Optimally Scheduling Satellite Communications

Brian Lemay
University of Michigan
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Co-authors: Prof Amy Cohn, Prof James Cutler, Jeremy Castaing, Robert Zideck
Motivations

**QB50 Mission**
**Expected to Launch in 2016**

<table>
<thead>
<tr>
<th>Year</th>
<th>Approx. Launches Per Year:</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006-2012</td>
<td>25</td>
</tr>
<tr>
<td>2013</td>
<td>92</td>
</tr>
<tr>
<td>2014</td>
<td>140</td>
</tr>
</tbody>
</table>

Goal and Outline

• Schedule downloads for a multi-satellite, multi-ground station system.

Inputs
- Orbits
- Contact Times

Model
- Decisions
- Constraints
- Objective

Results
- Run Times
- Analysis
Orbits and Ground Stations

Satellite ground tracks
Abstract Representation of Contact Times

\[ \sum_{i=1}^{t_1} \text{Ground Station } i \]

Ground Station 1
Ground Station 2
Ground Station 3

Satellite 1
Abstract Representation of Contact Times

$\sum_{i=1}^{3} t_i$

Ground Station 1

Ground Station 2

Ground Station 3

$i=1$  $i=2$  $i=3$

Satellite 1
Abstract Representation of Contact Times
Abstract Representation of Contact Times

- Ground Station 1
- Ground Station 2
- Ground Station 3

Satellite 1
Satellite 2
Abstract Representation of Contact Times

Ground Station 1

Ground Station 2

Ground Station 3

$t_0$ $t_1$ $t_2$ $t_3$ $t_4$

$i=1$ $i=2$ $i=3$ $i=4$

- Satellite 1
- Satellite 2
- Satellite 3
Abstract Representation of Contact Times

Ground Station 1

Ground Station 2

Ground Station 3

$i=1$  $i=2$  $i=3$  $i=4$

$t_0$  $t_1$  $t_2$  $t_3$  $t_4$

Satellite 1
Satellite 2
Satellite 3
Abstract Representation of Contact Times

$\sum_{i=1}^{4} t_i$

Ground Station 1

Ground Station 2

Ground Station 3

Satellite 1

Satellite 2

Satellite 3
**The Multi-Satellite, Multi-Ground Station Scheduling Problem (MMSP)**

- **Objective:** to maximize the total amount of data downloaded over the planning horizon

- **Subject to:**
  - Download opportunities
  - Conflicts
  - Energy & Data Dynamics
  - Ground Stations Characteristics:
    - Download Rate (bits/sec)
    - Download Cost (joules/bit)
    - Efficiency (percentage of download actually received)
Download Decisions

• $x_{sig}$ – Percentage of interval $i$ that satellite $s$ downloads to ground station $g$

• $q_{sig}$ – Amount of data downloaded from satellite $s$ during interval $i$ to ground station $g$
A Simple Schedule

<table>
<thead>
<tr>
<th>Ground Station 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval 1</td>
</tr>
<tr>
<td>( x )</td>
</tr>
<tr>
<td>Satellite 1</td>
</tr>
<tr>
<td>Satellite 2</td>
</tr>
</tbody>
</table>

![Diagram showing ground station and satellite connections]

- Ground Station 1
- Satellite 1
- Satellite 2

Download
Energy and Data Dynamics

\[ e_{(i+1)} = \min \{ e_{max}, e_i + \delta_{i}^{e+} - \delta_{i}^{e-} - \sum_{g} \alpha_{ig} q_{ig} \} \]

Energy available is based on previous energy + energy gained - energy spent unrelated to downloads - energy used for downloads

\[ d_{(i+1)} = \min \{ d_{max}, d_i + \delta_{i}^{d+} - \delta_{i}^{d-} - \sum_{g} \eta_{ig} q_{ig} \} \]

Data available is based on previous data + data gained - data lost - data successfully downloaded

Parameters:
- \( \delta_{i}^{e+} \): net amount of energy acquired (joules)
- \( \delta_{i}^{e-} \): net amount of energy acquired (joules)
- \( \delta_{i}^{d+} \): net amount of data acquired (bits)
- \( \delta_{i}^{d-} \): net amount of data acquired (bits)
- \( \alpha_{ig} \): Download cost (joules/bit)
- \( \eta_{ig} \): Download efficiency (% received)

Variables:
- \( q_{ig} \): Amount of data downloaded (bits)
- \( e_i \): Energy available (joules)
- \( d_i \): Data available (bits)
What do we want to look at?

- **Tractability**: Can it solve real-world problems in a reasonable amount of time?

- **Quality**: How much value does it add over a simpler, more traditional scheduling approach?

- **Applications**: How can it be used as a mechanism for conducting analysis on a mission?
Generating Data for Testing

• Generated orbital information for each of the 50 satellites for the QB 50 mission
  – Determine which ground stations are in view
  – Determine when in view of the Sun

• Ground stations are located at each participating university and are randomly categorized as good, average, or poor
Solve Times
(50 Satellites, 50 Ground Stations)

MMSP model can be solved quickly for realistic planning horizons
Comparison Method

• Greedy Heuristic
  – For each time interval:
    • Identify maximum possible download for each satellite
    • Schedule the maximum download
    • Eliminate the participating satellite and ground station from the list of potential downloads
    • Repeat until no more feasible downloads
## Value of Optimization

<table>
<thead>
<tr>
<th>Energy Gain (joules/sec)</th>
<th>Optimization Total Download (MB)</th>
<th>Greedy Total Download (MB)</th>
<th>Improvement Over Greedy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>104.97</td>
<td>104.91</td>
<td>0.1%</td>
</tr>
<tr>
<td>0.04</td>
<td>104.97</td>
<td>104.91</td>
<td>0.1%</td>
</tr>
<tr>
<td>0.03</td>
<td>104.97</td>
<td>104.91</td>
<td>0.1%</td>
</tr>
<tr>
<td>0.025</td>
<td>104.97</td>
<td>102.73</td>
<td>2.2%</td>
</tr>
<tr>
<td>0.02</td>
<td>104.97</td>
<td>86.27</td>
<td>21.7%</td>
</tr>
<tr>
<td>0.01</td>
<td>104.97</td>
<td>69.36</td>
<td>51.3%</td>
</tr>
<tr>
<td>0.005</td>
<td>54.94</td>
<td>35.08</td>
<td>56.6%</td>
</tr>
<tr>
<td>0.004</td>
<td>27.47</td>
<td>17.55</td>
<td>56.5%</td>
</tr>
<tr>
<td>0.003</td>
<td>21.98</td>
<td>14.04</td>
<td>56.5%</td>
</tr>
<tr>
<td>0.002</td>
<td>16.48</td>
<td>10.53</td>
<td>56.6%</td>
</tr>
<tr>
<td>0.001</td>
<td>10.99</td>
<td>7.02</td>
<td>56.5%</td>
</tr>
</tbody>
</table>

1. Satellites can collect less energy
2. Optimization provides significant value when they do
## Value of Optimization

<table>
<thead>
<tr>
<th>Data Gain</th>
<th>Improvement Over Greedy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case (0.3 Mb/Day)</td>
<td>0.2%</td>
</tr>
<tr>
<td>x5</td>
<td>0.2%</td>
</tr>
<tr>
<td>x10</td>
<td>0.2%</td>
</tr>
<tr>
<td>x15</td>
<td>4.5%</td>
</tr>
<tr>
<td>x20</td>
<td>32.8%</td>
</tr>
<tr>
<td>x25</td>
<td>61.0%</td>
</tr>
<tr>
<td>x30</td>
<td>61.0%</td>
</tr>
<tr>
<td>x35</td>
<td>61.0%</td>
</tr>
</tbody>
</table>

Optimization provides significant value when satellites collect more data than what is currently planned.
Analysis of Deployment Options
Analysis of Deployment Options

Data Downloaded (bits)

Week Number

Total Download (MB)

1-minute 5-minutes 10-minutes

126.7 141.3 151.8

Improvement (%)

- 11.5% 19.8%

10-minute spacing enables more data to be downloaded
Conclusions and Future Work

• Capable of quickly solving real problems with clear benefits over more traditional methods
  – Enabled sensitivity analysis for evaluating system capabilities and bottlenecks

• MMSP adds the most value in scenarios where energy is a limiting resource

• Future Work: Stochastic instances, prioritized downloads, fairness of downloads
Thank You!

Contact info:
Brian Lemay: blemay@umich.edu
Amy Cohn: amycohn@umich.edu
James Cutler: jcutler@umich.edu
Jeremy Castaing: jctg@umich.edu
Robert Zideck: robzidek@umich.edu

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National Science Foundation *(CNS-1035236)*
Air Force Research Laboratory *(FA9550-12-1-0401)*
Parameters:
\( \gamma_{sig} \): Indicator if in view during \( i \)
\( \eta_{ig} \): Download efficiency (% received)
\( t_i \): Duration of interval (sec)
\( \phi_{ig} \): Download rate (bits/sec)
\( \alpha_{ig} \): Download cost (joules/bit)
\( \delta_{si}^{e} \): net amount of energy acquired (joules)
\( \delta_{si}^{d} \): net amount of data acquired (bits)

Variables:
\( x_{sig} \): Percent of interval used for download
\( q_{sig} \): Amount of data downloaded (bits)
\( e_{si} \): Energy available (joules)
\( d_{si} \): Data available (bits)
\( h_{si}^{e} \): excess energy spilled
\( h_{si}^{d} \): excess data spilled

Objective: Maximize total download over the planning horizon

\[
\max \sum_{s \in S} \sum_{i \in I} \sum_{g \in G} \eta_{ig} q_{sig}
\]
Parameters:
\( \gamma_{\text{sig}} \): Indicator if in view during \( i \)
\( \eta_{ig} \): Download efficiency (% received)
\( t_i \): Duration of interval (sec)
\( \phi_{ig} \): Download rate (bits/sec)
\( \alpha_{ig} \): Download cost (joules/bit)
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\( e_{si} \): Energy available (joules)
\( d_{si} \): Data available (bits)
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\( h_{si}^{d} \): excess data spilled

\[
\max \sum_{s \in S} \sum_{i \in I} \sum_{g \in G} \eta_{ig} q_{sig} \\
\quad \quad x_{\text{sig}} \leq \gamma_{\text{sig}} \quad \forall s \in S, i \in I, g \in G \quad (1)
\]

Can only download if satellite is in range of ground station
\[
\max \sum_{s \in S} \sum_{i \in I} \sum_{g \in G} \eta_{ig} q_{sig}
\]

\[
x_{sig} \leq \gamma_{sig} \quad \forall s \in S, i \in I, g \in G
\]

\[
\sum_{s \in S} x_{sig} \leq 1 \quad \forall i \in I, g \in G
\]

*Ground stations cannot receive data for more than 100% of each interval*

**Parameters:**
- \( \gamma_{sig} \): Indicator if in view during \( i \)
- \( \eta_{ig} \): Download efficiency (% received)
- \( t_i \): Duration of interval (sec)
- \( \phi_{ig} \): Download rate (bits/sec)
- \( \alpha_{ig} \): Download cost (joules/bit)
- \( \delta_{si}^e \): net amount of energy acquired (joules)
- \( \delta_{si}^d \): net amount of data acquired (bits)

**Variables:**
- \( x_{sig} \): Percent of interval used for download
- \( q_{sig} \): Amount of data downloaded (bits)
- \( e_{si} \): Energy available (joules)
- \( d_{si} \): Data available (bits)
- \( h_{si}^e \): excess energy spilled
- \( h_{si}^d \): excess data spilled
\[
\max \sum_{s \in S} \sum_{i \in I} \sum_{g \in G} \eta_{ig} q_{sig}
\]

\[
x_{sig} \leq \gamma_{sig} \\
\sum_{s \in S} x_{sig} \leq 1 \\
\sum_{g \in G} x_{sig} \leq 1
\]

\textbf{Satellites cannot transmit data for more than 100\% of each interval}

**Parameters:**
- \(\gamma_{sig}\): Indicator if in view during \(i\)
- \(\eta_{ig}\): Download efficiency (% received)
- \(t_i\): Duration of interval (sec)
- \(\phi_{ig}\): Download rate (bits/sec)
- \(\alpha_{ig}\): Download cost (joules/bit)
- \(\delta_{si}^e\): net amount of energy acquired (joules)
- \(\delta_{si}^d\): net amount of data acquired (bits)

**Variables:**
- \(x_{sig}\): Percent of interval used for download
- \(q_{sig}\): Amount of data downloaded (bits)
- \(e_{si}\): Energy available (joules)
- \(d_{si}\): Data available (bits)
- \(h_{si}^e\): excess energy spilled
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\( \eta_{ig} \): Download efficiency (% received)
\( t_i \): Duration of interval (sec)
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\( q_{sig} \): Amount of data downloaded (bits)
\( e_{si} \): Energy available (joules)
\( d_{si} \): Data available (bits)
\( h_{si}^e \): excess energy spilled
\( h_{si}^d \): excess data spilled

\[
\max \sum_{s \in S} \sum_{i \in I} \sum_{g \in G} \eta_{ig} q_{sig}
\]

\[
x_{sig} \leq \gamma_{sig} \quad \forall s \in S, i \in I, g \in G \tag{1}
\]

\[
\sum_{s \in S} x_{sig} \leq 1 \quad \forall i \in I, g \in G \tag{2}
\]

\[
\sum_{g \in G} x_{sig} \leq 1 \quad \forall s \in S, i \in I \tag{3}
\]

\[
q_{sig} \leq t_i \phi_{ig} x_{sig} \quad \forall s \in S, i \in I, g \in G \tag{4}
\]

Download amount is limited by length of interval and download rate
\[
\max \sum_{s \in S} \sum_{i \in I} \sum_{g \in G} \eta_{ig} q_{sig}
\]

\begin{align*}
    x_{sig} & \leq \gamma_{sig} & \forall s \in S, i \in I, g \in G \quad (1) \\
    \sum_{s \in S} x_{sig} & \leq 1 & \forall i \in I, g \in G \quad (2) \\
    \sum_{g \in G} x_{sig} & \leq 1 & \forall s \in S, i \in I \quad (3) \\
    q_{sig} & \leq t_i \phi_{ig} x_{sig} & \forall s \in S, i \in I, g \in G \quad (4) \\
    e_{s0} & = e_{\text{start}} & \forall s \in S \quad (5) \\
\end{align*}

Initial energy available

Parameters:
\begin{itemize}
    \item $\gamma_{sig}$: Indicator if in view during $i$
    \item $\eta_{ig}$: Download efficiency (% received)
    \item $t_i$: Duration of interval (sec)
    \item $\phi_{ig}$: Download rate (bits/sec)
    \item $\alpha_{ig}$: Download cost (joules/bit)
    \item $\delta_{si}^e$: net amount of energy acquired (joules)
    \item $\delta_{si}^d$: net amount of data acquired (bits)
\end{itemize}

Variables:
\begin{itemize}
    \item $x_{sig}$: Percent of interval used for download
    \item $q_{sig}$: Amount of data downloaded (bits)
    \item $e_{si}$: Energy available (joules)
    \item $d_{si}$: Data available (bits)
    \item $h_{si}^e$: excess energy spilled
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\end{itemize}
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\[
\max \sum_{s \in S} \sum_{i \in I} \sum_{g \in G} \eta_{ig} q_{sig}
\]

\[
x_{sig} \leq \gamma_{sig} \quad \forall s \in S, i \in I, g \in G \quad (1)
\]

\[
\sum_{s \in S} x_{sig} \leq 1 \quad \forall i \in I, g \in G \quad (2)
\]

\[
\sum_{g \in G} x_{sig} \leq 1 \quad \forall s \in S, i \in I \quad (3)
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\[
q_{sig} \leq t_i \phi_{ig} x_{sig} \quad \forall s \in S, i \in I, g \in G \quad (4)
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\[
e_{s0} = e_{start} \quad \forall s \in S \quad (5)
\]

\[
e_{min} \leq e_{si} \leq e_{max} \quad \forall s \in S, i \in I \quad (6)
\]

Energy buffer size: lower and upper bound on stored energy
\[
\max \sum_{s \in S} \sum_{i \in I} \sum_{g \in G} \eta_{ig} q_{sig}
\]

\[x_{sig} \leq \gamma_{sig} \quad \forall s \in S, i \in I, g \in G \tag{1}\]
\[\sum_{s \in S} x_{sig} \leq 1 \quad \forall i \in I, g \in G \tag{2}\]
\[\sum_{g \in G} x_{sig} \leq 1 \quad \forall s \in S, i \in I \tag{3}\]
\[q_{sig} \leq t_i \phi_{ig} x_{sig} \quad \forall s \in S, i \in I, g \in G \tag{4}\]
\[e_{s0} = e_{\text{start}} \quad \forall s \in S \tag{5}\]
\[e_{\text{min}} \leq e_{si} \leq e_{\text{max}} \quad \forall s \in S, i \in I \tag{6}\]
\[e_{s,i+1} = e_{si} + \delta_{si}^e - \sum_{g \in G} \alpha_{ig} q_{sig} - h_{si}^e \quad \forall s \in S, i \in I \tag{7}\]

Energy available is based on previous energy + energy gained - energy used - any excess energy that must be spilled

Parameters:
\- \(\gamma_{sig}\): Indicator if in view during \(i\)
\- \(\eta_{ig}\): Download efficiency (% received)
\- \(t_i\): Duration of interval (sec)
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- \( e_{si} \): Energy available (joules)
- \( d_{si} \): Data available (bits)
- \( h_{si}^{e} \): excess energy spilled
- \( h_{si}^{d} \): excess data spilled

\( x_{sig} \leq \gamma_{sig} \) \hspace{1cm} \forall s \in S, i \in I, g \in G \hspace{1cm} (1)

\( \sum_{s \in S} x_{sig} \leq 1 \) \hspace{1cm} \forall i \in I, g \in G \hspace{1cm} (2)

\( \sum_{g \in G} x_{sig} \leq 1 \) \hspace{1cm} \forall s \in S, i \in I \hspace{1cm} (3)

\( q_{sig} \leq t_i \phi_{ig} x_{sig} \) \hspace{1cm} \forall s \in S, i \in I, g \in G \hspace{1cm} (4)

\( e_{s0} = e_{start} \) \hspace{1cm} \forall s \in S \hspace{1cm} (5)

\( e_{min} \leq e_{si} \leq e_{max} \) \hspace{1cm} \forall s \in S, i \in I \hspace{1cm} (6)

\( e_{s,i+1} = e_{si} + \delta_{si}^{e} - \sum_{g \in G} \alpha_{ig} q_{sig} - h_{si}^{e} \) \hspace{1cm} \forall s \in S, i \in I \hspace{1cm} (7)

\( d_{s0} = d_{start} \) \hspace{1cm} \forall s \in S \hspace{1cm} (8)

Initial data available
\[
\max \sum_{s \in S} \sum_{i \in I} \sum_{g \in G} \eta_{ig} q_{sig} \\
\]

\[
x_{sig} \leq \gamma_{sig} \quad \forall s \in S, i \in I, g \in G \tag{1}
\]
\[
\sum_{s \in S} x_{sig} \leq 1 \quad \forall i \in I, g \in G \tag{2}
\]
\[
\sum_{g \in G} x_{sig} \leq 1 \quad \forall s \in S, i \in I \tag{3}
\]
\[
qu_{sig} \leq t_i \phi_{ig} x_{sig} \quad \forall s \in S, i \in I, g \in G \tag{4}
\]
\[
e_{s0} = e_{start} \quad \forall s \in S \tag{5}
\]
\[
e_{min} \leq e_{si} \leq e_{max} \quad \forall s \in S, i \in I \tag{6}
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e_{s, i+1} = e_{si} + \delta_{si}^e - \sum_{g \in G} \alpha_{ig} q_{sig} - h_{si}^e \quad \forall s \in S, i \in I \tag{7}
\]
\[
d_{s0} = d_{start} \quad \forall s \in S \tag{8}
\]
\[
0 \leq d_{si} \leq d_{max} \quad \forall s \in S, i \in I \tag{9}
\]

**Data buffer size:** lower and upper bound on stored energy

**Parameters:**
- \( \gamma_{sig} \): Indicator if in view during \( i \)
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**Variables:**
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q_{sig} \leq t_i \phi_{ig} x_{sig} \quad \forall s \in S, i \in I, g \in G
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e_{s0} = e_{start} \quad \forall s \in S
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e_{min} \leq e_{si} \leq e_{max} \quad \forall s \in S, i \in I
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e_{s,i+1} = e_{si} + \delta_{si}^e - \sum_{g \in G} \alpha_{ig} q_{sig} - h_{si}^e \quad \forall s \in S, i \in I
\]

\[
d_{s0} = d_{start} \quad \forall s \in S
\]

\[
0 \leq d_{si} \leq d_{max} \quad \forall s \in S, i \in I
\]

\[
d_{s,i+1} = d_{si} + \delta_{si}^d - \sum_{g \in G} \eta_{i,g} q_{sig} - h_{si}^d \quad \forall s \in S, i \in I
\]

Data available is based on previous data + data gained - data used - any excess data that must be spilled

**Parameters:**
- \(\gamma_{sig}\): Indicator if in view during \(i\)
- \(\eta_{ig}\): Download efficiency (% received)
- \(t_i\): Duration of interval (sec)
- \(\phi_{ig}\): Download rate (bits/sec)
- \(\alpha_{ig}\): Download cost (joules/bit)
- \(\delta_{si}^e\): net amount of energy acquired (joules)
- \(\delta_{si}^d\): net amount of data acquired (bits)

**Variables:**
- \(x_{sig}\): Percent of interval used for download
- \(q_{sig}\): Amount of data downloaded (bits)
- \(e_{si}\): Energy available (joules)
- \(d_{si}\): Data available (bits)
- \(h_{si}^e\): excess energy spilled
- \(h_{si}^d\): excess data spilled
\[
\max \sum_{s \in S} \sum_{i \in I} \sum_{g \in G} \eta_{ig} q_{sig}
\]

\[
x_{sig} \leq \gamma_{sig} \quad \forall s \in S, \ i \in I, \ g \in G
\]

\[
\sum_{s \in S} x_{sig} \leq 1 \quad \forall i \in I, \ g \in G
\]

\[
\sum_{g \in G} x_{sig} \leq 1 \quad \forall s \in S, \ i \in I
\]

\[
q_{sig} \leq t_i \phi_{ig} x_{sig} \quad \forall s \in S, \ i \in I, \ g \in G
\]

\[
e_{s0} = e_{start} \quad \forall s \in S
\]

\[
e_{min} \leq e_{si} \leq e_{max} \quad \forall s \in S, \ i \in I
\]

\[
e_{s,i+1} = e_{si} + \delta_{si}^e - \sum_{g \in G} \alpha_{ig} q_{sig} - h_{si}^e \quad \forall s \in S, \ i \in I
\]

\[
d_{s0} = d_{start} \quad \forall s \in S
\]

\[
0 \leq d_{si} \leq d_{max} \quad \forall s \in S, \ i \in I
\]

\[
d_{s,i+1} = d_{si} + \delta_{si}^d - \sum_{g \in G} \eta_{i,g} q_{sig} - h_{si}^d \quad \forall s \in S, \ i \in I
\]

\[
0 \leq x_{sig} \leq 1 \quad \forall s \in S, \ i \in I, \ g \in G
\]

\[
q_{sig}, e_{si}, d_{si}, h_{si}^e, h_{si}^d \in \mathbb{R}^+ \quad \forall s \in S, \ i \in I, \ g \in G
\]

**Parameters:**
- \( \gamma_{sig} \): Indicator if in view during \( i \)
- \( \eta_{ig} \): Download efficiency (% received)
- \( t_i \): Duration of interval (sec)
- \( \phi_{ig} \): Download rate (bits/sec)
- \( \alpha_{ig} \): Download cost (joules/bit)
- \( \delta_{si}^e \): net amount of energy acquired (joules)
- \( \delta_{si}^d \): net amount of data acquired (bits)

**Variables:**
- \( x_{sig} \): Percent of interval used for download
- \( q_{sig} \): Amount of data downloaded (bits)
- \( e_{si} \): Energy available (joules)
- \( d_{si} \): Data available (bits)
- \( h_{si}^e \): excess energy spilled
- \( h_{si}^d \): excess data spilled