Analysis of Stochastic Mixed-Integer Linear Programming Models for the Outpatient Appointment Scheduling Problem

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Stochastic Outpatient Scheduling Problem (SOPSP)

- **Outpatient clinic managers** must schedule start times and order for a day's worth of patients
- Each patient has a **known type** and a **random** (non-negative) service duration that follows a **known probability distribution**
- **GOAL**: Given uncertainty in patient service durations, **minimize the expectation of a weighted sum** of total patient **waiting time, total idle time, and clinic overtime**

Complex Stochastic Combinatorial Optimization Problem

Key Contributions

- We **propose a new stochastic mixed-integer linear programming** (SMILP) model for SOPSP
- We **compare our model** with those of **Berg et al., 2014[1] (B)** and **Mancilla and Storer., 2013[2] (M)**, which are the only SMILPs for SOPSP and similar SASS problems
- SOPSP is a (well-known) **single-server stochastic appointment sequencing and scheduling** (SASS) problem with applications including scheduling of surgeries in an operating room, ships in a port, exams in an examination facility, and more

New SMILP for SOPSP

Indices and sets

P	patients to be scheduled, $p = 1, ..., P$
I	positions in the sequence, $i = 1, ..., P$
N	scenarios to be considered, $n = 1, ..., N$

Parameters

λ_i^w	waiting time penalty for appointment i
λ_i^g	penalty for idle time between appointments i and $i+1$
λ^o	overtime penalty
$\mathcal{L}$	planned length of clinic day
d_p^n	duration of procedure p in scenario n

Scenario-independent (first-stage) variables

$x_{i,p}$	binary assignment variable indicating whether patient p is assigned to appointment i
t_i	scheduled start time of appointment i

Scenario-dependent (second-stage) variables

s_i^n	actual start time of appointment i in scenario n
g_i^n	idle time after appointment i in scenario n
o^n	overtime in scenario n.

Our New SMILP for SOPSP

$$\text{minimize } \frac{1}{N}\sum_{n=1}^{N}\left[\sum_{i=1}^{P}\lambda_i^w \cdot (s_i^n - t_i) + \sum_{i=1}^{P}\lambda_i^g \cdot g_i^n + \lambda^o \cdot o^n\right] \quad (1)$$

$$\text{subject to } \sum_{i=1}^{P} x_{ip} = 1 \quad \forall p \quad (2)$$

$$\sum_{p=1}^{P} x_{ip} = 1 \quad \forall i \quad (3)$$

$$s_i^n \geq t_i \quad \forall i, n \quad (4)$$

$$s_i^n \geq s_{i-1}^n + \sum_{p=1}^{P} d_p^n \cdot x_{i-1,p} \quad \forall (i \geq 2, n) \quad (5)$$

$$g_i^n = s_{i+1}^n - \left(s_i^n + \sum_{p=1}^{P} d_p^n \cdot x_{i,p}\right) \quad \forall (i < P, n) \quad (6)$$

$$o^n \geq \left(s_P^n + \sum_{p=1}^{P} d_p^n \cdot x_{P,p}\right) - \mathcal{L} \quad \forall n \quad (7)$$

$$(t_i, g_i^n, s_i^n, o^n) \geq 0 \quad \forall (i, n) \quad (8)$$

$$x_{i,p} \in \{0, 1\} \quad \forall (i, p) \quad (9)$$

SMILP Model Details:

Objective function:

(1) The sample average of the weighted linear combination of the total waiting time, total idle time, and overtime

First-stage:

(2-3) Ensure that each patient is assigned to one appointment and each appointment is assigned to one patient

Second stage: for each scenario n

(4-5) Require the start time of the ith appointment, s_i^n, to be no smaller than the scheduled start time, t_i, and the completion time of the preceding appointment

(6) Define the idle time as the gap between the actual start time of an appointment and the completion time of the preceding one

(7) Define the overtime as the positive difference between the completion time of the last appointment and the clinic scheduled closing time, L

(8-9) Define the feasible ranges of the decision variables

Theoretical Analysis of the SMILP for SOPSP

Sizes of SOPSP Formulations

Table 1: Sizes of formulations of the SOPSP with P procedures and N scenarios.

	(B)	(M)	(S)
# Binary variables	$2P^2 + 4P + 2$	P^2	P^2
# Continuous variables	$P + 1 + N(2P^2 + 4P + 4)$	$P + N(2P^2 + 2)$	$P + N(2P + 1)$
# First-stage constraints	$P^3 + 5P^2 + 11P + 10$	$P^2 + 3P$	$P^2 + 3P$
# Second-stage constraints	$N(4P^2 + 9P + 5)$	$N(4P^2 + P + 2)$	$5NP$

The tightness of SOPSP Formulations

Theorem 1. *The linear programming relaxation (LPR) of the (S) and (M) models are equivalent. Furthermore, the LPR of (S) model is tighter than that of (B)*

Computational Analysis of the SMILP for SOPSP

Description of Experiments

- 14 different SOPSP instances with 12 patients types and 4-20 patients
- Three different weight structures
- 420 sample average approximations (SAA), each with 1,000 scenarios
- Time limit: 2 hours
- Using a **standard optimization modeling tool,** AMPL, and a **commercial MILP solver**, CPLEX, with default settings

Results

- **Using our model (S)**, we were able to solve **all of the 420 SAAs** instances in **less than 20 minutes**
- **Comparison with model (B)**

Table 2: Ratios of optimal objective values of LP relaxations of (S) and (B).

$\lambda^w = \lambda^g = \lambda^o$			$\lambda^w = 1, \lambda^g = 0, \lambda^o = 10$			$\lambda^w = 1, \lambda^g = 5, \lambda^o = 7.5$		
Min	Avg±stdv	Max	Min	Avg±stdv	Max	Min	Avg±stdv	Max
1.95	2.62±0.41	3.48	1.11	1.38±0.26	2.08	1.27	1.64±0.33	2.49

  - Using **model (B)**, We were able to solve **only 160** of the 420 SAAs
  - It took **6-138 time longer** than our model to solve these 160 SAAs
  - The **remaining 260 SAAs** that were not solved **terminated with relative MIP gaps** ($\frac{UB-LB}{UB} \times 100\%$) **between 16% and 70%**
- **Comparison with model (M)**
  - **Using model (M),** we were able to solve **only 320** of the 420 SAAs
  - It took **2-43 time longer** than our model to solve these 320 SAAs
  - The remaining **80 SAAs** that were not solved **terminated with relative MIP gaps between 15% and 25%.**

Future Work & Conclusions

- We presented a new SMILP for the basic (yet still challenging) single-resource stochastic appointment sequencing and scheduling problem
- We also compare this model to two closely-related formulations in the literature and analyze them both empirically and theoretically
- Computational results demonstrated where significant improvements in performance could be gained with our proposed model
- We plan to:
  - extend our approach to include additional sources of uncertainty, particularly variability in patient arrival time
  - develop templates and policies for scheduling patients dynamically as they randomly request future appointments.

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References

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2. Mancilla, C., Storer, R., 2012. A sample average approximation approach to stochastic appointment sequencing and scheduling. IIE Transactions 44 (8), 655–670.