Stochastic Optimization to Reduce Wait Times in an Outpatient Infusion Center

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Our Collaborators

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  - Louise Salamin, Nurse Manager
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Motivations

- **Current state:**

  Average waiting time from arrival to infusion area to beginning of treatment is 42 minutes

- **Goal:**

  Generate appointment schedules that reduce patient waiting times and total length of day of operations

Outline of the presentation

- Description of the problem
- Stochastic Optimization Model
- Decomposition Algorithm
- Future Research

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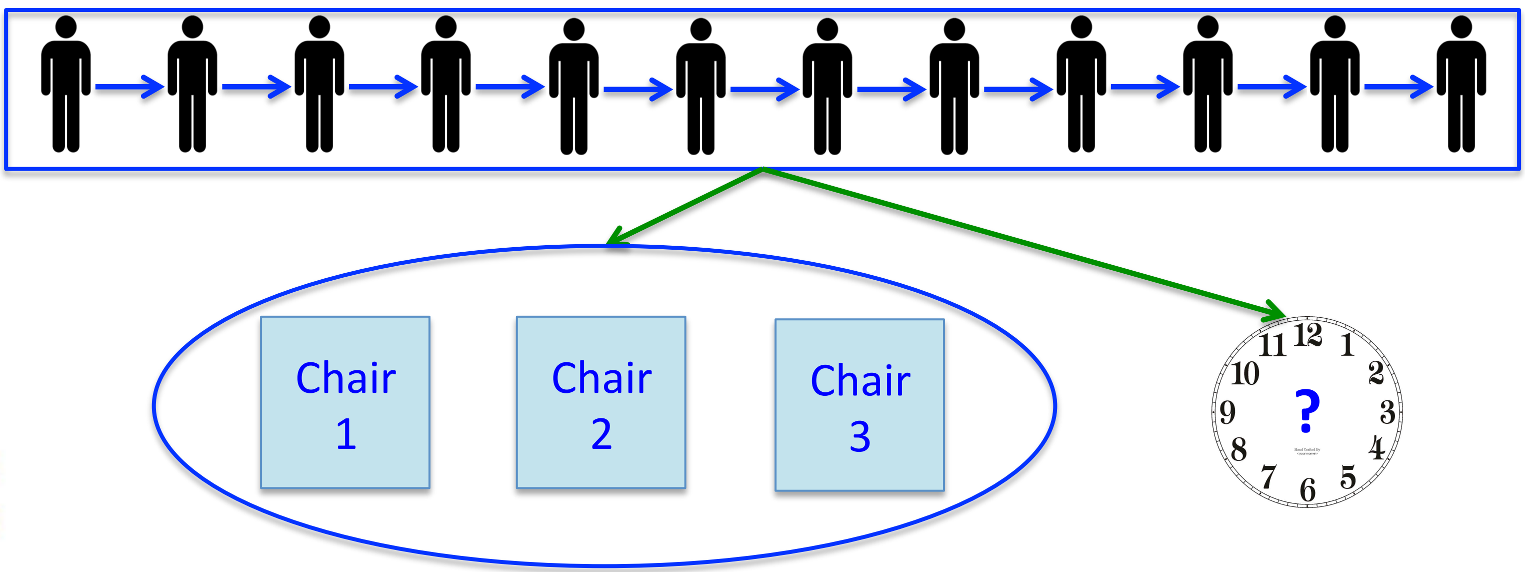
The Scheduling Process

- **Phase 1: Online Scheduling (Day-15 to Day-2)**
  – Patient/Physician calls to schedule an appointment
  – Scheduler assigns patient to a day and a slot
  – Scheduler gives approximate appointment time

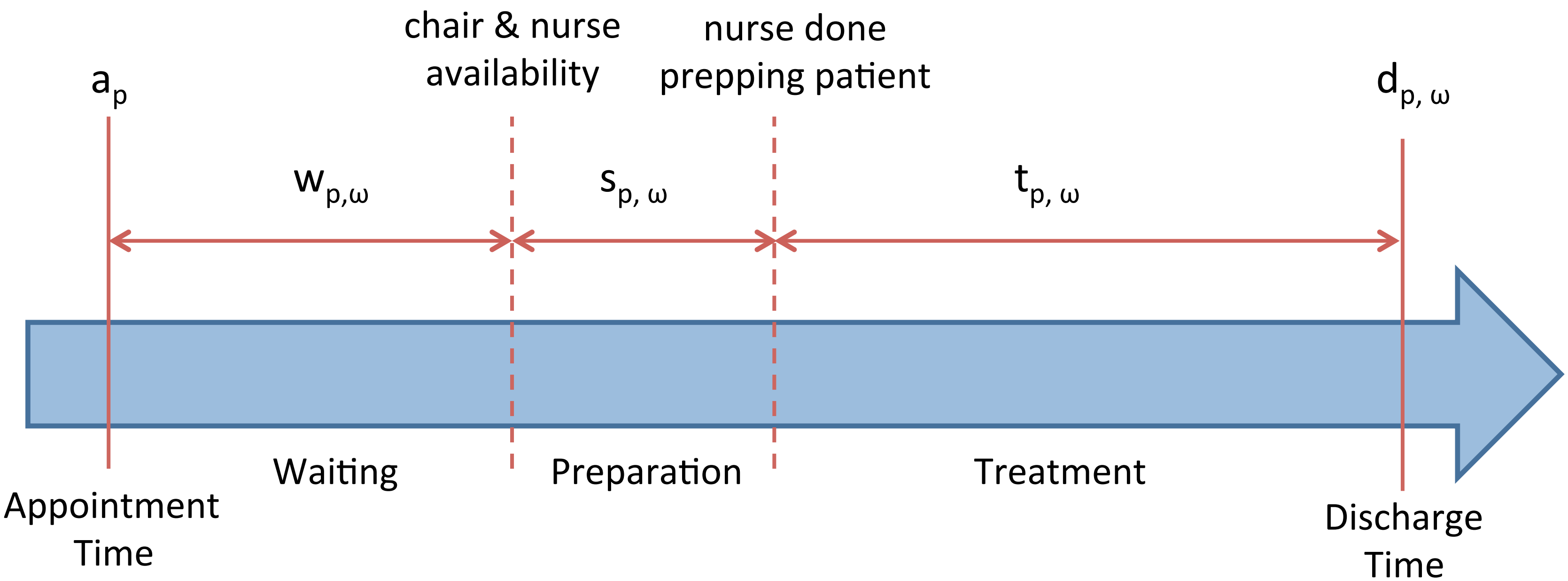
- **Phase 2: Fine-Tuning Optimization (Day-2)**
  – Once the list for a day is full, we set final appointment times
  – We preserve patients sequence so that final times are close to original estimates
  – We optimize appointment times to minimize patient waiting and staff overtime

Assumptions

- 12 patients have to be scheduled (according to a given sequence)
  - Appointment time
  - Chair assignment
- 3 chairs are available (infusion pod)
- 1 nurse is responsible for the patients assigned to those 3 chairs
- We assume that patients arrive on time



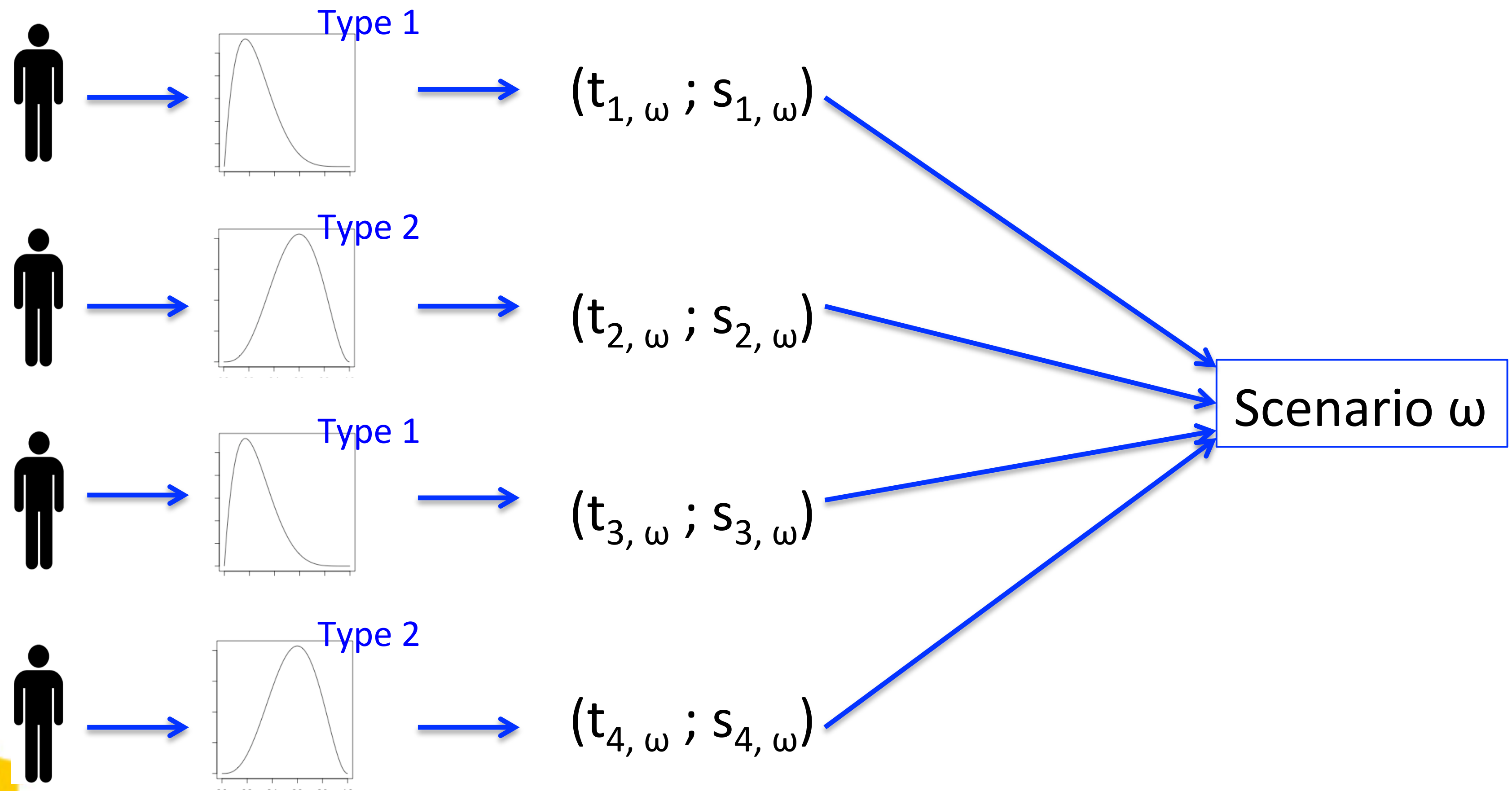
Patient timeline



- Treatment $t_{p,\omega}$ and preparation times $s_{p,\omega}$ are random parameters
- From historical data analysis we divide patients in 5 types
- Each type has specific distributions

Definition of Scenarios

- How to construct a scenario ω:



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Overview of the model

First Stage Decision:

Appointment Times: a_p	Continuous	≥0

Second Stage Decision:

Chair Assignment: x^{ω}_{pc}	**Binary**	1 iff patient p is assigned to chair c in scenario ω

Other Variables:

Waiting Time: w^{ω}_{p}	Continuous	≥0
End of Day: E^{ω}	Continuous	≥0
Discharge Time: d^{ω}_{p}	Continuous	≥0

$$\min_{a,x^\omega,d^\omega,w^\omega,E^\omega} \quad \lambda \sum_{p\in P} \sum_{\omega\in\Omega} w_p^\omega + (1-\lambda) \sum_{\omega\in\Omega} E^\omega \tag{0}$$

Objective: Trade-off between the total expected waiting time and the expected end of the day.

Variables :

a_p : appointment time of patient p
d_p^ω : discharge time of patient p in scenario ω
w_p^ω : waiting time of patient p in scenario ω
E^ω : end of the day in scenario ω

Parameters :

s_p^ω : treatment time of patient p in scenario ω
t_p^ω : treatment time of patient p in scenario ω
λ : weight in objective
M : big constant

$$\min_{a,x^{\omega},d^{\omega},w^{\omega},E^{\omega}} \quad \lambda \sum_{p\in P}\sum_{\omega\in\Omega} w_p^{\omega} + (1-\lambda)\sum_{\omega\in\Omega} E^{\omega} \tag{0}$$

$$\text{subject to} \quad \sum_{c\in C} x_{pc}^{\omega} = 1 \qquad \forall p \in P,\ \forall \omega \in \Omega \tag{1}$$

Each patient is assigned to exactly one chair in each scenario.

Variables :

a_p : appointment time of patient p
d_p^{ω} : discharge time of patient p in scenario ω
w_p^{ω} : waiting time of patient p in scenario ω
E^{ω} : end of the day in scenario ω

Parameters :

s_p^{ω} : treatment time of patient p in scenario ω
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$$
\begin{aligned}
\min_{a,x^\omega,d^\omega,w^\omega,E^\omega} \quad & \lambda \sum_{p\in P}\sum_{\omega\in\Omega} w_p^\omega + (1-\lambda)\sum_{\omega\in\Omega} E^\omega && && (0)\\
\text{subject to} \quad & \sum_{c\in C} x_{pc}^\omega = 1 && \forall p\in P,\ \forall\omega\in\Omega && (1)\\
& a_p + w_p^\omega + s_p^\omega + t_p^\omega = d_p^\omega && \forall p\in P,\ \forall\omega\in\Omega && (2)
\end{aligned}
$$

Definition of the discharge time of each patient in each scenario.

Variables :

a_p : appointment time of patient p
d_p^ω : discharge time of patient p in scenario ω
w_p^ω : waiting time of patient p in scenario ω
E^ω : end of the day in scenario ω

Parameters :

s_p^ω : treatment time of patient p in scenario ω
t_p^ω : treatment time of patient p in scenario ω
λ : weight in objective
M : big constant

$$\min_{a,x^\omega,d^\omega,w^\omega,E^\omega} \quad \lambda \sum_{p\in P}\sum_{\omega\in\Omega} w_p^\omega + (1-\lambda)\sum_{\omega\in\Omega} E^\omega \qquad (0)$$

$$\text{subject to} \quad \sum_{c\in C} x_{pc}^\omega = 1 \qquad \forall p \in P,\ \forall \omega\in\Omega \qquad (1)$$

$$a_p + w_p^\omega + s_p^\omega + t_p^\omega = d_p^\omega \qquad \forall p \in P,\ \forall \omega\in\Omega \qquad (2)$$

$$a_{p_1} + w_{p_1}^\omega + M(2 - x_{p_1c}^\omega - x_{p_2c}^\omega) \geq d_{p_2}^\omega \qquad \forall c\in C,\ \forall p_1 > p_2 \in P,\ \forall\omega\in\Omega \qquad (3)$$

A patient can sit on his chair only if all the previous patients assigned to his chair have been discharged.

Variables :

a_p : appointment time of patient p
d_p^ω : discharge time of patient p in scenario ω
w_p^ω : waiting time of patient p in scenario ω
E^ω : end of the day in scenario ω

Parameters :

s_p^ω : treatment time of patient p in scenario ω
t_p^ω : treatment time of patient p in scenario ω
λ : weight in objective
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$$
\begin{array}{lll}
\min\limits_{a,x^\omega,d^\omega,w^\omega,E^\omega} & \lambda \sum\limits_{p\in P} \sum\limits_{\omega\in\Omega} w_p^\omega + (1-\lambda) \sum\limits_{\omega\in\Omega} E^\omega & & (0)\\
\text{subject to} & \sum\limits_{c\in C} x_{pc}^\omega = 1 & \forall p \in P,\ \forall \omega \in \Omega & (1)\\
& a_p + w_p^\omega + s_p^\omega + t_p^\omega = d_p^\omega & \forall p \in P,\ \forall \omega \in \Omega & (2)\\
& a_{p_1} + w_{p_1}^\omega + M(2 - x_{p_1c}^\omega - x_{p_2c}^\omega) \geq d_{p_2}^\omega & \forall c \in C,\ \forall p_1 > p_2 \in P,\ \forall \omega \in \Omega & (3)\\
& a_{p_1} + w_{p_1}^\omega \geq a_{p_2} + w_{p_2}^\omega + s_{p_2}^\omega & \forall p_1 > p_2 \in P,\ \forall \omega \in \Omega & (4)
\end{array}
$$

A patient can sit on his chair if the nurse has finished to prepare all previous patients on his pod.

Variables :

a_p : appointment time of patient p

d_p^ω : discharge time of patient p in scenario ω

w_p^ω : waiting time of patient p in scenario ω

E^ω : end of the day in scenario ω

Parameters :

s_p^ω : treatment time of patient p in scenario ω

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\begin{array}{lll}
\min\limits_{a,x^\omega,d^\omega,w^\omega,E^\omega} & \lambda \sum\limits_{p\in P}\sum\limits_{\omega\in\Omega} w_p^\omega + (1-\lambda)\sum\limits_{\omega\in\Omega} E^\omega & & (0)\\
\text{subject to} & \sum\limits_{c\in C} x_{pc}^\omega = 1 & \forall p\in P,\ \forall\omega\in\Omega & (1)\\
& a_p + w_p^\omega + s_p^\omega + t_p^\omega = d_p^\omega & \forall p\in P,\ \forall\omega\in\Omega & (2)\\
& a_{p_1} + w_{p_1}^\omega + M(2 - x_{p_1c}^\omega - x_{p_2c}^\omega) \geq d_{p_2}^\omega & \forall c\in C,\ \forall p_1 > p_2\in P,\ \forall\omega\in\Omega & (3)\\
& a_{p_1} + w_{p_1}^\omega \geq a_{p_2} + w_{p_2}^\omega + s_{p_2}^\omega & \forall p_1 > p_2\in P,\ \forall\omega\in\Omega & (4)\\
& E^\omega \geq d_p^\omega & \forall p\in P,\ \forall\omega\in\Omega & (5)
\end{array}
$$

Definition of the end of the day in each scenario.

Variables :

a_p : appointment time of patient p
d_p^ω : discharge time of patient p in scenario ω
w_p^ω : waiting time of patient p in scenario ω
E^ω : end of the day in scenario ω

Parameters :

s_p^ω : treatment time of patient p in scenario ω
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λ : weight in objective
M : big constant

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\begin{aligned}
\min_{a, x^\omega, d^\omega, w^\omega, E^\omega} \quad & \lambda \sum_{p \in P} \sum_{\omega \in \Omega} w_p^\omega + (1-\lambda) \sum_{\omega \in \Omega} E^\omega && && (0)\\
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& a_p + w_p^\omega + s_p^\omega + t_p^\omega = d_p^\omega && \forall p \in P,\ \forall \omega \in \Omega && (2)\\
& a_{p_1} + w_{p_1}^\omega + M(2 - x_{p_1 c}^\omega - x_{p_2 c}^\omega) \geq d_{p_2}^\omega && \forall c \in C,\ \forall p_1 > p_2 \in P,\ \forall \omega \in \Omega && (3)\\
& a_{p_1} + w_{p_1}^\omega \geq a_{p_2} + w_{p_2}^\omega + s_{p_2}^\omega && \forall p_1 > p_2 \in P,\ \forall \omega \in \Omega && (4)\\
& E^\omega \geq d_p^\omega && \forall p \in P,\ \forall \omega \in \Omega && (5)\\
& x_{pc}^\omega \in \{0, 1\} && \forall c \in C,\ \forall p \in P,\ \forall \omega \in \Omega && (6)\\
& a_p \geq 0 && \forall p \in P && (7)\\
& w_p^\omega, d_p^\omega \geq 0 && \forall p \in P,\ \forall \omega \in \Omega && (8)
\end{aligned}
$$

Binary and non-negativity constraints.

Variables :

a_p : appointment time of patient p
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Parameters :

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Intractability of this model

- Large scale MIP (large number of scenarios)
- Weak relaxation bound

Number of Scenarios	**Solve Time**
1	1 sec
5	15 sec
10	1 min
20	3 min
50	15 min
100	*10 h**
500	*1000 h**

(* Estimates)

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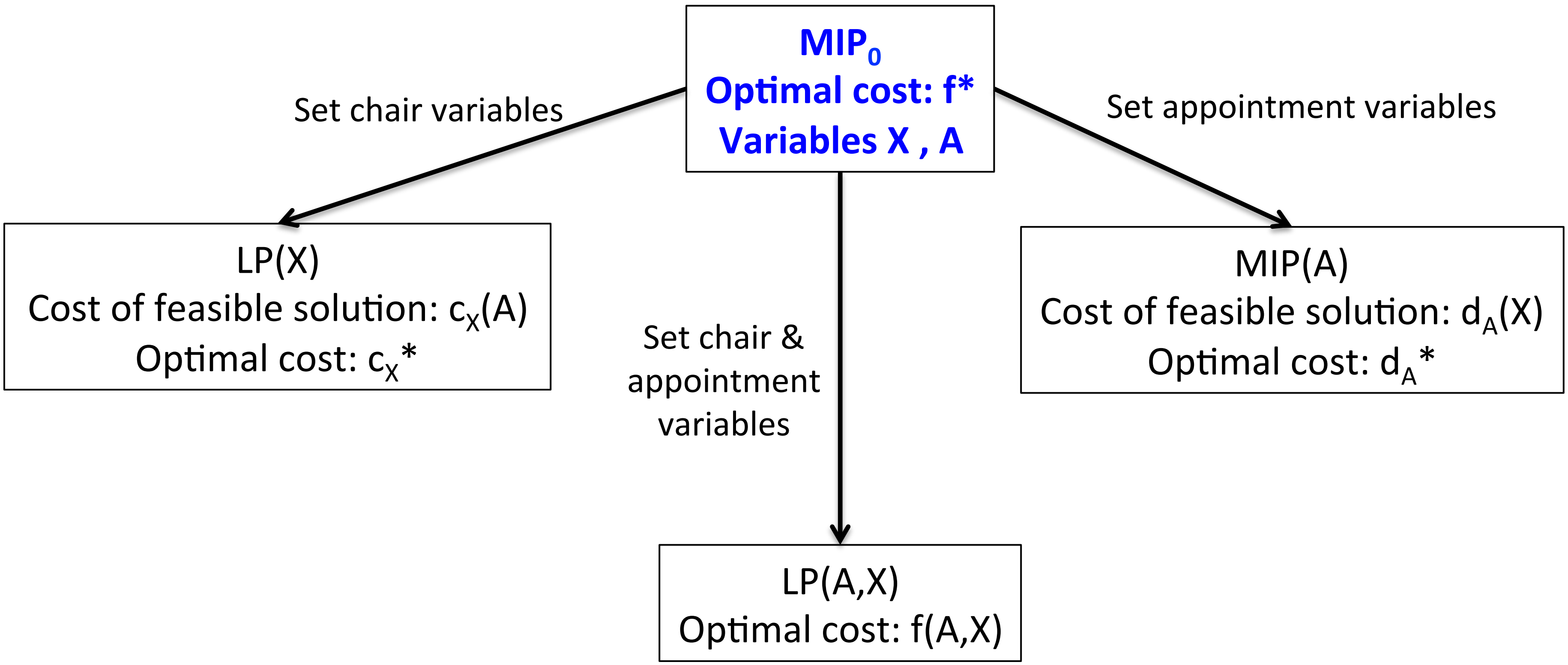
Decomposition

Appointment Times: $\mathbf{a_p}$

Chair Assignment: $\mathbf{x^{\omega}{}_{cp}}$

Decomposition



Decomposition

MIP$_0$
Optimal cost: f*
Variables X , A

Set chair variables

LP(X)
Cost of feasible solution: $c_X(A)$
Optimal cost: c_X^*

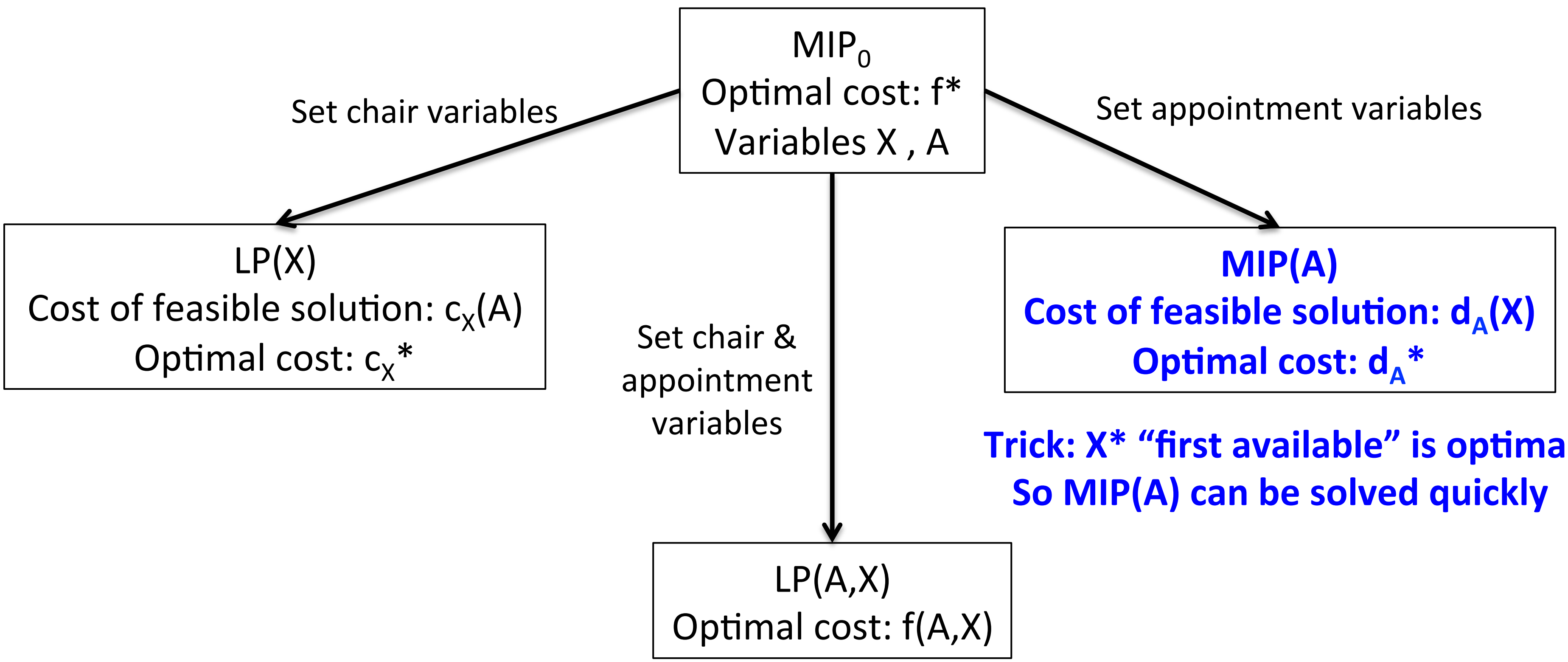
Set appointment variables

MIP(A)
Cost of feasible solution: $d_A(X)$
Optimal cost: d_A^*

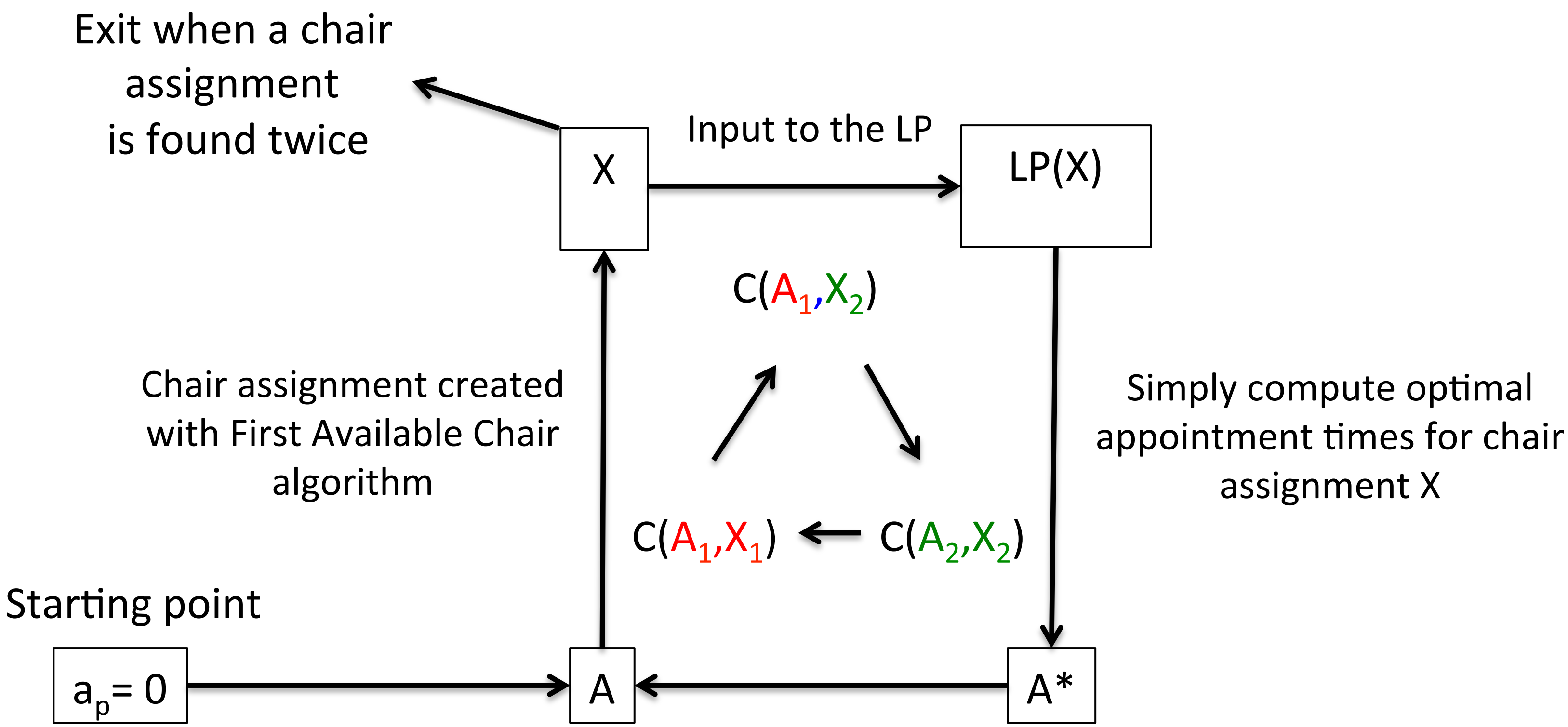
Set chair & appointment variables

LP(A,X)
Optimal cost: f(A,X)

Decomposition



The Fix-Unfix Algorithm

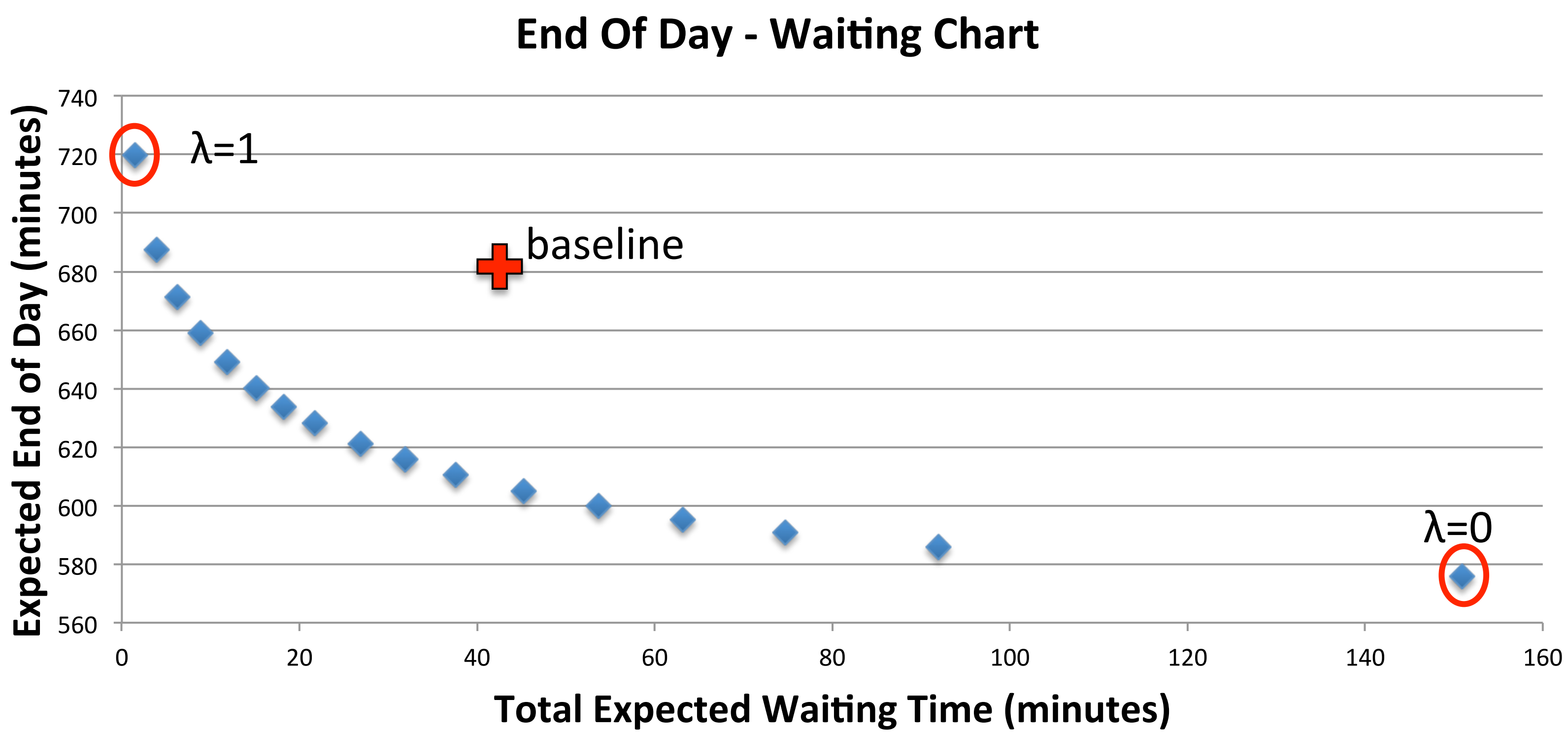


Improvement: $c(A_1,X_1) \geq c(A_1,X_2) \geq c(A_2,X_2)$

Termination: We iterate until we find the same chair assignment twice

Results of the Algorithm

- Instance with 1000 scenarios
- Runtimes < 4 sec
- Between 2 and 7 iterations before termination

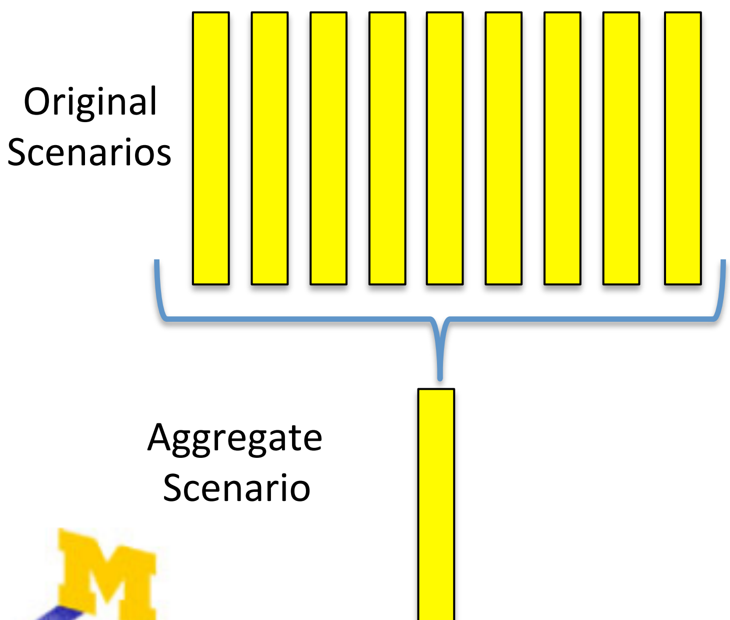


Generalized Jensen's Bound

<u>General Idea:</u> apply Jensen inequality to the objective function

Traditional Jensen's Bound

$$\min_{\substack{x^\omega : Ax^\omega = b \\ \forall \omega \in \Omega}} \sum_{\omega \in \Omega} p^\omega c^T x^\omega \geq \sum_{\omega \in \Omega} p^\omega \min_{x^\omega : Ax^\omega = b} c^T x^\omega$$



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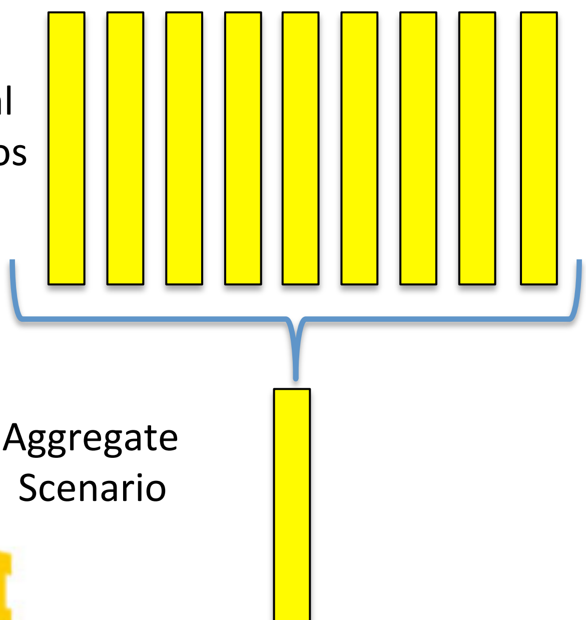
Original Scenarios



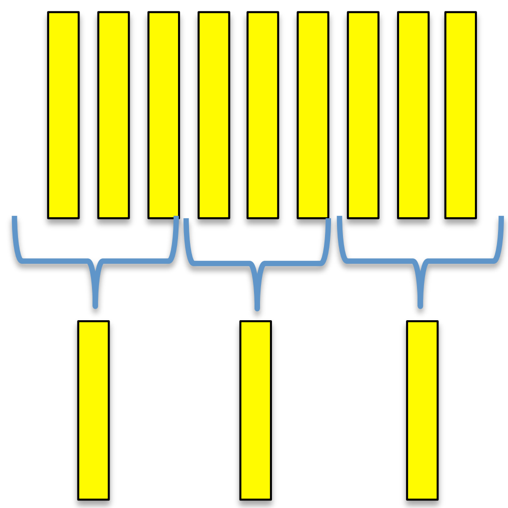
Aggregate Scenario

Generalized Jensen's Bound

$$\min_{\substack{x^\omega : Ax^\omega=b \\ \forall \omega\in\Omega}} \sum_{\omega\in\Omega} c^T x^\omega \geq \sum_{i\in[1,k]} p(G_i^\omega) \min_{\substack{x^\omega : Ax^\omega=b \\ \forall \omega\in G_i}} c^T x^\omega$$

$$\text{where } \{G_1 \dots G_k\} \text{ is a partition of } \Omega$$

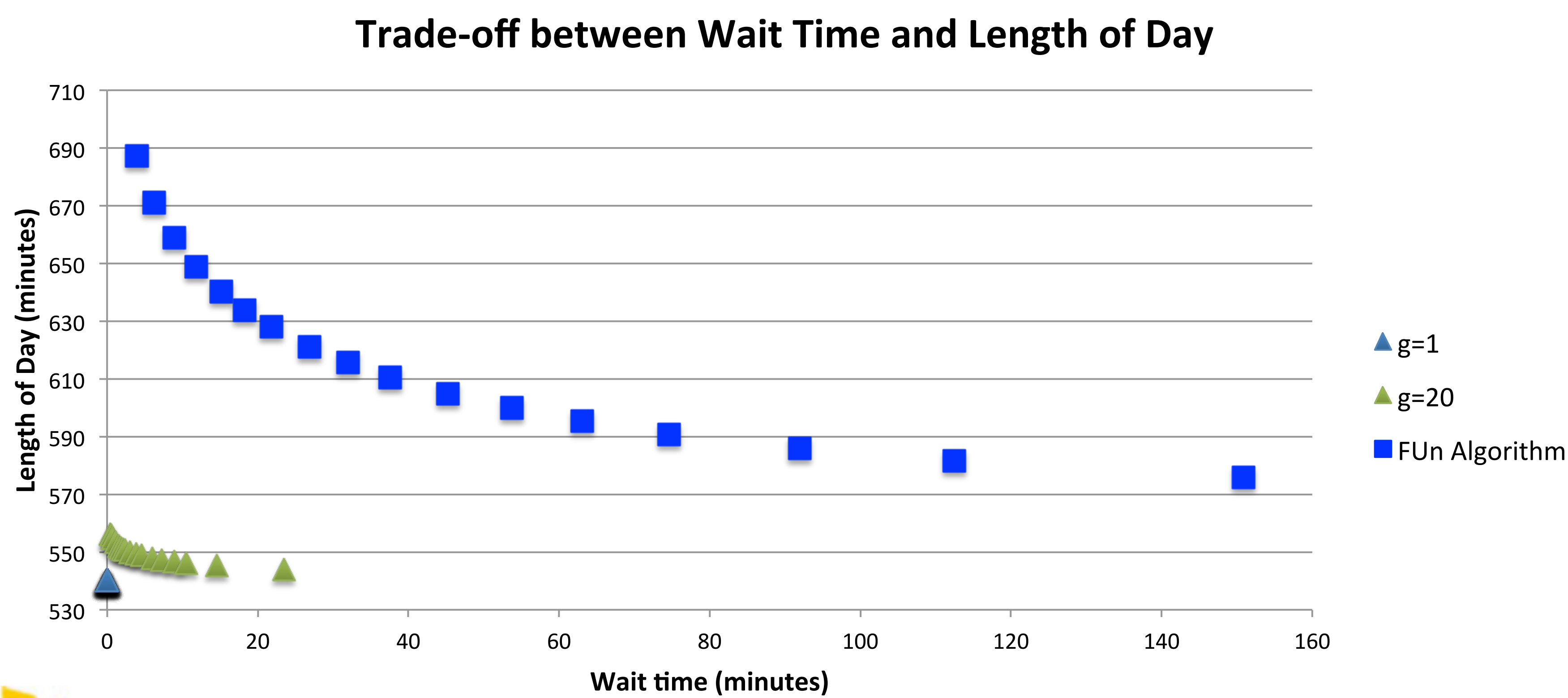
Original Scenarios



Aggregate Scenarios

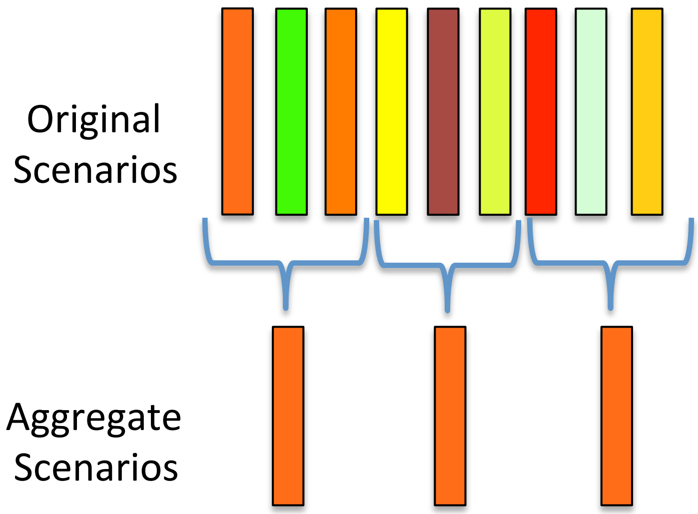
Comparison of the *FUn* Heuristic & Jensen's bound

- **Instance with 1000 scenarios**
- **Jensen's bound computed with 20 groups of 50 scenarios**



Generalized Jensen's Bound: An improvement

General Idea: Waiting is high due to variability of the scenarios

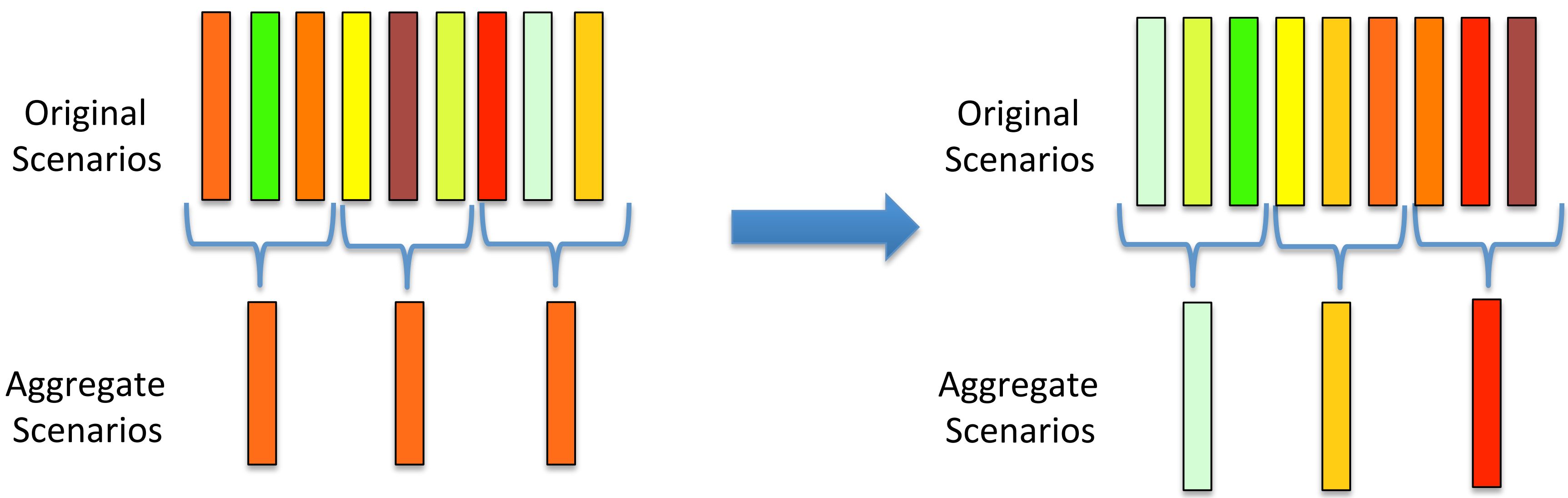


Weak Bound

Generalized Jensen's Bound: An improvement

<u>General Idea:</u> Waiting is high due to variability of the scenarios

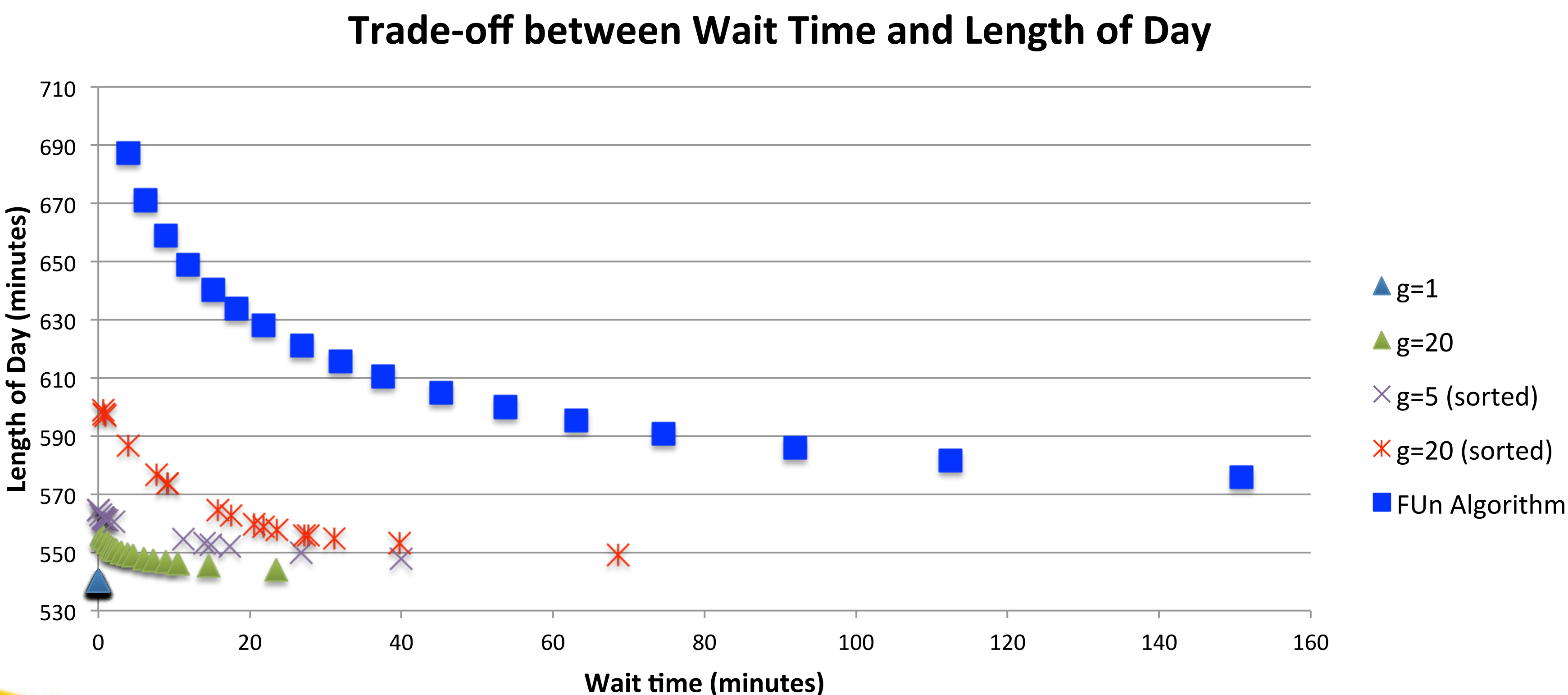
→ We should order the scenarios by total treatment length before grouping



Weak Bound **Good Bound**

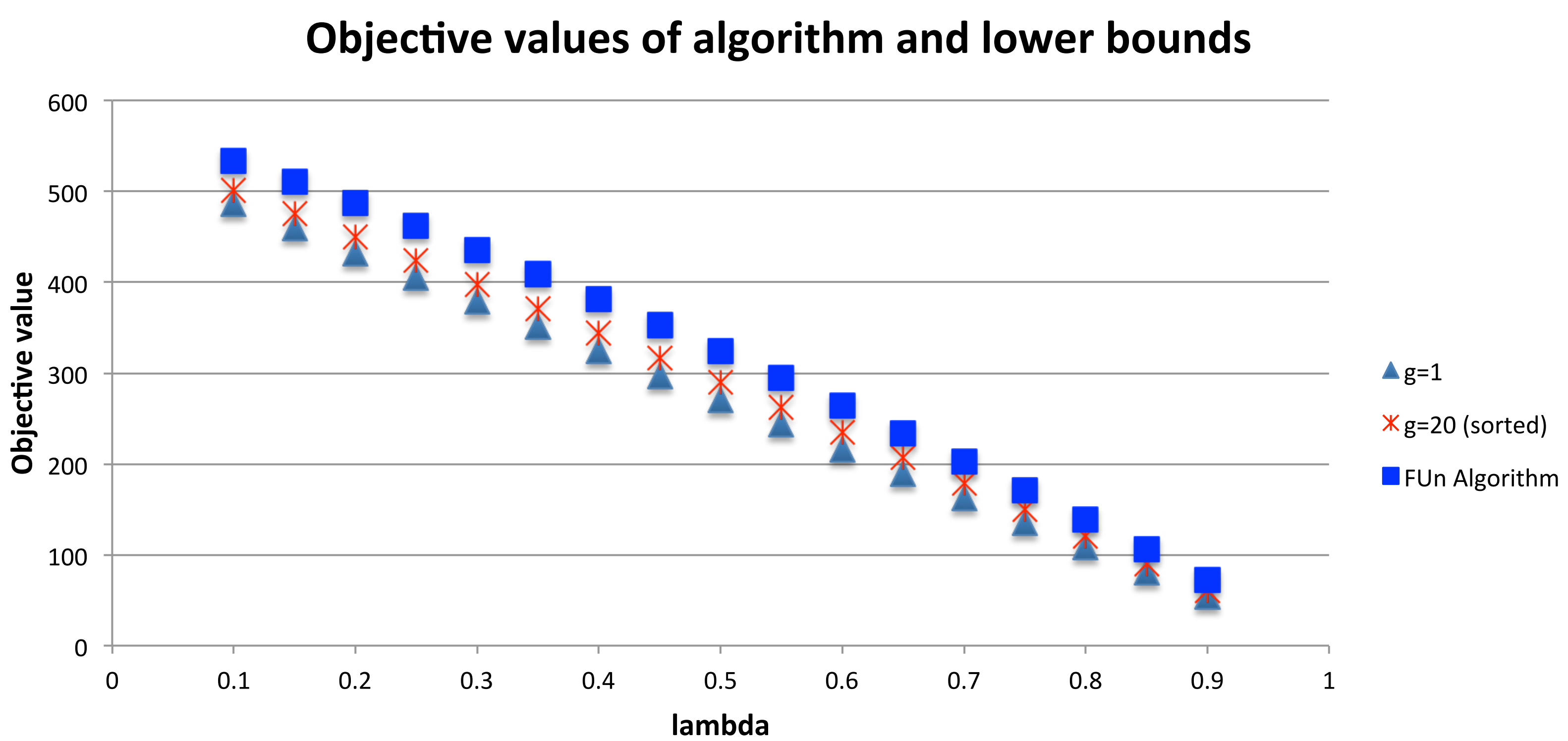
Comparison of the Algorithm & Jensen's bound

- **Instance with 1000 scenarios**
- **Jensen's bound with 5 and 20 groups of 50 scenarios (sorted method)**



Comparison of the Algorithm & Jensen's bound

- **Performance ratio** $\dfrac{\text{obj}(\textit{Algo}) - \text{LB}}{\text{LB}} \approx 10\%$



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Future Work

- **Theoretical Work:**
  - Understand why the *Fix-Unfix* algorithm works so well
  - Can this algorithm be successfully applied to some famous problems?
  - Merge scheduling phases 1 and 2 in a single optimization model
- **Towards an implementation at the Cancer Center:**
  - Define general rules/guidelines to help the phase 1 scheduling process
  - Find easy-to-implement good sequences: Longest (Shortest) Processing Time First, Shortest Variance First...

CHEPS and the HEPS Master's Program

- **CHEPS:** The Center for Healthcare Engineering and Patient Safety
- **HEPS:** Industrial and Operations Engineering (IOE) Master's Concentration in Healthcare Engineering and Patient Safety offered by CHEPS
- CHEPS and HEPS offer unique multidisciplinary teams from engineering, medicine, public health, nursing, and more collaborating with healthcare professionals to better provide and care for patients
- For more information, contact Amy Cohn at amycohn@umich.edu or visit the CHEPS website at: https://www.cheps.engin.umich.edu

CENTER FOR
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UNIVERSITY OF MICHIGAN

Thank you

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