Analysis of Stochastic Mixed-Integer Linear Programming Models for the Outpatient Appointment Scheduling Problem

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**Stochastic Outpatient Scheduling Problem (SOPSP)**

- Outpatient clinic managers must schedule start times and order for a day’s worth of patients.
- Each patient has a known type and a random (non-negative) service duration that follows a known probability distribution.
- GOAL: Given uncertainty in patient service durations, minimize the patient’s waiting time, total idle time, and clinic overtime.

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**Key Contributions**

- We propose a new stochastic mixed-integer linear programming (SMILP) model for SOPSP.
- We compare our model with those of Berg et al., 2014\textsuperscript{1} (B) and Mancilla and Storer, 2013\textsuperscript{2} (M), which are the only SMILPs for SOPSP and similar SASS problems.
- SOPSP is a (well-known) single-server stochastic appointment sequencing and scheduling (SASS) problem with applications including scheduling of surgeries in an operating room, ships in a port, exams in an examination facility, and more.

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**New SMILP for SOPSP**

- We define the time of the patient’s appointment, the patient’s waiting time, the idle time, and the clinic’s overtime.

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**Theoretical Analysis of the SMILP for SOPSP**

**Objective function:**

1. The sample average of the weighted linear combination of the total waiting time, total idle time, and clinic overtime.

**First-stage:**

2. Ensure that each patient is assigned to one appointment and that each appointment is assigned to one patient.

**Second-stage:**

3. For each scenario $n$.

4. Require the start time of the appointment, $s^i_n$, to be no smaller than the scheduled start time, $t_i$, and the completion time of the preceding appointment.

5. Define the idle time as the gap between the actual start time of an appointment and the completion time of the preceding appointment.

6. Define the overtime as the positive difference between the completion time of the last appointment and the clinic’s scheduled closing time, $L$.

**SOPSP Formulations**

**Sizes of SOPSP Formulations**

<table>
<thead>
<tr>
<th>Scenario (P)</th>
<th>Scheduling Formulation</th>
<th>Scheduling Model</th>
<th>Scheduling Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>P patients to be scheduled, $p = 1, ..., P$</td>
<td>$I$ positions in the sequence, $i = 1, ..., I$</td>
<td>$N$ scenarios to be considered, $n = 1, ..., N$</td>
<td>$L$ planning horizon of clinic day</td>
</tr>
</tbody>
</table>

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**The tightness of SOPSP Formulations**

Theorem 1. The linear programming relaxation (LPR) of the (S) and (M) models are equivalent. Furthermore, the LPR of (S) model is tighter than that of (B).

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**Computational Analysis of the SMILP for SOPSP**

**Description of Experiments**

- 14 different SOPSP instances with 12 patients and 4-20 patients.
- Three different weight structures.
- 420 sample average approximations (SAA), each with 1,000 scenarios.
- Time limit: 2 hours.
- Using a standard optimization modeling tool, AMPL, and a commercial MILP solver, CPLEX, with default settings.

**Results**

- Using our model (S), we were able to solve all of the 420 SAs instances in less than 20 minutes.
- Comparison with model (B).
  - Min $\lambda^+$ = 1.5 with $\lambda^+$ = 5, $\lambda^+$ = 7.5
  - Max $\lambda^+$ = 10 with $\lambda^+$ = 0, $\lambda^+$ = 10
  - Using model (B), we were able to solve only 160 of the 420 SAs.
  - It took 6-138 time longer than our model to solve these 160 SAs.
  - The remaining 260 SAs that were not solved terminated on relative MIP gaps ($\frac{Delta}{UB} \times 100\%$) between 16% and 70%.
- Comparison with model (M).
  - Using model (M), we were able to solve only 320 of the 420 SAs.
  - It took 2-43 time longer than our model to solve these 320 SAs.
  - The remaining 80 SAs that were not solved terminated on relative MIP gaps between 15% and 25%.

**Future Work & Conclusions**

- We present a new SMILP for the basic (yet still challenging) single-resource stochastic appointment sequencing and scheduling problem.
- We also compare this model to two closely-related formulations in the literature and analyze them both empirically and theoretically.
- Computational results demonstrated significant improvements in performance could be gained with our proposed model.
- We plan to:
  - Extend our approach to include additional sources of uncertainty, particularly variability in patient arrival time.
  - Develop templates and policies for scheduling patients dynamically as they randomly request future appointments.

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**References**