ \text{LBShifts}_r \leq \sum_{s \in S} \sum_{d \in D} x_{r,s,d} \leq \text{UBShifts}_r, \quad \forall r \in R \]

\[ \text{LBNites}_r \leq \sum_{s \in S} \sum_{d \in D} x_{r,s,d} \leq \text{UBNites}_r, \quad \forall r \in R \]
External requirements

Cannot create work assignments that conflict with outside commitments

\[ x_{r,s,d} = 0, \quad \forall r \in R, d \in D, s \in \{ \text{conferences, clinics, vacations, etc.} \} \]
Pediatric paired shifts

Ensure that at least 1 of 2 shifts in a pair is covered by a Pediatric resident each day

\[
\sum_{r \in \{\text{PED}\}} \sum_{s \in \mathcal{P}} x_{r,s,d} \geq 1,
\]

\forall d \in D, \mathcal{P} = \{\{7a,9a\}, \{4p,5p\}, \{8p,11p\}\}
Senior-only shifts

Certain shifts must be covered by senior-level residents

\[ \sum_{r \in \{\text{interns}\}} \sum_{d \in D} x_{rsd} = 0, \]

\[ \forall s \in \{7a, 11p\} \]
Work-rest rules

Residents must get at least 10 hours off-duty between ending one shift and beginning another

\[ x_{rsd} + \sum_{(s',d') \in \{\text{within 10 hrs of }(s,d)\}} x_{rs'd'} \leq 1, \]

\[ \forall r \in R, s \in S, d \in D \]
Multi-criteria objective

Multi-criteria schedule

- Total shift equity (TSE)
- Night shift equity (NSE)
- Bad sleep patterns (BSP)
- Post-continuity clinic shifts (PCC)
- ...

Preferences? Weights? Trade-off?

Multi-objective Mathematical Programming
Multi-criteria objective

**Optimization problem**

\[
\text{Min } w_1(TSE) + w_2(NSE) + w_3(BSP) + w_4(PCC)
\]

s.t.

- "rules/requirements"
- \( x_{rsd} \in \{0,1\} \)

Quantifying preferences \((w_i)\) is difficult
- Subjective weights
- Alternative measures
- Non-linearity
Multi-criteria objective

Feasibility Optimization problem

\[
\min w_1(TSE) + w_2(NSE) + w_3(BSP) + w_4(PCC)
\]

s.t. "rules/requirements"

\[
x_{r,s,d} \in \{0,1\}
\]

\[
\begin{align*}
lb_{TSE} & \leq (TSE) \leq ub_{TSE} \\
lb_{NSE} & \leq (NSE) \leq ub_{NSE} \\
lb_{BSP} & \leq (BSP) \leq ub_{BSP} \\
lb_{PCC} & \leq (PCC) \leq ub_{PCC}
\end{align*}
\]

Benefits of a feasibility problem

– Flexibility

– Speed: < 2 seconds per iteration

Given: 20 residents / 7 shifts daily / 35 days
### Iterative improvement

<table>
<thead>
<tr>
<th>Resident Name</th>
<th>Number of Shifts</th>
<th>Number of Night Shifts</th>
<th>Number of Post-CC Shifts</th>
<th>Number of Bad Sleep Patterns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smith</td>
<td>8 (7,9)</td>
<td>2 (2,3)</td>
<td>0 (0,1)</td>
<td>0 (0,0)</td>
</tr>
<tr>
<td>Sanchez</td>
<td>8 (7,10)</td>
<td>2 (2,3)</td>
<td>0 (0,1)</td>
<td>0 (0,0)</td>
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<tr>
<td>Chen</td>
<td>8 (7,9)</td>
<td>2 (2,3)</td>
<td>1 (0,1)</td>
<td>0 (0,0)</td>
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<tr>
<td>Shah</td>
<td>14 (13,15)</td>
<td>4 (3,5)</td>
<td>1 (0,1)</td>
<td>0 (0,0)</td>
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</table>
Implementation results

Reduced time to create schedules

20 hours per month → 1 hour per month

Statistically significant improvement in 3 of 4 major metrics
Total shift equity

<table>
<thead>
<tr>
<th>Month</th>
<th>2010-11: 0.0761 ± 0.0214</th>
<th>2012-13: 0.0665 ± 0.0367</th>
<th>2013-14: 0.0801 ± 0.0231</th>
<th>2014-15: 0.0743 ± 0.0238</th>
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</thead>
<tbody>
<tr>
<td>July</td>
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<td>May</td>
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</tr>
<tr>
<td>June</td>
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</tr>
</tbody>
</table>
Bad sleep patterns

<table>
<thead>
<tr>
<th>Month</th>
<th>2010-11: 6.9167 ± 2.8749</th>
<th>2013-14: 0.0833 ± 0.2887</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012-13: 1.1667 ± 4.0415</td>
<td>2014-15: 0.3333 ± 0.8876</td>
<td></td>
</tr>
</tbody>
</table>
Implementation summary

Months with poor metrics tend to have:

– Fewer residents overall
– Fewer senior residents
– Fewer Pediatrics residents
Simulation study

<table>
<thead>
<tr>
<th>Total Residents</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
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<tbody>
<tr>
<td>20</td>
<td>5.4%</td>
<td>33.0%</td>
<td>66.8%</td>
<td>84.8%</td>
<td>92.6%</td>
<td>95.9%</td>
<td>95.2%</td>
<td>96.4%</td>
<td>95.7%</td>
<td>96.1%</td>
</tr>
<tr>
<td>19</td>
<td>6.2%</td>
<td>32.4%</td>
<td>60.7%</td>
<td>79.7%</td>
<td>89.5%</td>
<td>93.1%</td>
<td>94.0%</td>
<td>93.5%</td>
<td>94.2%</td>
<td>94.3%</td>
</tr>
<tr>
<td>18</td>
<td>4.1%</td>
<td>25.8%</td>
<td>55.2%</td>
<td>76.2%</td>
<td>87.6%</td>
<td>88.9%</td>
<td>91.4%</td>
<td>91.1%</td>
<td>92.2%</td>
<td>92.6%</td>
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<tr>
<td>17</td>
<td>3.8%</td>
<td>25.0%</td>
<td>48.8%</td>
<td>71.4%</td>
<td>81.9%</td>
<td>86.4%</td>
<td>89.3%</td>
<td>87.8%</td>
<td>86.9%</td>
<td>89.1%</td>
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<tr>
<td>16</td>
<td>2.2%</td>
<td>20.0%</td>
<td>45.6%</td>
<td>65.5%</td>
<td>77.0%</td>
<td>81.0%</td>
<td>80.0%</td>
<td>83.3%</td>
<td>82.4%</td>
<td>82.9%</td>
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<td>15</td>
<td>2.1%</td>
<td>16.6%</td>
<td>35.2%</td>
<td>55.7%</td>
<td>69.2%</td>
<td>75.4%</td>
<td>74.0%</td>
<td>76.2%</td>
<td>76.7%</td>
<td>75.7%</td>
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<tr>
<td>14</td>
<td>1.2%</td>
<td>11.4%</td>
<td>29.2%</td>
<td>47.9%</td>
<td>58.9%</td>
<td>63.2%</td>
<td>66.9%</td>
<td>67.9%</td>
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<td>22.9%</td>
<td>36.4%</td>
<td>48.5%</td>
<td>55.5%</td>
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<td>34.2%</td>
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<td>40.8%</td>
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<tr>
<td>11</td>
<td>0.3%</td>
<td>3.4%</td>
<td>8.8%</td>
<td>15.5%</td>
<td>22.4%</td>
<td>27.5%</td>
<td>27.5%</td>
<td>25.9%</td>
<td>28.1%</td>
<td>28.1%</td>
</tr>
</tbody>
</table>

Percentage Feasible (of 2,000 Iterations)

<table>
<thead>
<tr>
<th>Pr(Senior Standing)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
</tr>
<tr>
<td>10%</td>
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</tbody>
</table>
Motivation

C.S. Mott Emergency Room shift scheduling

Residency rotation scheduling

Conclusions and potential opportunities
Rotation scheduling

Assigning residents to services over the course of the academic year

Must simultaneously satisfy service coverage needs and academic requirements

Typically month-long rotations
Pediatric Residency Program

Training in inpatient and ambulatory settings

Integration with combined programs

**Service pair:**
an ordered couplet of services that may be worked during the same month

| Service Pair $p$ | $1^{st}$ Half Service | $2^{nd}$ Half Service | Hard Rotation? $(1 = Yes, 0 = No)$ |
Residencies and fellowships in

- General
- Vascular
- Plastic
- Thoracic
- Anesthesiology
- Many more…

Service and education integration with numerous other programs and institutions
Decision variables

Whether to assign resident $r$ to service $s$ (or pair $p$) on month $m$:

**Pediatric Residency Program**

$$x_{rpm} \in \{0, 1\},$$

$$\forall r \in R, p \in P, m \in M$$

**Department of Surgery**

$$x_{rsm} \in \{0, 1\},$$

$$\forall r \in R, s \in S, m \in M$$
Monthly rotation assignment

Each resident is assigned one service (pair) per month

**Pediatric Residency Program**
\[ \sum_{p \in P} x_{rpm} = 1, \]
\[ \forall r \in R, m \in M \]

**Department of Surgery**
\[ \sum_{s \in S} x_{rsm} = 1, \]
\[ \forall r \in R, m \in M \]
Service coverage

Each service must have between a minimum L and maximum U number of residents (fitting a certain category c) at any time

**Pediatric Residency Program**

\[
L_{sm} \leq \sum_{p \in P_{sh}} x_{rpm} \leq U_{sm},
\]

\(\forall s \in S, m \in M, h \in \{1, 2\}\)

**Department of Surgery**

\[
L_{csm} \leq \sum_{r \in R} a_{rc} x_{rsm} \leq U_{csm},
\]

\(\forall c \in C, s \in S, m \in M\)
Resident education

Each resident must work between a minimum $\lambda$ and maximum $\mu$ number of months on each service throughout the year.

**Pediatric Residency Program**

$$\lambda_{rs} \leq \sum_{p \in P} \sum_{m \in M} a_{ps} x_{rpm} \leq \mu_{rs},$$

$\forall \ r \in R, \ s \in S$

**Department of Surgery**

$$\lambda_{rs} \leq \sum_{m \in M} x_{rsm} \leq \mu_{rs},$$

$\forall \ r \in R, \ s \in S$
PEDIATRIC RESIDENCY PROGRAM-SPECIFIC CONSTRAINTS
Residents may work a limited number of sequences of 3 hard service pairs $h_p$ in a row

$$\sum_{p \in P} h_p x_{rpm} + h_p x_{rp(m+1)} + h_p x_{rp(m+2)} \leq y_{rm} + 2,$$

$\forall r \in R, m \in \{1, \ldots, |M| - 2\}$

$$\sum_{m \in M} y_{rm} \leq H_r,$$

$\forall r \in R$
DEPARTMENT OF SURGERY-
SPECIFIC CONSTRAINTS
Extended rotations

Residents assigned to services in extended rotation rule $e$ must be assigned for consecutive months equal to the specified duration $d^e$

\[ x_{rs}[d^e \times (i-1)] = x_{rs} \{[d^e \times (i-1)] + j\}, \]

\[ \forall e \in E, r \in R^e, s \in S^e, \]
\[ i \in \{1, \ldots, |M|/d^e\}, \]
\[ j \in \{1, \ldots, d^e - 1\} \]
Resident included in sequencing rule $q$ must be assigned to certain services prior to being assigned to a particular service $s_q'$.

$$L^q \leq \left[ \sum_{s \in S^q} \sum_{m \in M^q} x_{rsm} \right] - \sum_{r \in R^n} x_{rs_q'} m_{q'}$$

$\forall q \in Q, r \in R^q$
Residents must not be assigned to a certain service more than once in a certain timeframe

\[ \sum_{m \in M^a} x_{rsm} \leq 1, \quad \forall a \in A, r \in R^a, s \in S^a \]
Resident pairing

Assigning residents in resident pair rule $n$ from group $\mathcal{R}^n$ to services $\mathcal{S}^n$ requires also assigning residents from group $\mathcal{R}^n$ to services $\mathcal{S}^n$

$$\ell_{sm}^n \leq \sum_{r \in \mathcal{R}^n} \sum_{s \in \mathcal{S}^n} x_{rsm} - \sum_{r \in \mathcal{R}^n} \sum_{s \in \mathcal{S}^n} x_{rsm} \leq u_{sm}^n$$

$\forall n \in \mathbb{N}, s \in \mathcal{S}^n, m \in \mathcal{M}^n$
Implementation process

1. Formulate
2. Encode
3. Load
4. Solve
5. Review
**Pediatric Residency Program**

- **Residents**: 99
- **Programs**: 4
- **Services**: 14

**Two-phase schedule creation**
- Senior phase
- Intern phase

- **≤3 minutes/iteration**

**Satisfied 238/242 (98.3%)** of requests made for Pediatric Residency Program

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**Department of Surgery**

- **Residents**: 118
- **Programs**: 10
- **Services**: 62

**Multi-phase schedule creation**
- Program lock-ins
- Individual lock-ins

- **≤5 minutes/iteration**

**Facilitated customized program tracking for 12 residents**
Presentation outline

Motivation

*Emergency Department shift scheduling*

*Residency rotation scheduling*

Conclusions and potential opportunities
Conclusions

Significantly reduced time and improved metrics for ED shift schedules

Lingering scheduling challenges may derive from the rotation schedule

Significantly improved satisfaction of time preferences for rotation schedules
Generalize models into universal formulation

Extend model to address other residency programs’ needs

Apply algorithm to identify maximally feasible sets of requests
Acknowledgements

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- University of Michigan Health System
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- The Doctors Company
Questions [ ? ] and comments [ ! ]

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