# Optimally Scheduling Satellite Communications

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# Motivations


*QB50 Mission*
*Expected to Launch*
*in 2016*

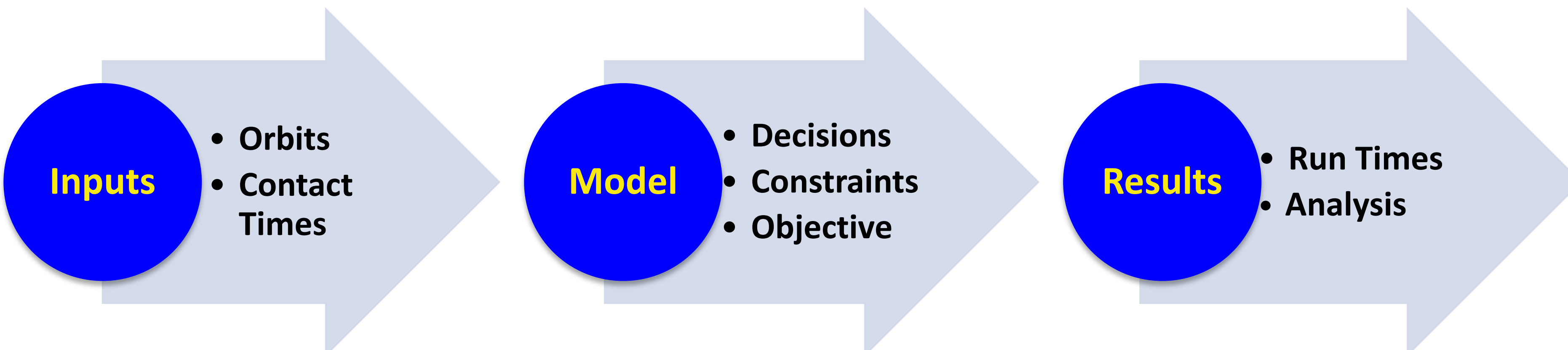
| 1 - 50 kg Satellite Launches [1] | |
|---|---|
| **Year:** | **Approx. Launches Per Year:** |
| 2006-2012 | 25 |
| 2013 | 92 |
| 2014 | 140 |

*Ground station*
*antennas*

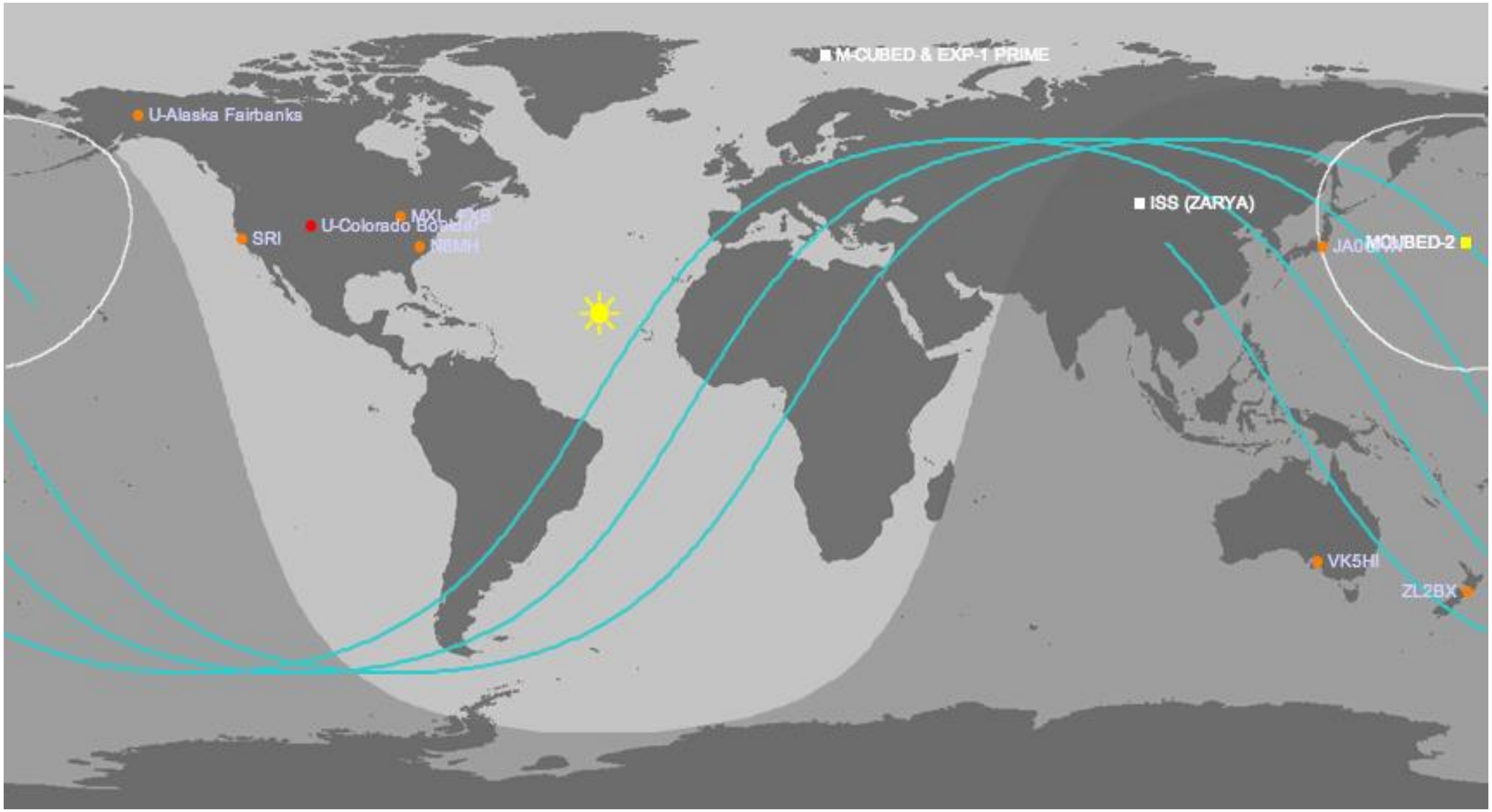
[1] Elizabeth Buchen and Dominic DePasquale. Nano/Microsatellite Market Assesment. SpaceWorks Enterprises, Inc. 2014

# Goal and Outline

- Schedule downloads for a multi-satellite, multi-ground station system.

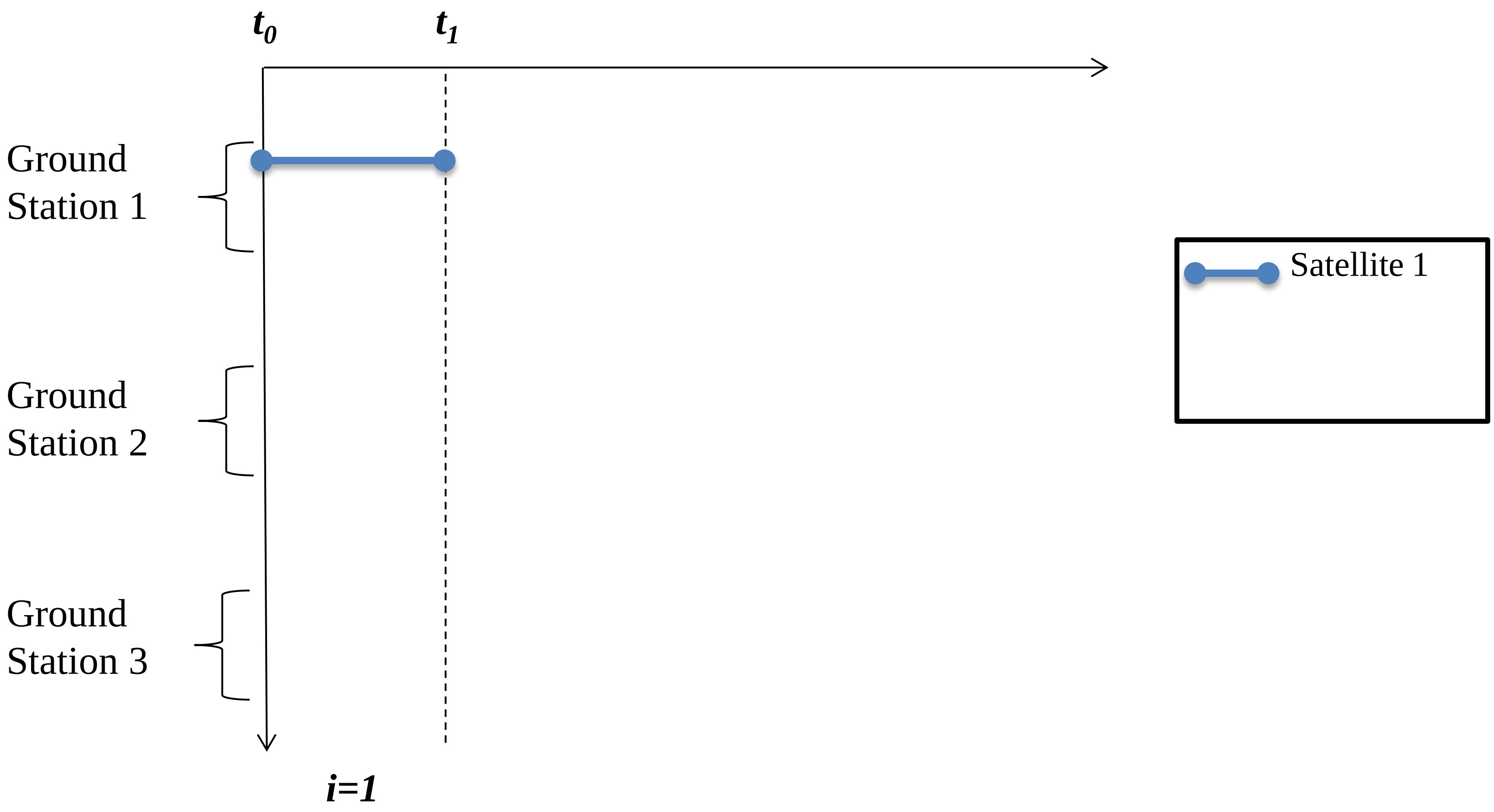


# Orbits and Ground Stations




Satellite ground tracks

# Abstract Representation of Contact Times



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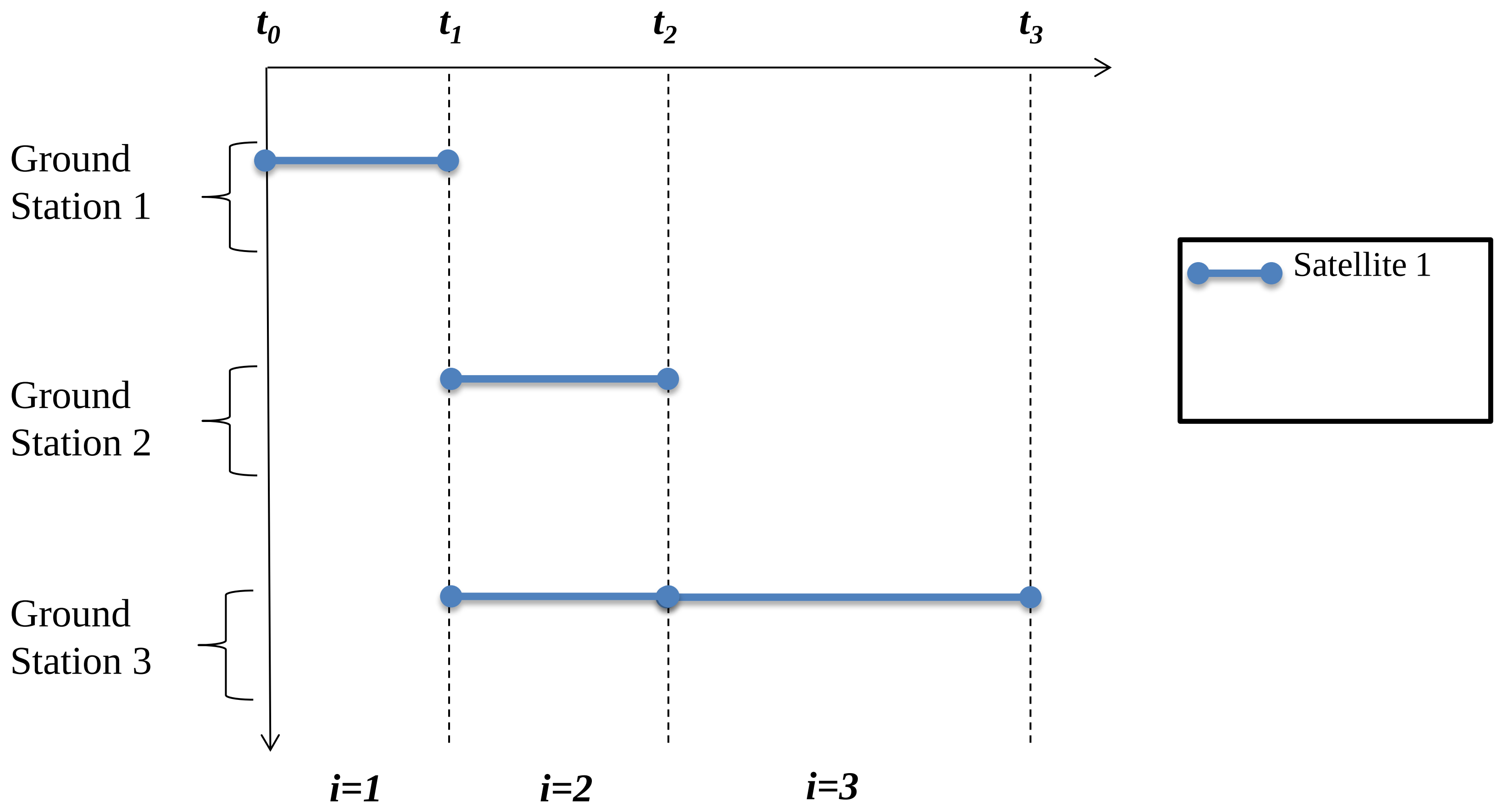

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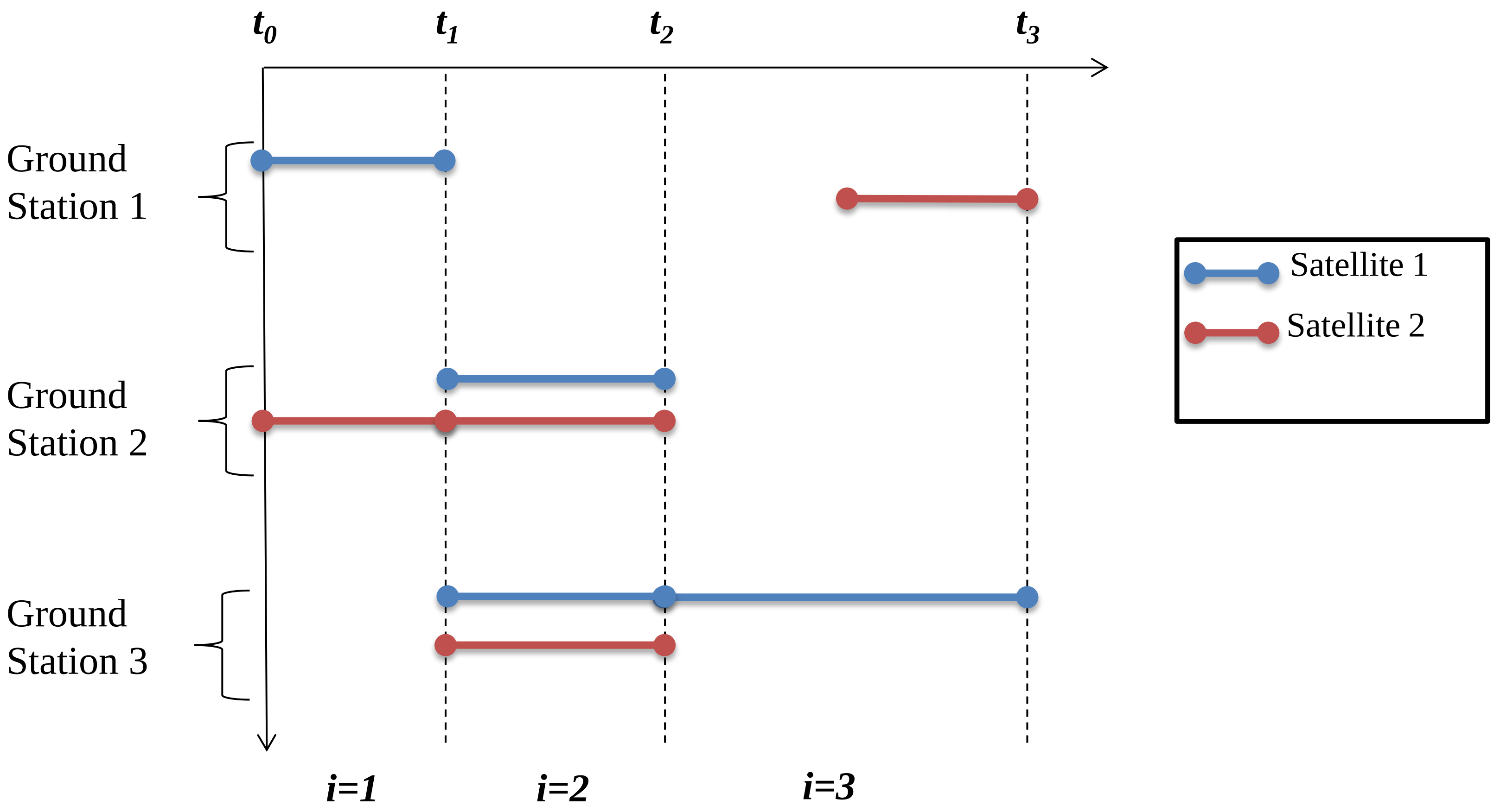

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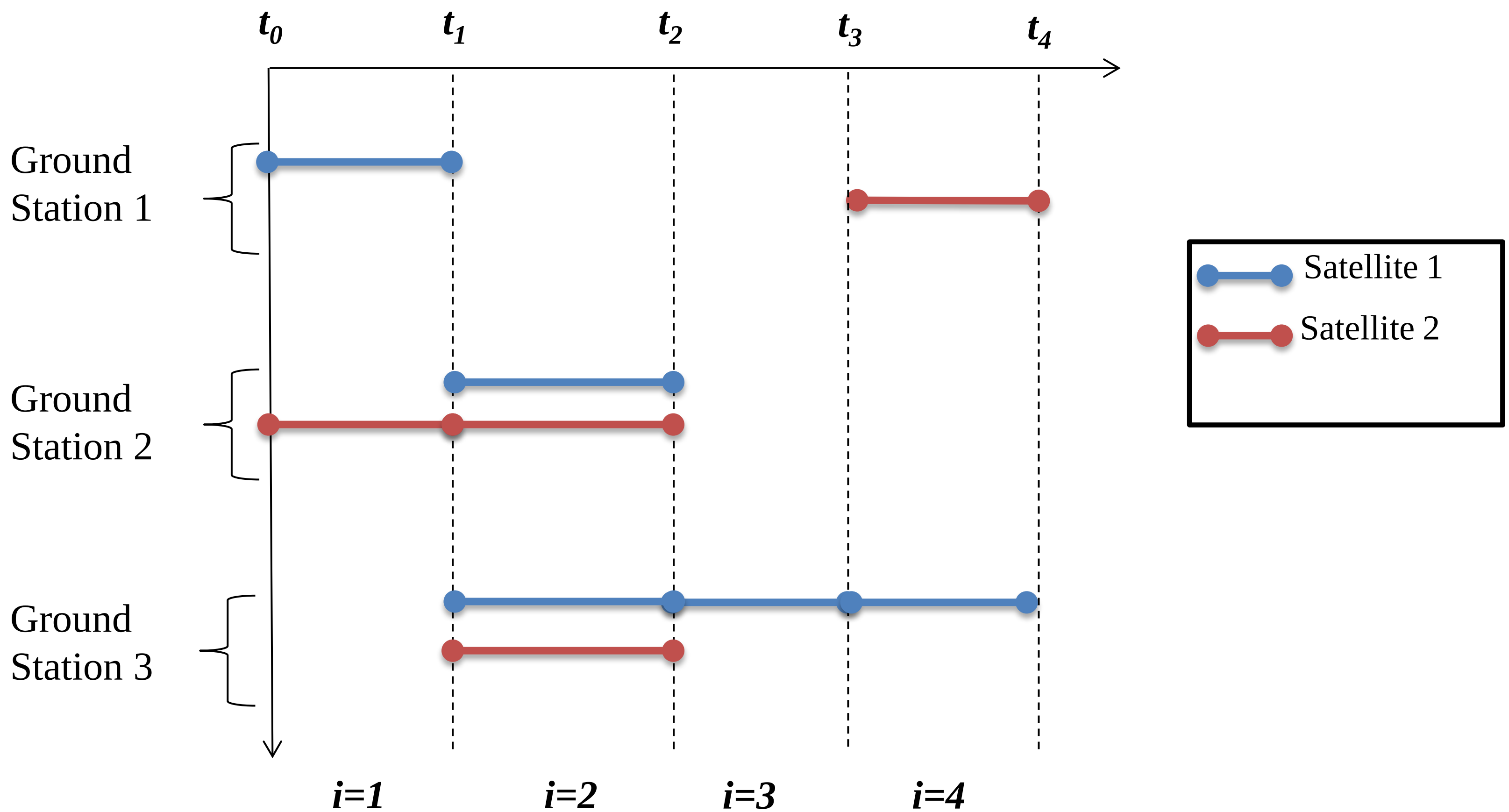

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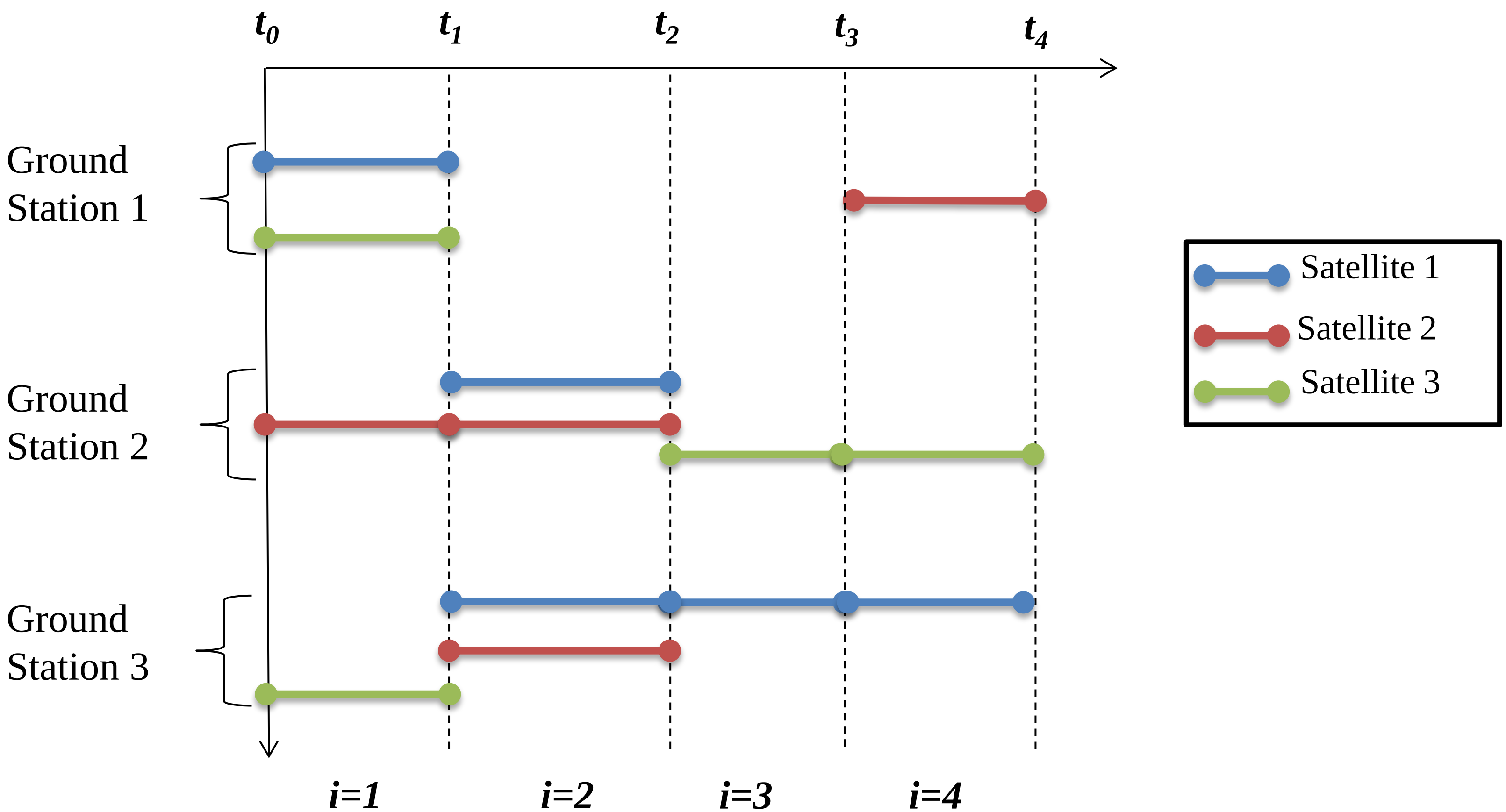

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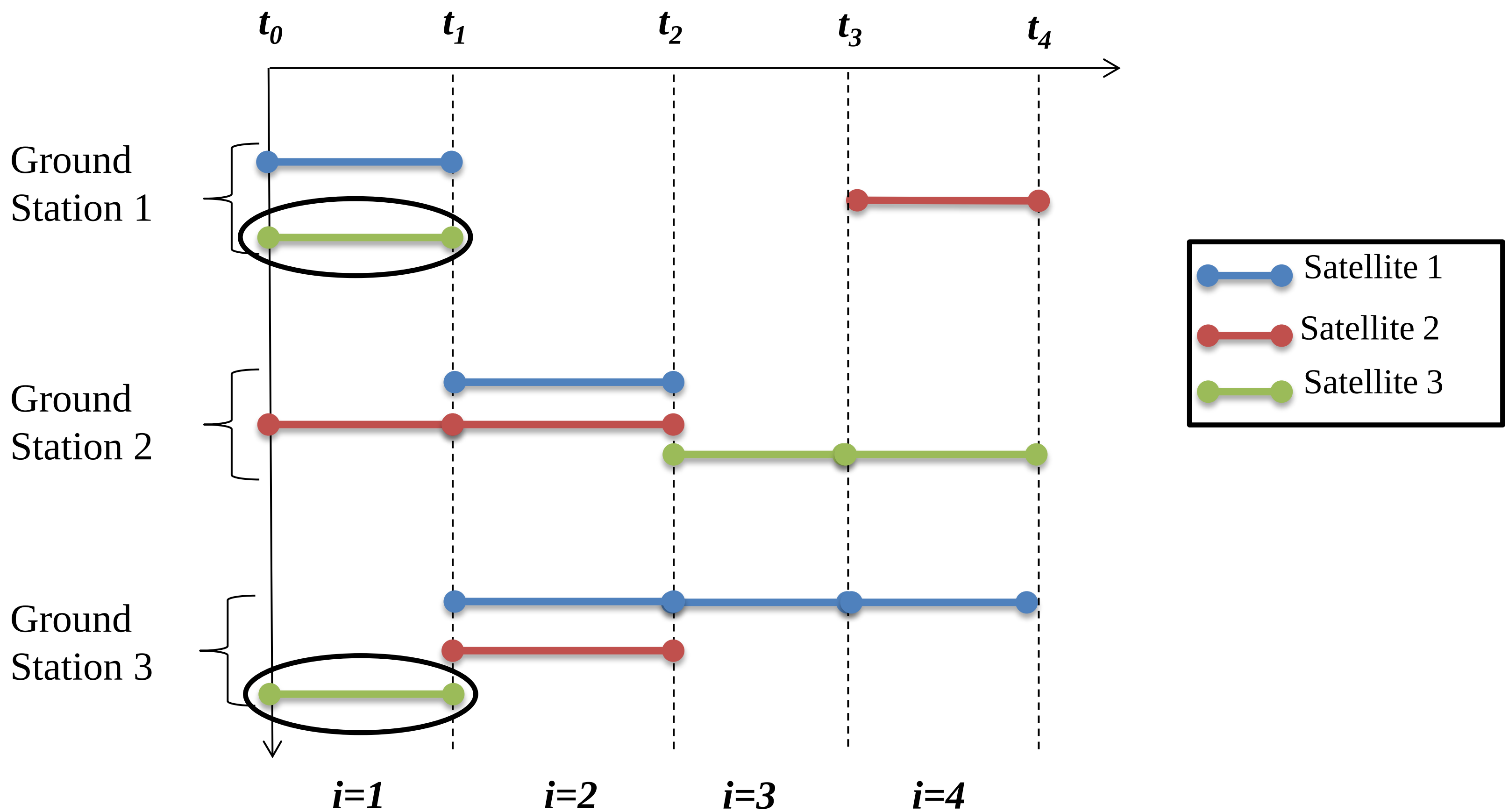

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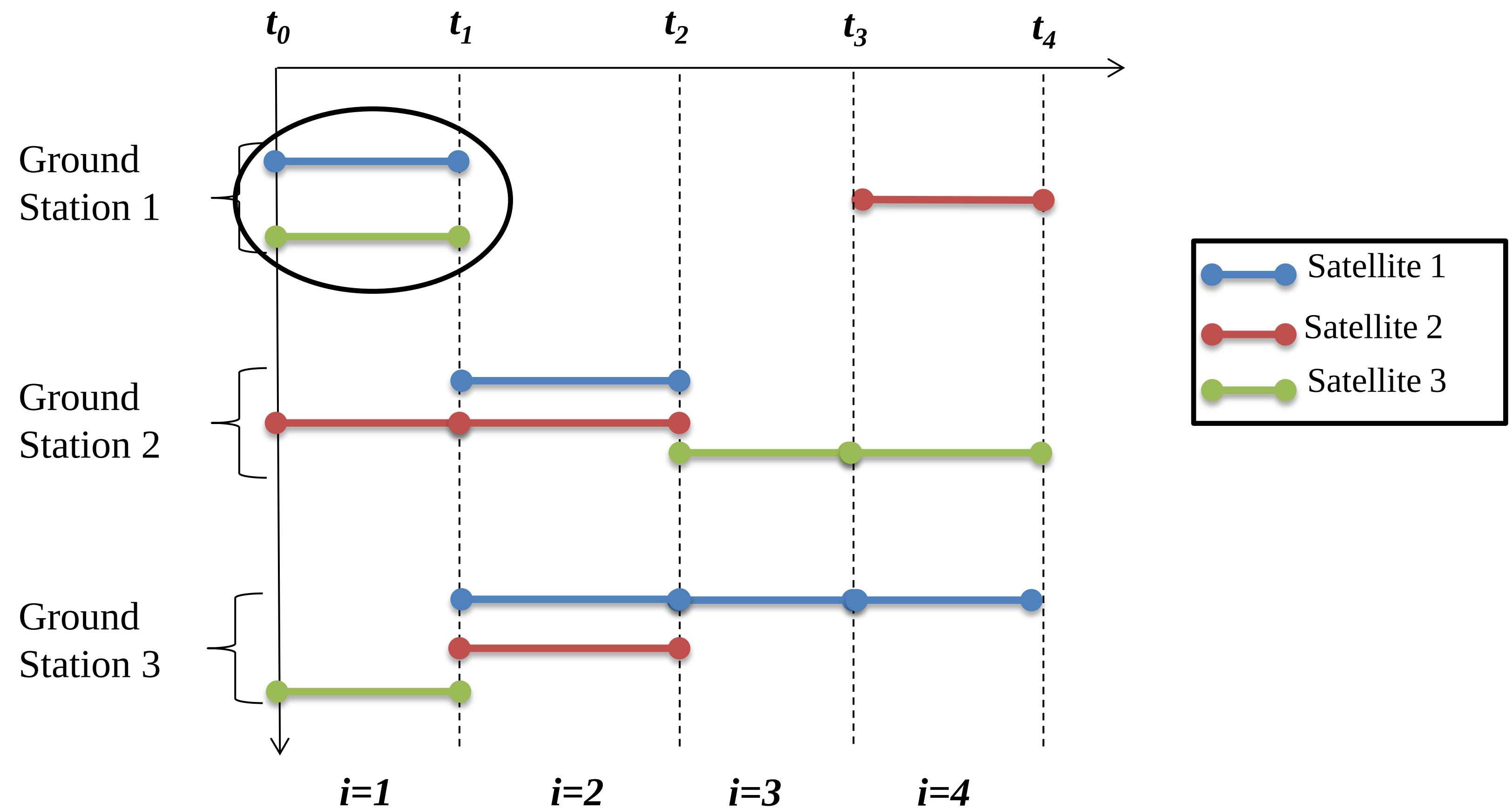


# The Multi-Satellite, Multi-Ground Station Scheduling Problem (MMSP)

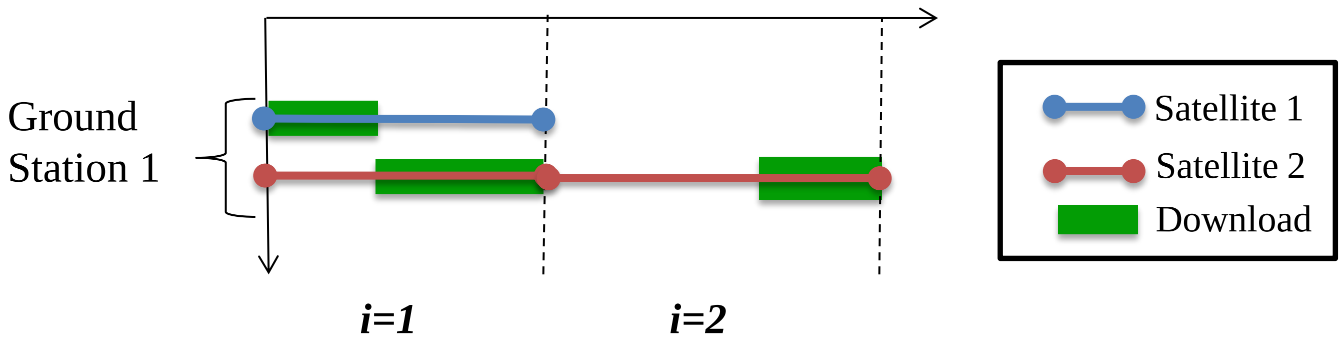
- **Objective:** to maximize the total amount of data downloaded over the planning horizon
- **Subject to**:
  - Download opportunities
  - Conflicts
  - Energy & Data Dynamics
  - Ground Stations Characteristics:
    - Download Rate (bits/sec)
    - Download Cost (joules/bit)
    - Efficiency (percentage of download actually received)

# Download Decisions

- $x_{sig}$ – Percentage of interval $i$ that satellite $s$ downloads to ground station $g$
- $q_{sig}$ – Amount of data downloaded from satellite $s$ during interval $i$ to ground station $g$

# A Simple Schedule

| | Ground Station 1 | | | |
|---|---|---|---|---|
| | Interval 1 | | Interval 2 | |
| | ***x*** | ***q*** | ***x*** | ***q*** |
| **Satellite 1** | 40% | 2 Mb | 0% | 0 Mb |
| **Satellite 2** | 60% | 3 Mb | 30% | 1 Mb |



# Energy and Data Dynamics

$$e_{(i+1)} = \min\{e_{max},\ e_i + \delta_i^{e+} - \delta_i^{e-} - \sum_g \alpha_{ig} q_{ig}\}$$

*Energy available is based on previous energy + energy gained - energy spent unrelated to downloads - energy used for downloads*

$$d_{(i+1)} = \min\{d_{max},\ d_i + \delta_i^{d+} - \delta_i^{d-} - \sum_g \eta_{ig} q_{ig}\}$$

*Data available is based on previous data + data gained - data lost - data successfully downloaded*

Parameters:

$\delta_i^{e+}$: net amount of energy acquired (joules)

$\delta_i^{e-}$: net amount of energy acquired (joules)

$\delta_i^{d+}$: net amount of data acquired (bits)

$\delta_i^{d-}$: net amount of data acquired (bits)

$\alpha_{ig}$: Download cost (joules/bit)

$\eta_{ig}$: Download efficiency (% received)

Variables:

$q_{ig}$: Amount of data downloaded (bits)

$e_i$: Energy available (joules)

$d_i$: Data available (bits)

# What do we want to look at?

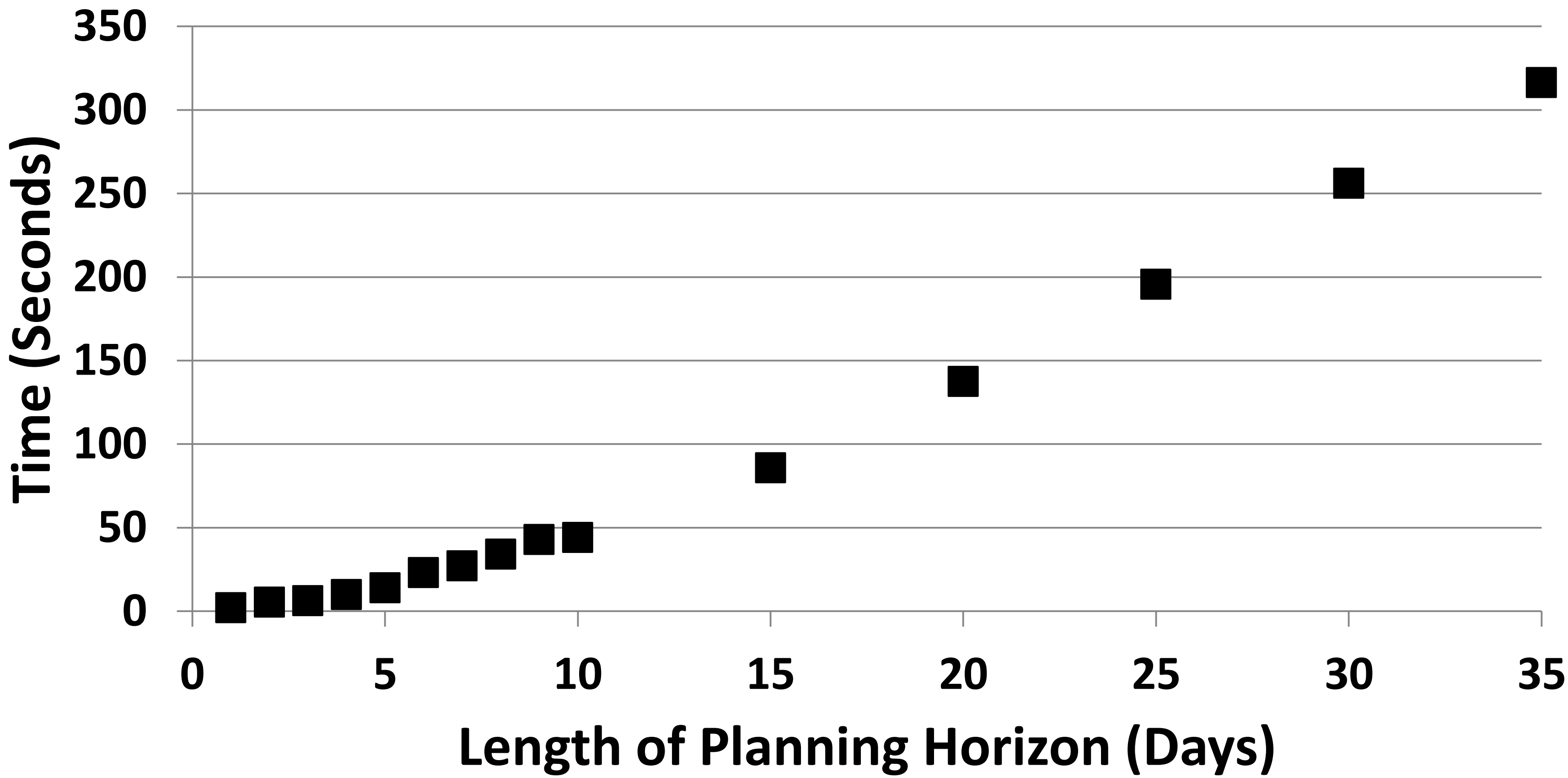
- **Tractability:** Can it solve real-world problems in a reasonable amount of time?
- **Quality:** How much value does it add over a simpler, more traditional scheduling approach?
- **Applications:** How can it be used as a mechanism for conducting analysis on a mission?

# Generating Data for Testing

- Generated orbital information for each of the 50 satellites for the QB 50 mission
  - Determine which ground stations are in view
  - Determine when in view of the Sun
- Ground stations are located at each participating university and are randomly categorized as good, average, or poor

# Solve Times

## (50 Satellites, 50 Ground Stations)




**MMSP model can be solved quickly for realistic planning horizons**

# Comparison Method

- Greedy Heuristic
  - For each time interval:
    - Identify maximum possible download for each satellite
    - Schedule the maximum download
    - Eliminate the participating satellite and ground station from the list of potential downloads
    - Repeat until no more feasible downloads

# Value of Optimization

| Energy Gain (joules/sec) | Optimization Total Download (MB) | Greedy Total Download (MB) | Improvement Over Greedy |
|---|---|---|---|
| 0.05 | 104.97 | 104.91 | 0.1% |

1. **Satellites can collect less energy**
2. **Optimization provides significant value when they do**

# Value of Optimization

| Data Gain | Improvement Over Greedy |
|---|---|
| Base Case (0.3 Mb/Day) | 0.2% |
| x5 | 0.2% |
| x10 | 0.2% |
| x15 | 4.5% |
| x20 | 32.8% |
| x25 | 61.0% |
| x30 | 61.0% |
| x35 | 61.0% |

**Optimization provides significant value when satellites collect more data than what is currently planned**

# Analysis of Deployment Options

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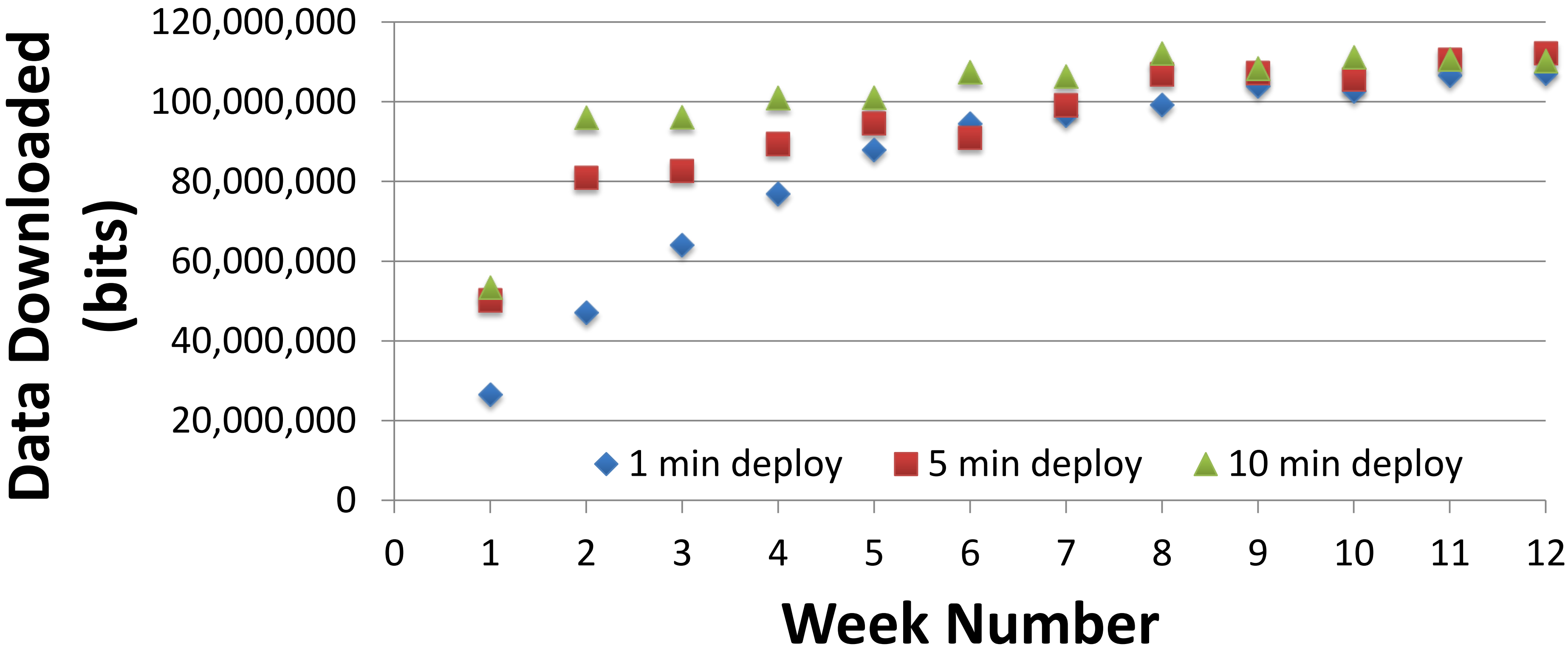


| | 1-minute | 5-minutes | 10-minutes |
|---|---|---|---|
| Total Download (MB) | 126.7 | 141.3 | 151.8 |
| Improvement (%) | - | 11.5% | 19.8% |

**10-minute spacing enables more data to be downloaded**

# Conclusions and Future Work

- Capable of quickly solving real problems with clear benefits over more traditional methods
  - Enabled sensitivity analysis for evaluating system capabilities and bottlenecks
- MMSP adds the most value in scenarios where energy is a limiting resource
- Future Work: Stochastic instances, prioritized downloads, fairness of downloads

# Thank You!

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$$\max \sum_{s \in S} \sum_{i \in I} \sum_{g \in G} \eta_{ig} q_{sig}$$

*Objective: Maximize total download over the planning horizon*

<u>Parameters:</u>

$\gamma_{sig}$: Indicator if in view during i
$\eta_{ig}$: Download efficiency (% received)
$t_i$: Duration of interval (sec)
$\phi_{ig}$: Download rate (bits/sec)
$\alpha_{ig}$: Download cost (joules/bit)
$\delta_{si}^{e}$: net amount of energy acquired (joules)
$\delta_{si}^{d}$: net amount of data acquired (bits)

<u>Variables:</u>

$x_{sig}$: Percent of interval used for download
$q_{sig}$: Amount of data downloaded (bits)
$e_{si}$: Energy available (joules)
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$h_{si}^{e}$: excess energy spilled
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$$\max \sum_{s\in S} \sum_{i\in I} \sum_{g\in G} \eta_{ig} q_{sig}$$

$$x_{sig} \leq \gamma_{sig} \qquad \forall s \in S, i \in I, g \in G \qquad (1)$$

*Can only download if satellite is in range of ground station*

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$$\sum_{s \in S} x_{sig} \leq 1 \qquad \forall i \in I, g \in G \qquad (2)$$

*Ground stations cannot receive data for more than 100% of each interval*

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*Satellites cannot transmit data for more than 100% of each interval*

Parameters:

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$\eta_{ig}$: Download efficiency (% received)
$t_i$: Duration of interval (sec)
$\phi_{ig}$: Download rate (bits/sec)
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$\delta_{si}^{e}$: net amount of energy acquired (joules)
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Variables:

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$$q_{sig} \le t_i \phi_{ig} x_{sig} \qquad \forall s \in S, i \in I, g \in G \qquad (4)$$

*Download amount is limited by length of interval and download rate*

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$$e_{s0} = e_{start} \qquad \forall s \in S \qquad (5)$$

*Initial energy available*

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$$e_{min} \le e_{si} \le e_{max} \qquad \forall s \in S, i \in I \qquad (6)$$

*Energy buffer size: lower and upper bound on stored energy*

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$$
\begin{array}{lll}
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q_{sig} \leq t_i \phi_{ig} x_{sig} & \forall s \in S, i \in I, g \in G & (4) \\
e_{s0} = e_{start} & \forall s \in S & (5) \\
e_{min} \leq e_{si} \leq e_{max} & \forall s \in S, i \in I & (6) \\
e_{s,i+1} = e_{si} + \delta^e_{si} & & \\
\qquad - \sum_{g\in G} \alpha_{ig} q_{sig} - h^e_{si} & \forall s \in S, i \in I & (7)
\end{array}
$$

*Energy available is based on previous energy + energy gained - energy used - any excess energy that must be spilled*

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& e_{s,i+1} = e_{si} + \delta^e_{si} && && \\
& \qquad - \sum_{g\in G} \alpha_{ig} q_{sig} - h^e_{si} && \forall s \in S, i \in I && (7)\\
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*Initial data available*

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*Data buffer size: lower and upper bound on stored energy*

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$$e_{s,i+1} = e_{si} + \delta^e_{si} - \sum_{g\in G}\alpha_{ig}q_{sig} - h^e_{si} \qquad \forall s\in S, i\in I \qquad (7)$$

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& d_{s0} = d_{start} && \forall s \in S && (8)\\
& 0 \le d_{si} \le d_{max} && \forall s \in S, i \in I && (9)\\
& d_{s,i+1} = d_{si} + \delta^d_{si} && && \\
& \qquad - \sum_{g\in G} \eta_{i,g} q_{sig} - h^d_{si} && \forall s \in S, i \in I && (10)\\
& 0 \le x_{sig} \le 1 && \forall s \in S, i \in I, g \in G && (11)\\
& q_{sig}, e_{si}, d_{si}, h^e_{si}, h^d_{si} \in \mathbb{R}^+ && \forall s \in S, i \in I, g \in G && (12)
\end{aligned}$$

*Variable limits*

Parameters:

$\gamma_{sig}$: Indicator if in view during i

$\eta_{ig}$: Download efficiency (% received)

$t_i$: Duration of interval (sec)

$\phi_{ig}$: Download rate (bits/sec)

$\alpha_{ig}$: Download cost (joules/bit)

$\delta_{si}^e$: net amount of energy acquired (joules)

$\delta_{si}^d$: net amount of data acquired (bits)

Variables:

$x_{sig}$: Percent of interval used for download

$q_{sig}$: Amount of data downloaded (bits)

$e_{si}$: Energy available (joules)

$d_{si}$: Data available (bits)

$h_{si}^e$: excess energy spilled

$h_{si}^d$: excess data spilled