Shift Scheduling in Pediatric Emergency Medicine

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Content

- Background
- Motivation
- Formulations
  - Weighted Sum method
  - Metric constraints method
- Result
- Future Research
  - Pareto method

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Resident Responsibilities in the U-M Pediatric Emergency Department

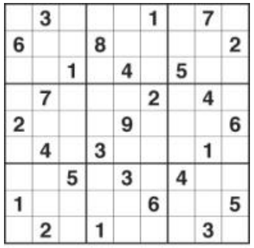
- 3-7 year medical training program
  - Responsibilities differ by residency year
- Balancing patient care and educational requirements
  - In hospital
    - Caring for patients
    - Teaching medical students
    - Learning from attending physicians
  - Out of hospital
    - Community clinics
    - Conferences
    - Other educational requirements

Pediatric ED: Scheduling Considerations

- All shifts assigned to a resident
- Appropriate coverage
  - e.g. certain shifts require a senior resident
- ACGME rules (similar to ABET for engineering)
  - e.g. 10 hour break rule
- Several different residency programs
  - Pediatrics (PED)
  - Family practice (FP)
  - Emergency medicine (EM)
- And others

Motivation

- **Scheduling residents**
  - Complicated requirements (UM Pediatric ED)
    - 25 governing rules and preferences
      - Educational goals
      - Patient care
      - Regularization / Safety
  - Chief resident built monthly schedule by hand
    - Time consuming process: 20 - 25 hours / month
    - Transfer every year: no scheduling experience in July
    - Guess and check: errors / tedious correction process



Mixed Integer Programming

Motivation

- **Practical Significance**
  - Poor-quality schedule
    - Residents: decreased interest in learning
    - Patients: adverse health events

    (Smith-Coggins R, et. al. (1994) : "Relationship of day versus night sleep to physician performance and mood." Annals of Emergency Medicine)
- **Goals**
  - Solves for feasible schedule quickly
  - Create a good quality schedule with no violations

Quality

Time

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Formulation: Problem Size

- **Sets**
  - R: set of residents
    - 15-25 residents
  - D: set of days in the schedule
    - 35 days
  - S: set of shifts
    - 8 shifts
- **Decision Variables**
  - Binary: $x_{rds} \in \{0, 1\}$
    - 1 if resident r works shift s on day d
    - 0 otherwise

Residents Name				
Smith	Sanchez	Chen	Shah	...

	27th	...	1st	...	31st
7a-4p	Shah	...		...	
9a-6p	Joe	...		...	Shah
10a-7p		...		...	
12p-9p	Chen	...		...	Chen
4p-1a	Smith	...	Sanchez	...	
5p-2a		...		...	Sanchez
8p-5a	Sanchez	...	Smith	...	Smith
11p-8a		...	Chen	...	Joe

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Objectives: Shift Fairness

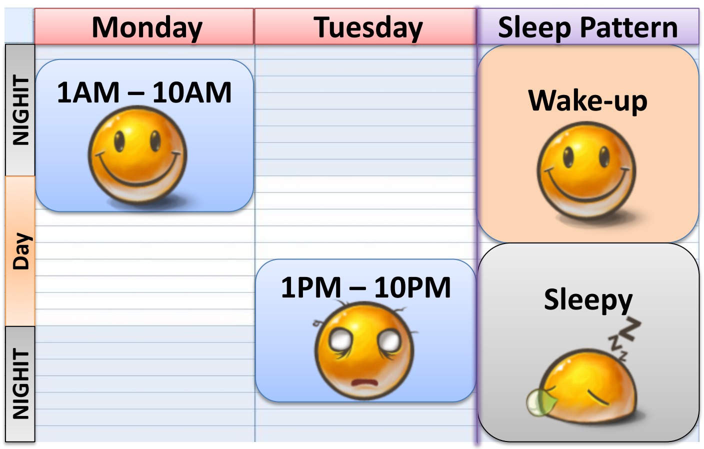
- **Total / night shift equity**
  - Equal opportunities for training
  - Improved morale and learning ability

Resident Name	Smith	Jones	Chen	Joe
Night Shifts / Total Shifts	0 / 7	1 / 7	1 / 7	5 / 7
Fairness				

  - Total shift equity (TSE): $(\sum t_{ij}\ , t_{ij} = \left|D_i - D_j\right|,\ i > j)$
  - Night shift equity (NSE): $(\sum n_{ij}\ ,\ n_{ij} = \left|N_i - N_j\right|,\ i > j)$

Objectives: Undesired Shift

- **Bad sleep patterns and post-clinic shifts**
  - **Improves resident quality of life**
  - **Increases patient safety**



Bad sleep pattern

Post-Clinic shift

  - **Minimum bad sleep patterns (BSP):** $\left(\sum count \right)$

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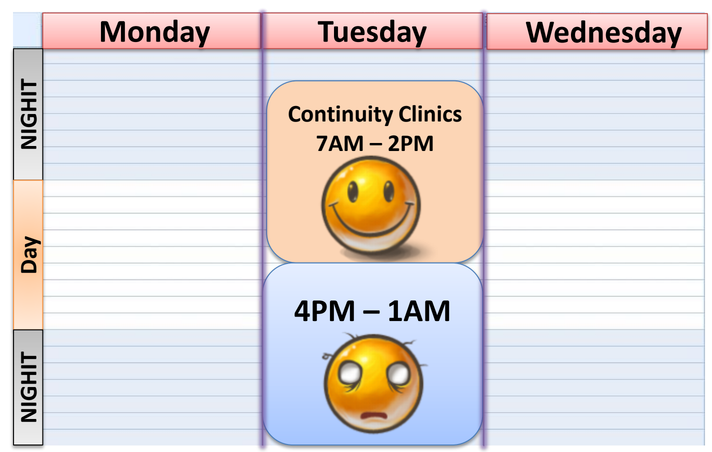
Bad sleep pattern



Post-Clinic shift

  - **Minimum post-clinic shifts (PCC):** $(\sum count)$

Formulation: Constraints

- **Constraints (rules/requirements)**
  - **One resident assigned to each shift in the month**
    - $\sum_{r \in \{\text{all}\}} x_{rds} = 1, \quad \forall d, \forall s$
  - **Meets shift requests**
    - $x_{rds} = 0, \quad \forall r, \forall d, s \in \{\text{day off, conferences, continuity clinic}\}$
  - **Ensure resident type appropriate for shift**
    - $\sum_{r \in \{\text{PED}\}} \sum_{s \in \text{P}} x_{rsd} \geq 1, \forall \text{d}, \text{P} = \{\{7\text{a},9\text{a}\}, \{4\text{p},5\text{p}\}, \{8\text{p},11\text{p}\}\}$
  - **Intern-forbidden shifts**
    - $\sum_{r \in \{\text{interns}\}} \sum_{d} x_{rsd} = 0, \forall s \in \{7\text{a}, 11\text{p}\}$
  - **And others**

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Multi-Criteria Problem

- **Multi-Criteria schedule**
  - Metrics for UM Pediatric Emergency Department
    - Total shift equity (TSE)
    - Night shift equity (NSE)
    - Minimum bad sleep patterns (BSP)
    - Minimum post-clinic shifts (PCC)
    - ⋮

Weights?
Preferences?
Trade-off?

Multi-objective Mathematical Programming

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Weighted Sum Method

$$\begin{aligned} &\mathbf{Min}\ \boldsymbol{w_1}(TSE) + \boldsymbol{w_2}(NSE) + \boldsymbol{w_3}(BSP) + \boldsymbol{w_4}(PCC) \\ &\mathbf{s.t.} \qquad\qquad \text{"rules/requirements"} \\ &\qquad\qquad\qquad x_{rds} \in \{0,1\} \end{aligned}$$

- **Quantifying preferences (w_i) is difficult**
  - **Weights are subjective and difficult to quantify**
    - **Resulting schedule does not match their intentions**
  - **Various measurement units**
    - **Equity (σ , $\mathrm{Max}|\mathrm{diff_{ij}}|$, $\sum|\mathrm{diff_{ij}}|$, …)**

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Optimized Residency Scheduling Assistant (ORSA): Metrics Formulation

- **Feasibility problem**
  - Constraint on metrics

$$
\begin{aligned}
&\textbf{Min } \color{red}{\boldsymbol{w_1}}(TSE) + \color{red}{\boldsymbol{w_2}}(NSE) + \color{red}{\boldsymbol{w_3}}(BSP) + \color{red}{\boldsymbol{w_4}}(PCC) \\
&\textbf{s.t.} \qquad\qquad \text{"rules/requirements"} \\
&\qquad\qquad\quad\;\; x_{rds} \in \{0,1\}
\end{aligned}
$$

- **Benefits of a feasibility problem**
  - More flexible
  - Faster to solve: < 2 sec.
    - Given: 35 days / 20 PEDs / 8 shifts

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&\textbf{s.t.} \quad \text{"rules/requirements"} \\
&\quad x_{rds} \in \{0,1\} \\
&\quad lb_{TSE} \le (TSE) \le ub_{TSE} \\
&\quad lb_{NSE} \le (NSE) \le ub_{NSE} \\
&\quad lb_{BSP} \le (BSP) \le ub_{BSP} \\
&\quad lb_{PCC} \le (PCC) \le ub_{PCC}
\end{aligned}$$

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Optimized Residency Scheduling Assistant (ORSA) : Interactive Improvement

- Example output of metrics
  - Value (Lower bound, Upper bound)

Resident Name	Number of Shifts	Number of Night Shifts	Number of Post CC	Number of Bad Sleep Templates
Smith	8 (7,9)	2 (0,10)	0 (0,1)	1 (0,1)
Sanchez	8 (7,10)	1 (0,10)	0 (0,1)	1 (0,1)
Chen	8 (7,9)	5 (0,10)	1 (0,1)	1 (0,1)
Shah	14 (13,15)	3 (0,10)	1 (0,1)	1 (0,1)
⋮	⋮	⋮	⋮	⋮

- Interactive approach engaging chief resident
  - Iteratively adjust bounds on metric constraints
  - Quickly build high quality schedule

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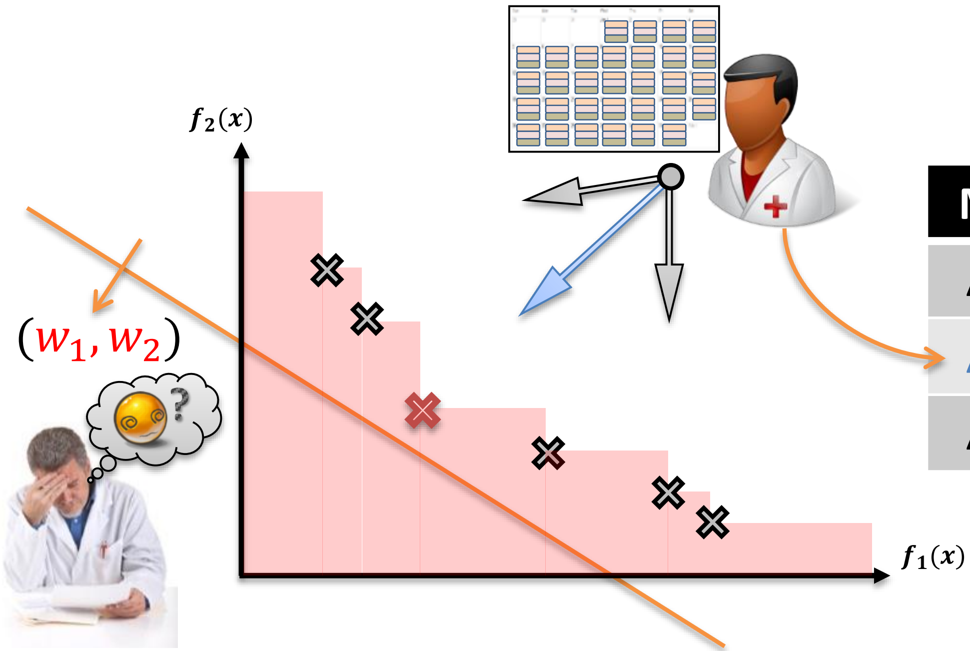
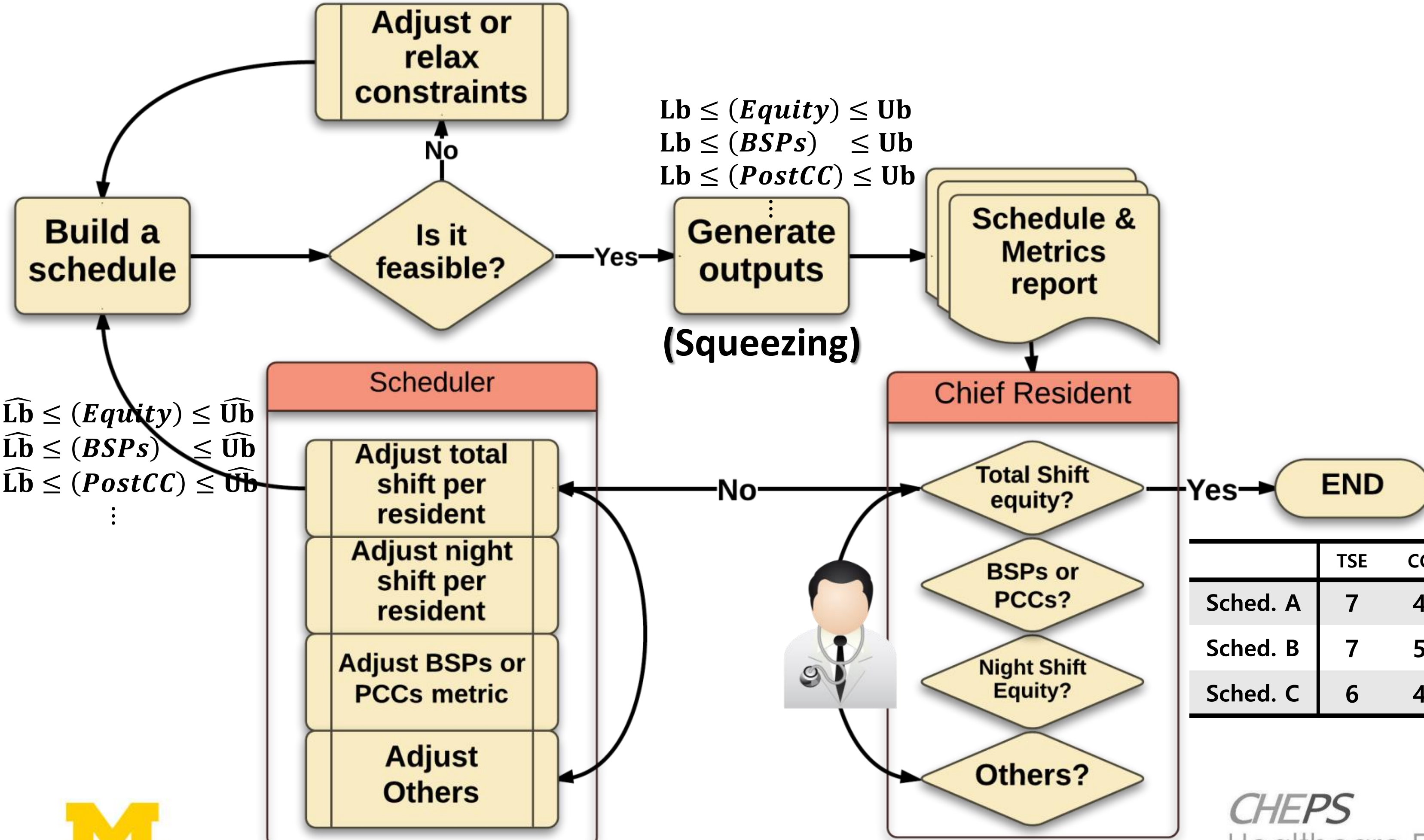
Optimized Residency Scheduling Assistant (ORSA) : Interactive Improvement

- **Interactive Feedback**
  - **Chief resident identifies undesirable qualitative characteristics**



Metrics	$f_1(x)$	$f_2(x)$	$f_3(x)$	…
ΔValue	↑	↓	↑	…
ΔValue	↓	↑	↓	…
ΔValue	↓	↑	↓	…

ORSA Methodology



	TSE	CC	BSPs	...
Sched. A	7	4	2	...
Sched. B	7	5	1	...
Sched. C	6	4	3	...

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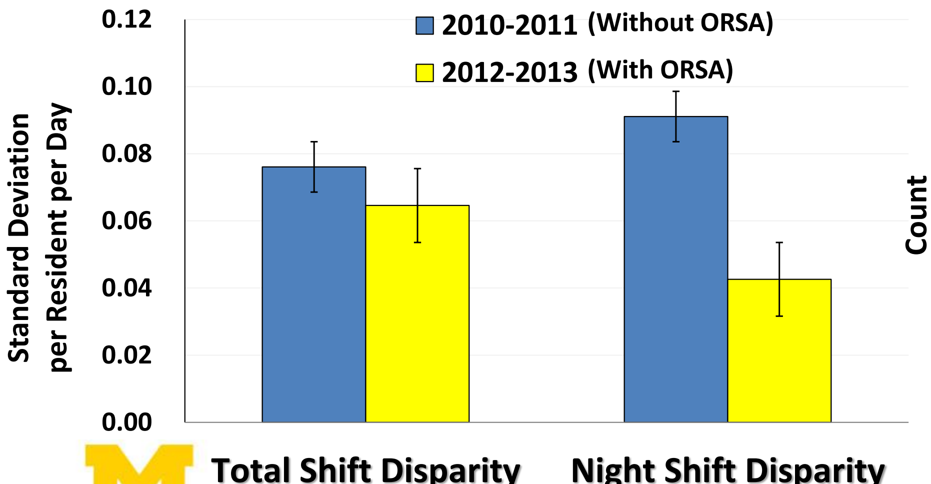
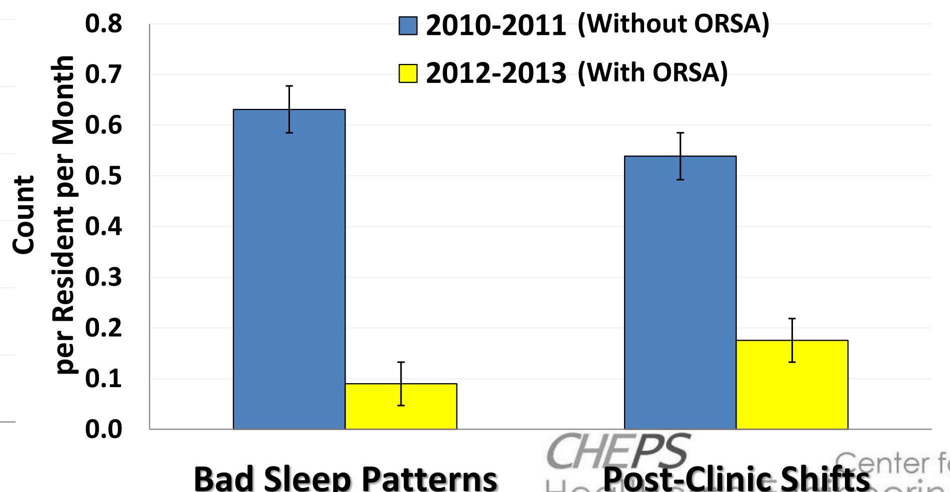
ORSA: Results

- **Our metrics-based scheduling tool:**
  - Reduces time to create schedules

20 hours /month	→	1 hour /month

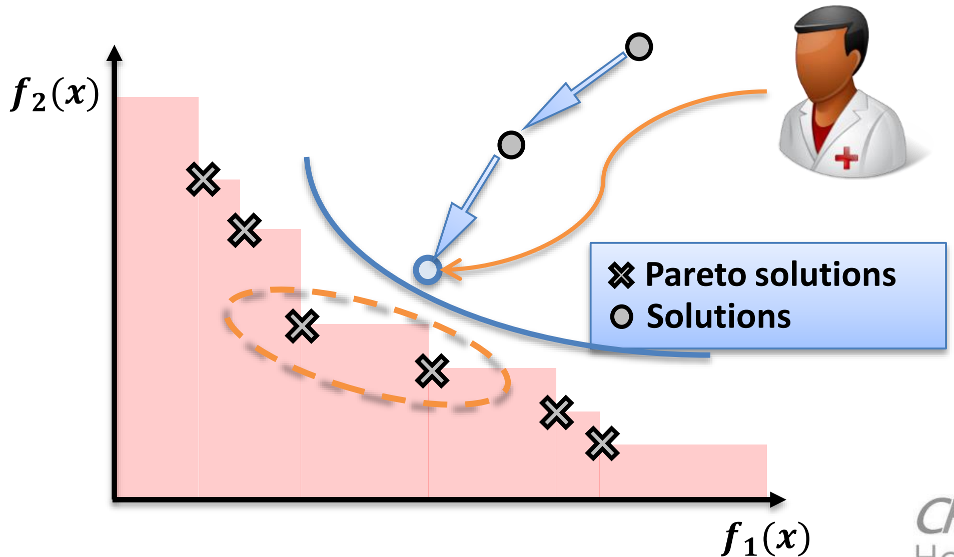
  - Solves a multi-criteria scheduling problem





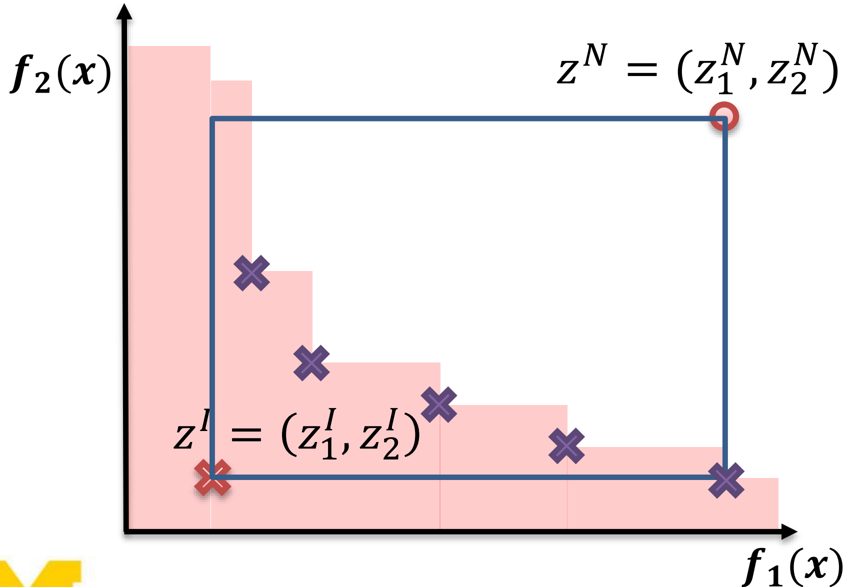
ORSA: Limitations

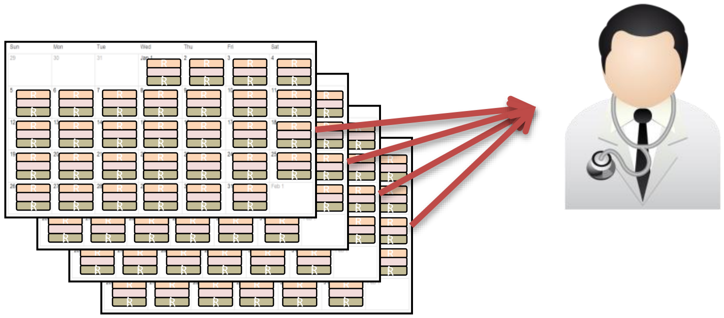
- **Myopic Solution**
  - **Non-Pareto solution could be selected by a chief residents**
    - **Never see the whole picture (the set of Pareto solutions)**
    - **The most preferred solution is "most preferred" with respect to their satisfaction**



Next Step

- **Pareto Solutions**
  - **Generate the Pareto solutions of the problem (all of them or a sufficient representation)**
    - **Select the most preferred one among them**





Efficient Schedules

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Pareto: Bi-Objective Problem

- **Notation**
  - $\mathcal{H}$: Solution Space, the set of feasible solutions
  - $\mathcal{P}$: Pareto Set
  - $z_i = f_i(x)$: ith integer objective function, $\in \mathbb{Z}$
  - Dominance ($\prec$) : $x \prec x'$ if and only if $z_i \leq z_i'$ where at least one inequality is strict
- **Bi-Objective Problem**

$$\min f(x) = (f_1(x), f_2(x))$$
$$s.t.\ x \in \mathcal{H}$$

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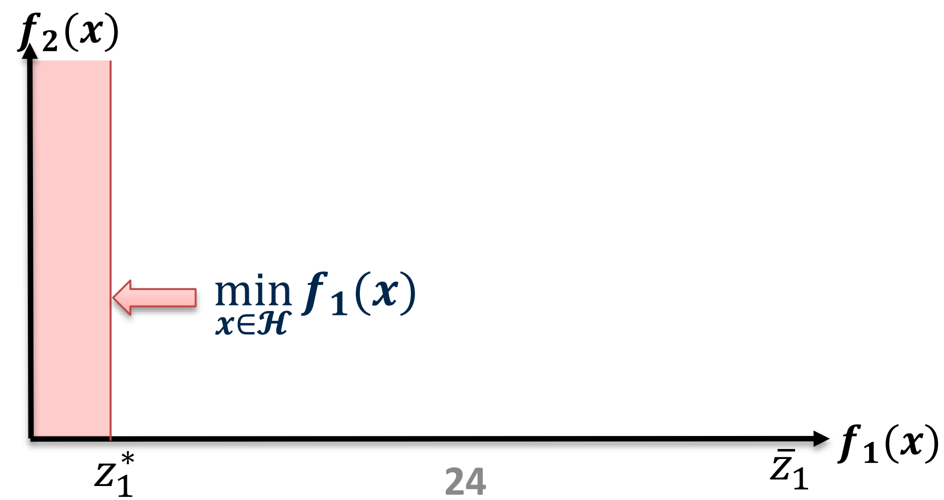
Pareto: Bi-Objective Problem

- **Pareto Square Region**
  - **Ideal Point:**
    - $\mathbf{z}_1^* = \min_{\boldsymbol{x} \in \mathcal{H}} \boldsymbol{f}_1(\boldsymbol{x})$ and $\mathbf{z}_2^* = \min_{x \in \mathcal{H}} f_2(x)$
  - **Nadir Point:**
    - $\bar{z}_1 = \min_{x \in \mathcal{H} \cap f_2(x) = z_2^*} f_1(x)$ and $\bar{z}_2 = \min_{x \in \mathcal{H} \cap f_1(x) = z_1^*} f_2(x)$



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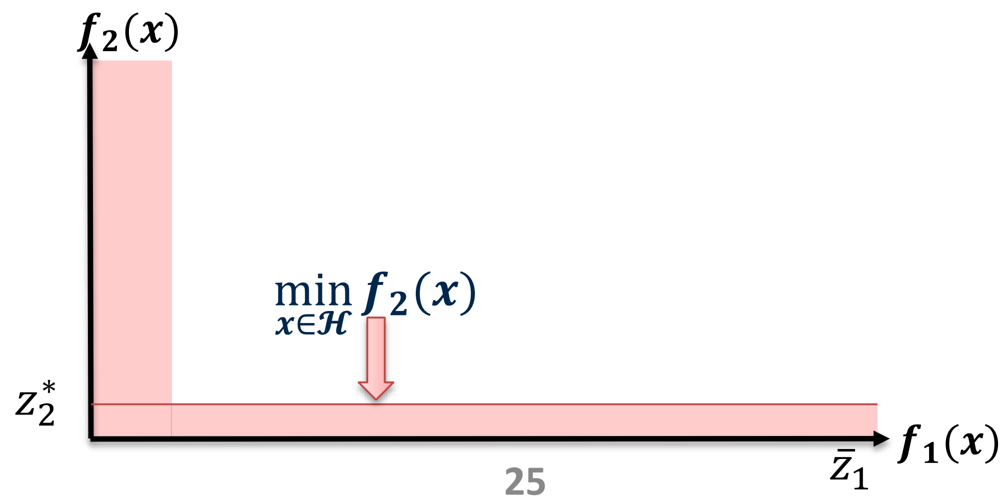

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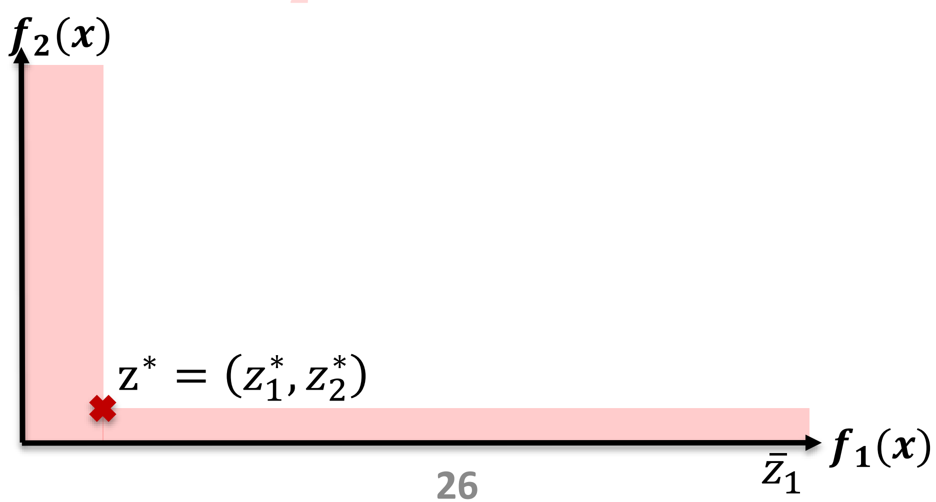

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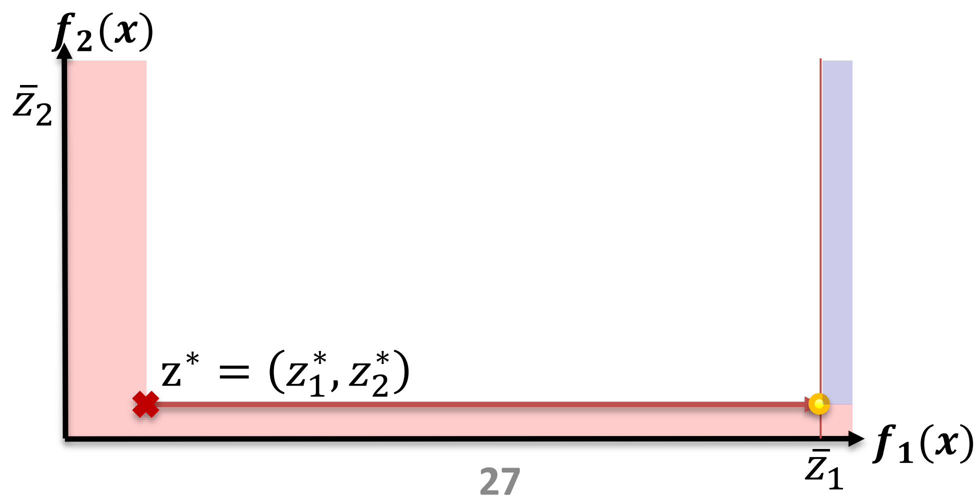

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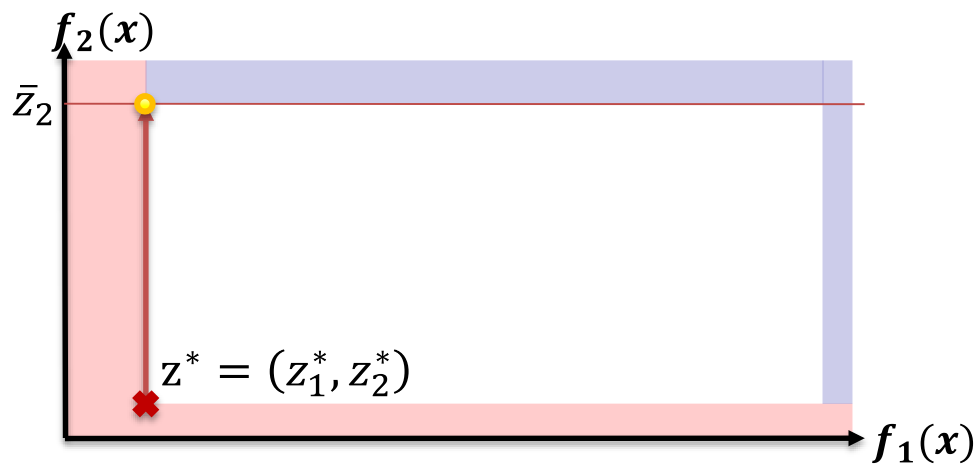

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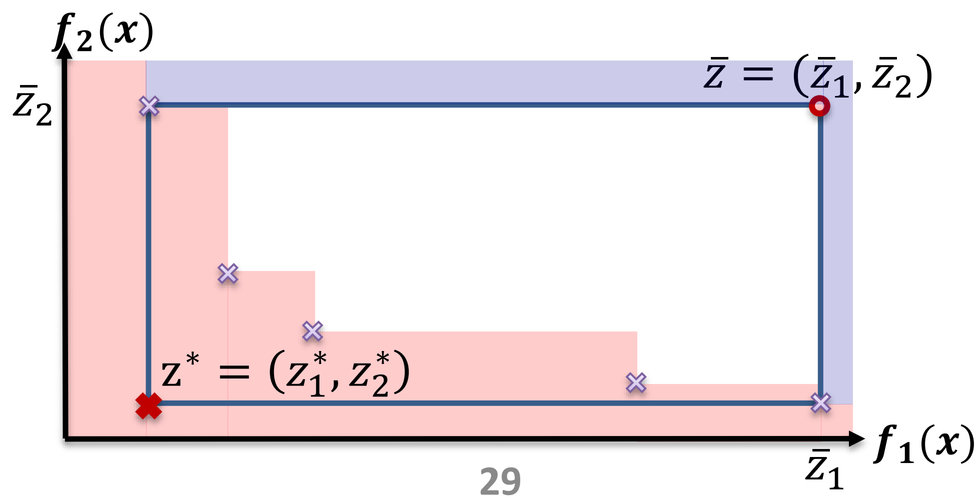

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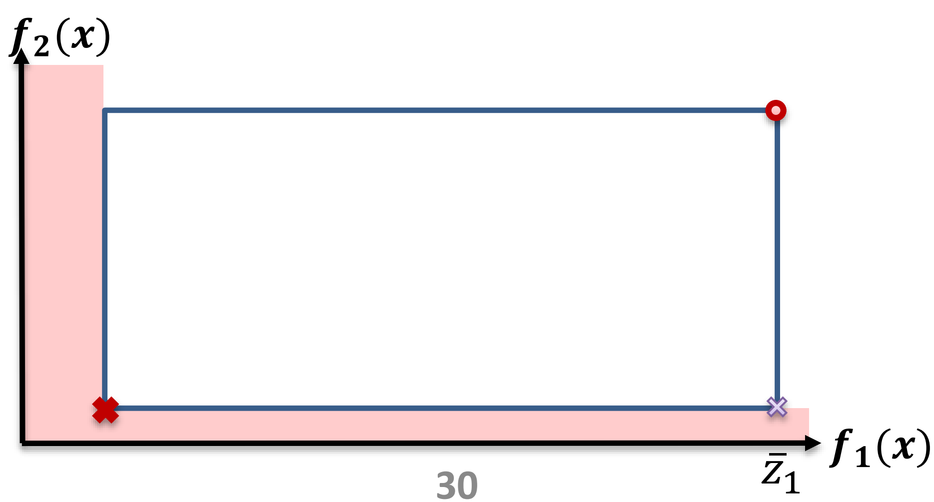
Pareto: Bi-Objective Problem

- **Pareto Squeezing Algorithm**

Algorithm 1 Exact squeezing algorithm for bi-objective problems

Let P is set of pareto solutions we've found;

Compute the ideal (z_1^*, z_2^*) and Nadir $(\bar{z}_1, \bar{z}_2)$ points;
Set $P := \{(\bar{z}_1, z_2^*)\}$ and $\delta := \bar{z}_1 - 1$;
WHILE $\delta \geq z_1^*$
    Solve $P_2(\delta)$ and get optimal solution (z_1^i, z_2^i) to $P_2(\delta)$;
    //Given (z_1^i, z_2^i), Find a left-botton corner $(\hat{z}_1^i)$ in the Pareto set;
    Solve $SQZ_1(z_2^i)$ and get optimal solution $(\hat{z}_1^i, z_2^i)$ to $SQZ_1(z_2^i)$;
END WHILE
Set $P := P + (\hat{z}_1^i, z_2^i)$ and $\delta = \hat{z}_1^i - 1$;
GO Step 2 UNTIL $z_1 = z_1^*$;



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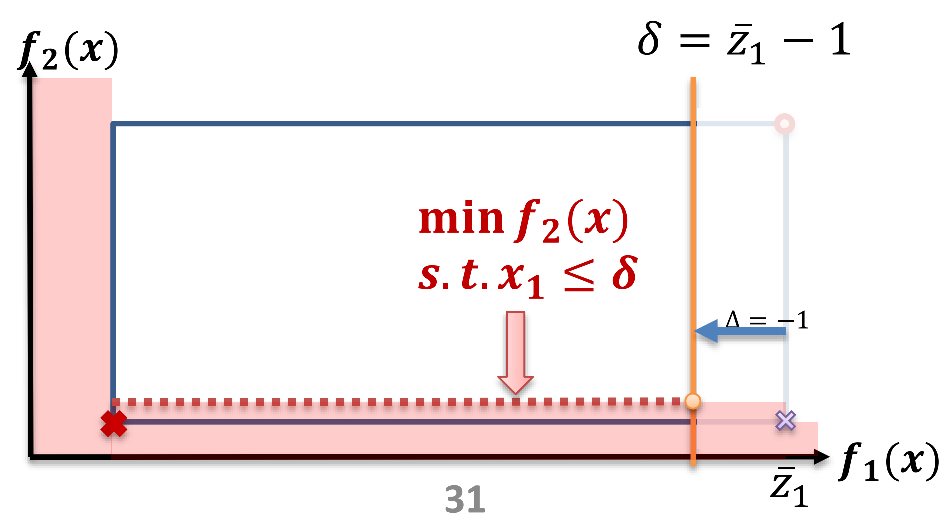

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Pareto: Bi-Objective Problem

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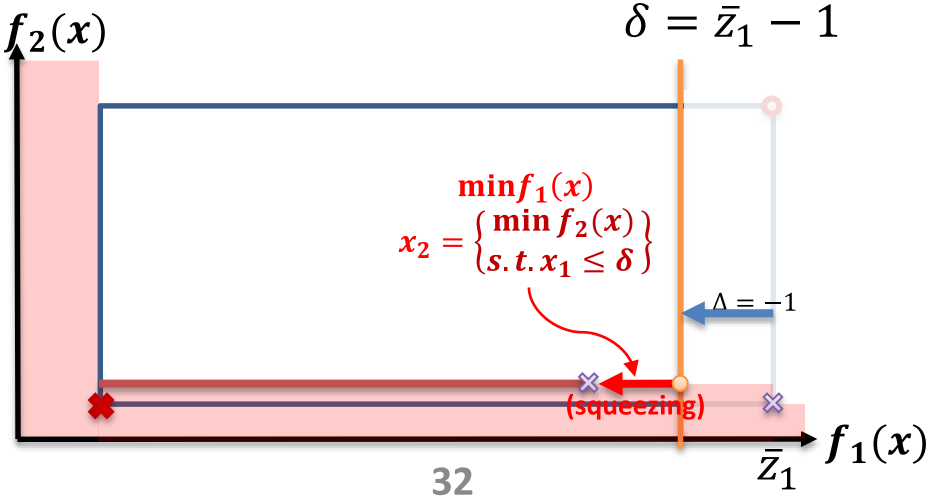

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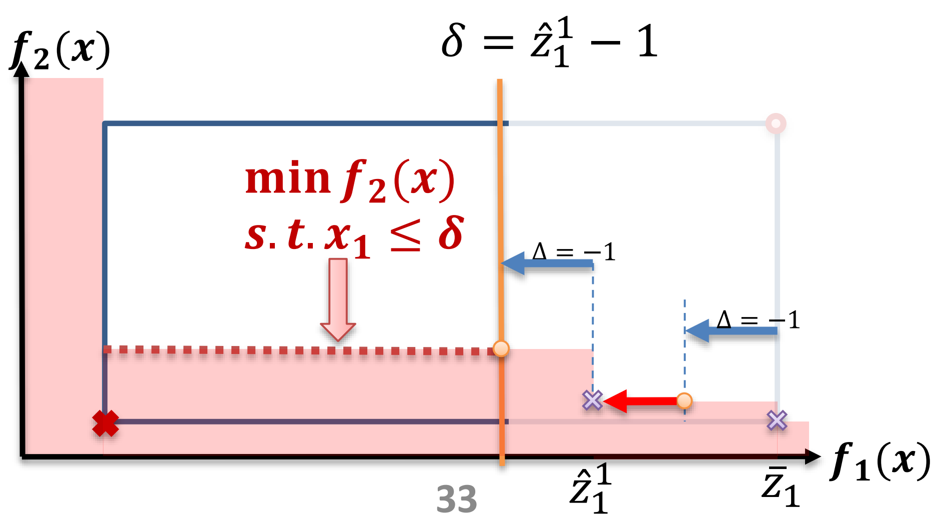
Let P is set of pareto solutions we've found;

Compute the ideal (z_1^*, z_2^*) and Nadir $(\bar{z}_1, \bar{z}_2)$ points;
Set $P := \{(\bar{z}_1, z_2^*)\}$ and $\delta := \bar{z}_1 - 1$;
WHILE $\delta \geq z_1^*$
    Solve $P_2(\delta)$ and get optimal solution (z_1^i, z_2^i) to $P_2(\delta)$;
    //Given (z_1^i, z_2^i), Find a left-botton corner $(\hat{z}_1^i)$ in the Pareto set;
    Solve $SQZ_1(z_2^i)$ and get optimal solution $(\hat{z}_1^i, z_2^i)$ to $SQZ_1(z_2^i)$;
END WHILE
Set $P := P + (\hat{z}_1^i, z_2^i)$ and $\delta = \hat{z}_1^i - 1$;
GO Step 2 UNTIL $z_1 = z_1^*$;



Pareto: Bi-Objective Problem

- ## Pareto Squeezing Algorithm

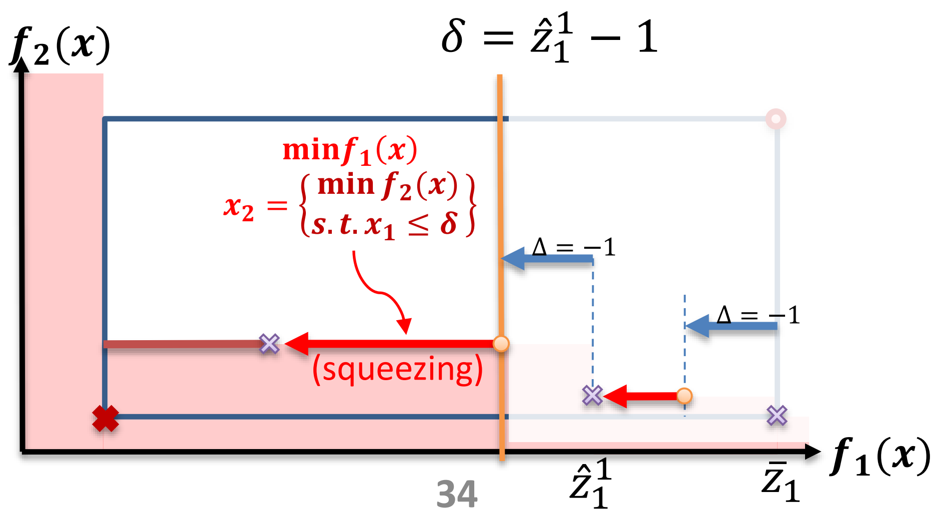
Algorithm 1 Exact squeezing algorithm for bi-objective problems

Let P is set of pareto solutions we've found;

Compute the ideal (z_1^*, z_2^*) and Nadir $(\bar{z}_1, \bar{z}_2)$ points;
Set $P := \{(\bar{z}_1, z_2^*)\}$ and $\delta := \bar{z}_1 - 1$;
WHILE $\delta \geq z_1^*$

Solve $P_2(\delta)$ and get optimal solution (z_1^i, z_2^i) to $P_2(\delta)$;
//Given (z_1^i, z_2^i), Find a left-botton corner $(\hat{z}_1^i)$ in the Pareto set;
Solve $SQZ_1(z_2^i)$ and get optimal solution $(\hat{z}_1^i, z_2^i)$ to $SQZ_1(z_2^i)$;

END WHILE
Set $P := P + (\hat{z}_1^i, z_2^i)$ and $\delta = \hat{z}_1^i - 1$;
GO Step 2 UNTIL $z_1 = z_1^*$;



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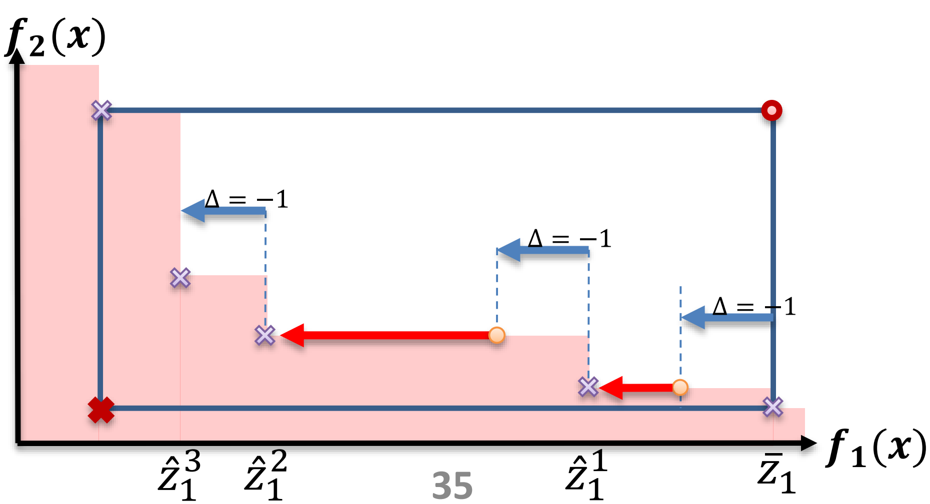
Pareto: Bi-Objective Problem

- **Pareto Squeezing Algorithm**

Algorithm 1 Exact squeezing algorithm for bi-objective problems

```
Let P is set of pareto solutions we've found;

    Compute the ideal (z_1^*, z_2^*) and Nadir (z̄_1, z̄_2) points;
    Set P := {(z̄_1, z_2^*)} and δ := z̄_1 − 1;
    WHILE δ ≥ z_1^*
        Solve P_2(δ) and get optimal solution (z_1^i, z_2^i) to P_2(δ);
        //Given (z_1^i, z_2^i), Find a left-botton corner (ẑ_1^i) in the Pareto set;
        Solve SQZ_1(z_2^i) and get optimal solution (ẑ_1^i, z_2^i) to SQZ_1(z_2^i);
    END WHILE
    Set P := P + (ẑ_1^i, z_2^i) and δ = ẑ_1^i − 1;
    GO Step 2 UNTIL z_1 = z_1^*;
```



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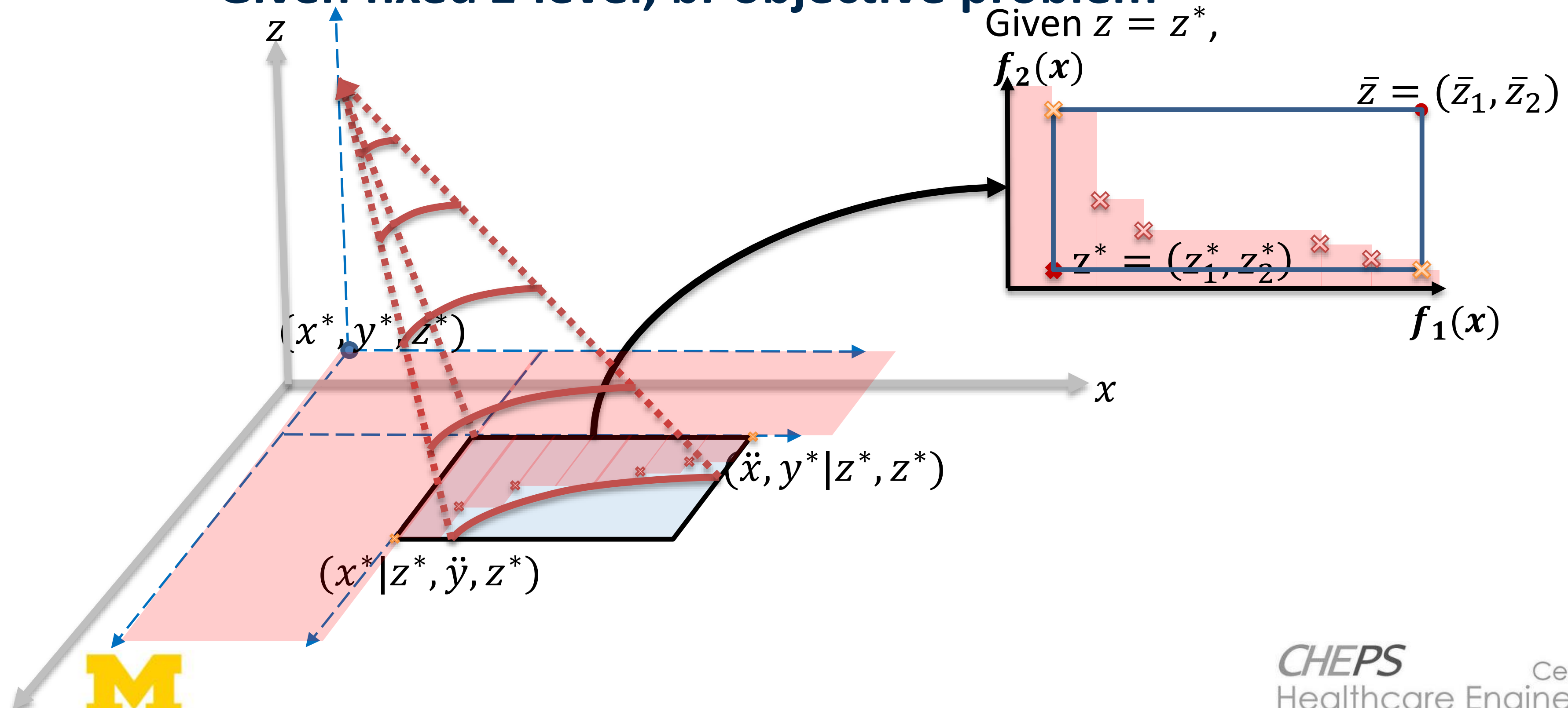
Pareto: Tri-Objective Problem

- "Pareto Cone"
  - Given fixed z level, bi-objective problem



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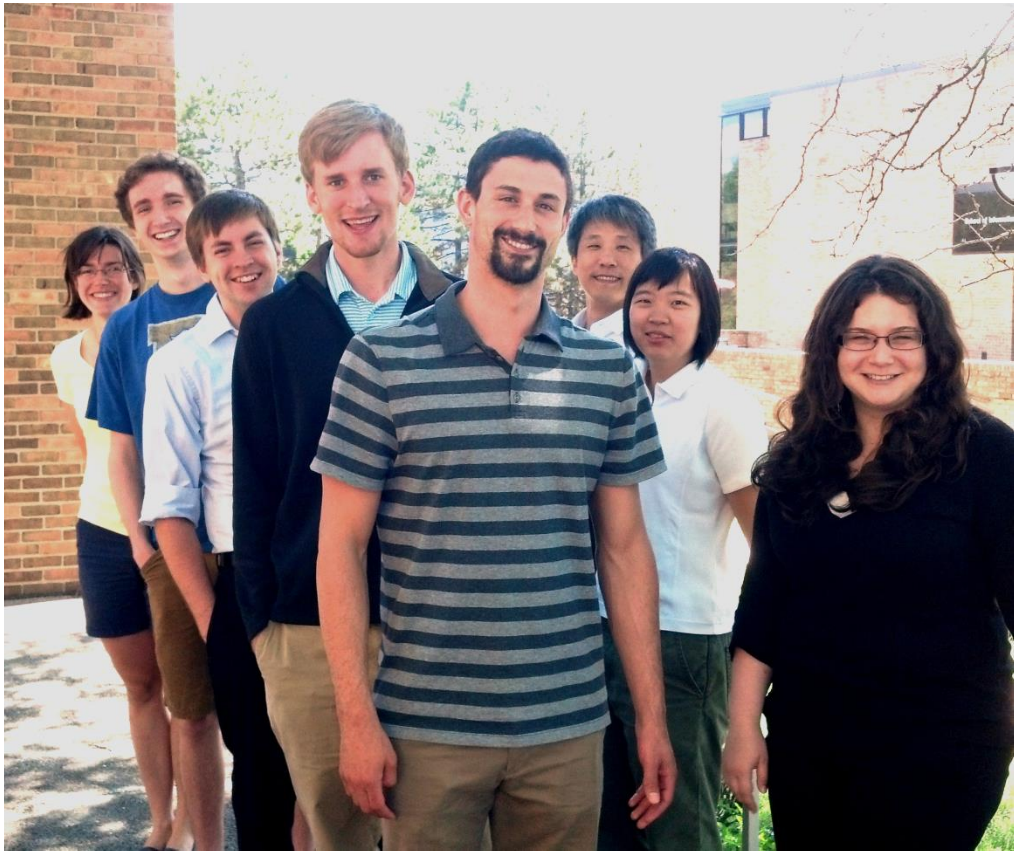
Thank You!



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