MICHIGAN ENGINEERING
UNIVERSITY OF MICHIGAN

Scheduling Colonoscopy Patients Under Uncertainty

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CENTER FOR
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UNIVERSITY OF MICHIGAN

Research Team

- Systems Concepts for the Optimization and Personalization of Endoscopy Scheduling (SCOPES) Team:
  - Dr. Amy Cohn, Associate Professor, Industrial and Operations Engineering
  - Dr. Sameer Saini and Dr. Jacob Kurlander, The University of Michigan Health System (UMHS) and the Vetrans Ann Arbor Healthcare System (VAAHS)
  - All CHEPS students who contribute to SCOPES

Presentation Outline

- Motivation and Overview
- The Offline Stochastic Colonoscopy Problem (OSSP)
- Solution Approach: Monte Carlo Optimization
- Numerical Experiment
- Conclusion and Future Directions

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Endoscopy Clinic

- Outpatient Procedure Clinic
- Conducting screening and surveillance procedures for diseases affecting the digestive system
  - abdominal pain, colitis, constipation, etc.
- Encountering exponentially increasing demand in a resource-constrained setting
  - 7.25 million procedures in 2010[1]

[1. Peery, Anne F., et al, 2012]

Colonoscopy Procedure

- The most common screening test for Colorectal Cancer (CRC)
  - 2nd leading cause of cancer-related death in the US[2]
  - 4.5 million age-eligible subjects in the US (≥50 years)[3]

[2. American Cancer Society] [3. Jain et al (2015)]

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- Medical procedure, usually performed by a gastroenterologist, allows for direct visual examination of the entire colon and rectum
  - Spot existing cancer, prompting treatment
  - Prevent future cancer (***polyps***)

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- Can help reduce CRC incidence by about 40% and mortality by about 50%[1]

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Colonoscopy Appointment Scheduling

- Rich literature about outpatient scheduling

Colonoscopy Appointment Scheduling

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- However, one special characteristic of colonoscopy scheduling is the unique bimodal duration structure

Type-Duration	Prep Quality	Health Conditions	No. of Polyps	Type of Sedation
Easy-Short	Good	Good	Low	Conscious
Complex-Long	Poor	Poor	High	Anesthesia

* Note: also for some cases procedure is not performed

Colonoscopy Scheduling

- Rich literature about outpatient scheduling
- However, one special characteristic of colonoscopy scheduling is the unique bimodal duration structure

Type-Duration	Prep Quality
Easy-Short	Good
Complex-Long	Poor

- Extra time flushing out colon

Colonoscopy Scheduling

- Multiple conflicting criteria that affect the quality of schedule
  - Waiting, idling, overtime (this talk)
  - Appointment Time Preferences (future talk)
  - Procedure quality (future talk)

[4. Rex et al. (2006)] [5. Klare et al. (2015)]

Research Goal

- Develop a Decision Support Tool to schedule colonoscopy patients while considering

1. The unique and bimodal duration structure
2. Patient absenteeism and unpunctuality
3. Multiple and conflicting criteria that affect the quality of schedule
   - Waiting, idling, and overtime
   - Patient outcomes and preferences (future talk)

Importance?

- On-time Schedule
  - Less provider fatigue
  - More efficient performance

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  - Better experience

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  - Less provider fatigue
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  - Better experience
- More appointments
- Better Outcome!

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The Offline Stochastic Colonoscopy Scheduling Problem (OSSP)

- Setup:
  - A set of P patients, each of a known type
  - Colonoscopy duration, patient arrival, and patient attendance are random variables
    - Each has a known distribution, is independent of scheduled time and from other patients
    - Observed on the day of service after the appointment decisions are made

The Offline Stochastic Colonoscopy Scheduling Problem (OSSP)

- Setup:
  - A set of *P* patients, each of a known type
  - Random characteristics
  - A single provider available during the clinic service hours (*P* slots)
- Goal:
  - Find optimal scheduling decisions for this set of patients, that minimizes a convex combination of the expected total patients waiting time and expected total idle time, and expected overtime

OSSP Formulation

Table 1. OSSP sets, parameters, and variables

Sets

$\{1.....P\}$: set of patients

$\{1.....P\}$: set of appointment intervals

Ω : set of scenarios

OSSP Formulation

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Sets	
$\{1.....P\}$	: set of patients
$\{1.....P\}$	: set of appointment intervals
Ω	: set of scenarios
Fixed (known) model parameters	
$\mathcal{L}$	: planned length of the shift/day
r	: no-show rate
λ^W	: weight assigned to total patient waiting time
λ^T	: weight assigned to idle time
λ^O	: weight assigned to overtime

OSSP Formulation

$w := [\tau, u, \eta]$

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Scenario dependent model parameters	
τ_p^w	: procedure duration of patient p in a scenario w
u_p^ω	: unpunctuality of patient p in a scenario ω
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Sequencing and Scheduling Variables	
x_{ip}	: 1 if patient p is the ith patient; 0 otherwise
t_i	: scheduled appointment time of the ith patient (continuous)

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Actual schedule decision variables	
a_i^ω	: arrival time of the ith scheduled patient under scenario ω
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o^ω	: overtime under scenario ω

OSSP $\quad v = \min f \;=\; \lambda^{W}\mathrm{E}[\text{total waiting}] + \lambda^{T}\mathrm{E}[\text{total idling}] + \lambda^{O}\mathrm{E}[\text{overtime}]$

s.t.

Parameters

$0 - \mathcal{L}$: clinic service hours
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Each patient is assigned is assigned to one position in the sequence

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& t_1 \geq 0 \\
& t_i \geq t_{i-1} && \text{for } i = 2, \ldots, P \\
& t_i \leq \mathcal{L} && \text{for } i = 1, \ldots, P
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Appointment times:
- obey the sequence
- within the clinic service hours

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$$a_i^\omega = t_i + \sum_{p=1}^{P} u_p^\omega \cdot x_{ip} \qquad \text{for } i = 1, \ldots, P, \omega \in \Omega$$

Arrival time= scheduled$\pm$unpunctuality

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$$s_i^\omega \geq a_i^\omega \qquad \text{for } i = 1, \ldots, P, \omega \in \Omega$$

start time=max(arrival, completion of previous patient)

$$s_i^\omega \geq s_{i-1}^\omega + \sum_{p=1}^{P} \tau_p^\omega \cdot x_{i-1p} \qquad \text{for } i = 2, \ldots, P, \omega \in \Omega$$

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Actual Schedule (a,s) $\neq$ (X, t)Planned Schedule

$$\text{Metrics} := \begin{cases} \text{waiting}(s-a) & : \text{failure to start and/or arrive as scheduled} \\ \text{overtime } (o) & : \text{failure to complete all procedures within the planned time limit } \mathcal{L} \\ \text{idling } (g) & : \text{free time between the completion of a procedure and arrival of the next scheduled patient} \end{cases}$$

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$$\text{OSSP} \quad v = \min f = \sum_{\omega \in \Omega} \phi(\omega) \Big[\lambda^W \sum_{i=1}^{P} (s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{P} g_i^\omega + \lambda^O o^\omega \Big]$$

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$$g_1^\omega \geq a_1^\omega - t_1 \qquad \forall \omega \in \Omega$$

$$g_i^\omega \geq a_i^\omega - \Big(s_{i-1}^\omega + \sum_{p=1}^{P} \tau_p^\omega \cdot x_{i-1p}\Big) \qquad \text{for } i = 2, \ldots, P, \omega \in \Omega$$

$$o^\omega \geq a_P^\omega + \sum_{j=1}^{P} \tau_j^\omega \cdot x_{Pj} - \mathcal{L} \qquad \forall \omega \in \Omega$$

Idle time before the *ith* patient

$$g^\omega, o^\omega \geq 0 \qquad \forall \omega \in \Omega$$

$$x_{ip} \in \{0, 1\} \qquad \forall (i, p) \in \{1, \ldots, P\}$$

overtime

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finite valued, piecewise linear , and convex on (X,t)

$$\text{OSSP(N)} \qquad v_N = \min \Big\{ f_N(\mathbf{X}, \mathbf{t}) := N^{-1} \sum_{n=1}^{N} \mathcal{F}(\mathbf{X}, \mathbf{t}, w^n) \Big\}$$

$$f_N(\mathbf{X}, \mathbf{t}) \to f_\Omega(\mathbf{X}, \mathbf{t}) \ \texttt{w.p.1.} \quad \texttt{as} \ N \to \infty$$

$$v_N \to v_\Omega^* \ \texttt{w.p.1.} \quad \texttt{as} \ N \to \infty$$

$$S_N := \Big\{ (\mathbf{X}, \mathbf{t}) \in (\mathcal{X}, \mathcal{T}) : f_N(\mathbf{X}, \mathbf{t}) \le v_N \Big\} \xrightarrow{\text{w.p.1. as } N \to \infty} S_\Omega := \Big\{ (\mathbf{X}, \mathbf{t}) \in (\mathcal{X}, \mathcal{T}) : f(\mathbf{X}, \mathbf{t}) \le v^* \Big\}$$

Parameters

- $0 - \mathcal{L}$: clinic service hours
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$\forall(i$ **Actual schedule variables**

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Solution Approach: Monte Carlo Optimization

$$N \in \mathcal{N} := \{\texttt{Set of Candidates}\}$$

for m=1:M **do**

Step 1. ***Generate a sample of size*** N

- Generate N scenarios of $\eta_p(\eta_p^1....\eta_p^N)$, $\forall p \in P$
- Generate N scenarios of $\tau_p(\tau_p^1....\tau_p^N)$, $\forall p \in P$
- Generate N scenarios of $u_p(u_p^1....u_p^N)$, $\forall p \in P$

Step 2. ***Solve Sample Average Problem (OSSP(N))***

Let v_N^m be the corresponding optimal objective value

Step 3. ***Evaluation of the true objective function using Monte Carlo simulation***

3.1 Generate Nscenarios, solve *OSSP(N)*, and obtain $(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m)$

3.2 Generate $N' >> N$ scenarios

3.2 Estimate the true value of the objective function $(\hat{v}_{N'}^m)$ using the schedule $(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m)$:

$$\hat{v}_{N'}^m(\widehat{\mathbf{x}}^m, \widehat{\mathbf{t}}^m) = \frac{1}{N'} \sum_{n=1}^{N'} \Big[\sum_{i=1}^{P} \big(s_i^n(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m) - a_i^n(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m) \big) + g_i^n(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m) + o^n(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m) \Big]$$

end

Compute the average of v_N^m ***and*** $\hat{v}_{N'}^m$ ***overall replication:***

$$\overline{v}_N = \frac{1}{M} \sum_{m=1}^{M} v_N^m \qquad \overline{v}_{N'} = \frac{1}{M} \sum_{m=1}^{M} \hat{v}_{N'}^m(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m)$$

Compute (Approximate) optimality indices

$$AOI_1 = \frac{\min\limits_{m \in M} (\hat{v}_{N'}^m) - \overline{v}_N}{\min\limits_{m \in M} (\hat{v}_{N'}^m)} \qquad AOI_2 = \frac{\overline{v}_{N'} - \overline{v}_N}{\overline{v}_{N'}}$$

Algorithm 1: *OSSP* Monte Carlo Optimization (OSSP-MCO)

for m=1:M **do**

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end

Compute the average of v_N^m ***and*** $\hat{v}_{N'}^m$ ***overall replication:***

$$\overline{v}_N = \frac{1}{M} \sum_{m=1}^{M} v_N^m \qquad \overline{v}_{N'} = \frac{1}{M} \sum_{m=1}^{M} \hat{v}_{N'}^m(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m)$$

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$$AOI_1 = \frac{\min\limits_{m \in M} (\hat{v}_{N'}^m) - \overline{v}_N}{\min\limits_{m \in M} (\hat{v}_{N'}^m)} \qquad AOI_2 = \frac{\overline{v}_{N'} - \overline{v}_N}{\overline{v}_{N'}}$$

Algorithm 1: *OSSP* Monte Carlo Optimization (OSSP-MCO)

for m=1:M **do**

Step 1. ***Generate a sample of size*** N

- Generate N scenarios of $\eta_p(\eta_p^1....\eta_p^N)$, $\forall p \in P$
- Generate N scenarios of $\tau_p(\tau_p^1....\tau_p^N)$, $\forall p \in P$
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Step 2. ***Solve Sample Average Problem (OSSP(N))***

Let v_N^m be the corresponding optimal objective value

Step 3. ***Evaluation of the true objective function using Monte Carlo simulation***

3.1 Generate Nscenarios, solve *OSSP(N)*, and obtain $(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m)$

3.2 Generate $N' >> N$ scenarios

3.2 Estimate the true value of the objective function $(\hat{v}_{N'}^m)$ using the schedule $(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m)$:

$$\hat{v}_{N'}^m(\widehat{\mathbf{x}}^m, \widehat{\mathbf{t}}^m) = \frac{1}{N'} \sum_{n=1}^{N'} \Big[\sum_{i=1}^{P} \big(s_i^n(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m) - a_i^n(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m) \big) + g_i^n(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m) + o^n(\widehat{\mathbf{X}}^m, \widehat{\mathbf{t}}^m) \Big]$$

end

Compute the average of v_N^m ***and*** $\hat{v}_{N'}^m$ ***overall replication:***

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OSSP Parameters: Type-based Duration Model

Type	Prep Quality
Easy-Short	Good
Complex-Long	Poor

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- Type-duration likelihood 0.75

OSSP Parameters: Type-based Duration Model

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- Discrete

OSSP Parameters: Type-based Duration Model

Type	Duration
Easy-Short	$\sim f(\cdot)^{short}$
Complex-Long	$\sim g(\cdot)^{long}$

- Type-duration likelihood $\mathbf{0.75}$
- Discrete
- Continuous
  - e.g., If the predicted type is easy, then duration $\sim f(\cdot)^{short}$ with 0.75 probability or $\sim g(\cdot)^{long}$ with 0.25 probability

OSSP Parameters: Type-based Duration Model

Type	Lowest conceivable value (LCV)	Mode	Highest conceivable value (HCV)
Easy-Short	20	30	60
Complex-Long	60	90	120

- Type-duration likelihood $\mathbf{0.75}$
- Discrete
- Continuous
  - e.g., If the predicted type is easy, then duration $\sim f(\cdot)^{short}$ with 0.75 probability or $\sim g(\cdot)^{long}$ with 0.25 probability
    - ➢ $\lceil |N(\mu_{s(l)}, \sigma^2_{s(l)})| \rceil$
    - ➢ **Three Sigma Rule**

$$\sigma_{s(l)} = \frac{HCV - LCV}{6}$$

OSSP Parameters: Type-based Duration Model

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➢ $\boldsymbol{f(\cdot)^{short} = [|N(30, 6.67)|]}$

➢ $\boldsymbol{g(\cdot)^{long} = [|N(90, 10)|]}$

Presentation Outline

- Motivation and Problem Overview
- The Offline Stochastic Colonoscopy Problem (OSSP)
- Solution Approach: Monte Carlo Optimization
- Numerical Experiment
- Conclusion and Future Directions

Tested Problem

Table 1: Characteristics and parameters of tested problem

Parameter	Distribution
Number of Patients (P)	7 Patients; (easy-short, complex-long)=(4, 3)

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Duration models parameters	short$\sim$ $\lvert Normal(30, 6.67)\rvert$
	long$\sim$ $\lvert Normal(90, 10)\rvert$
Patient unpunctuality (u)	$Normal(\mu_u = 0, -10, -20, -30, \sigma_u = 0, 15, 20)$
No-show rate (r)	18% (Berg et al. 2014)
No-show probability (η)	Uniform(0,1)

Sample Size

Sample Size: Discrete

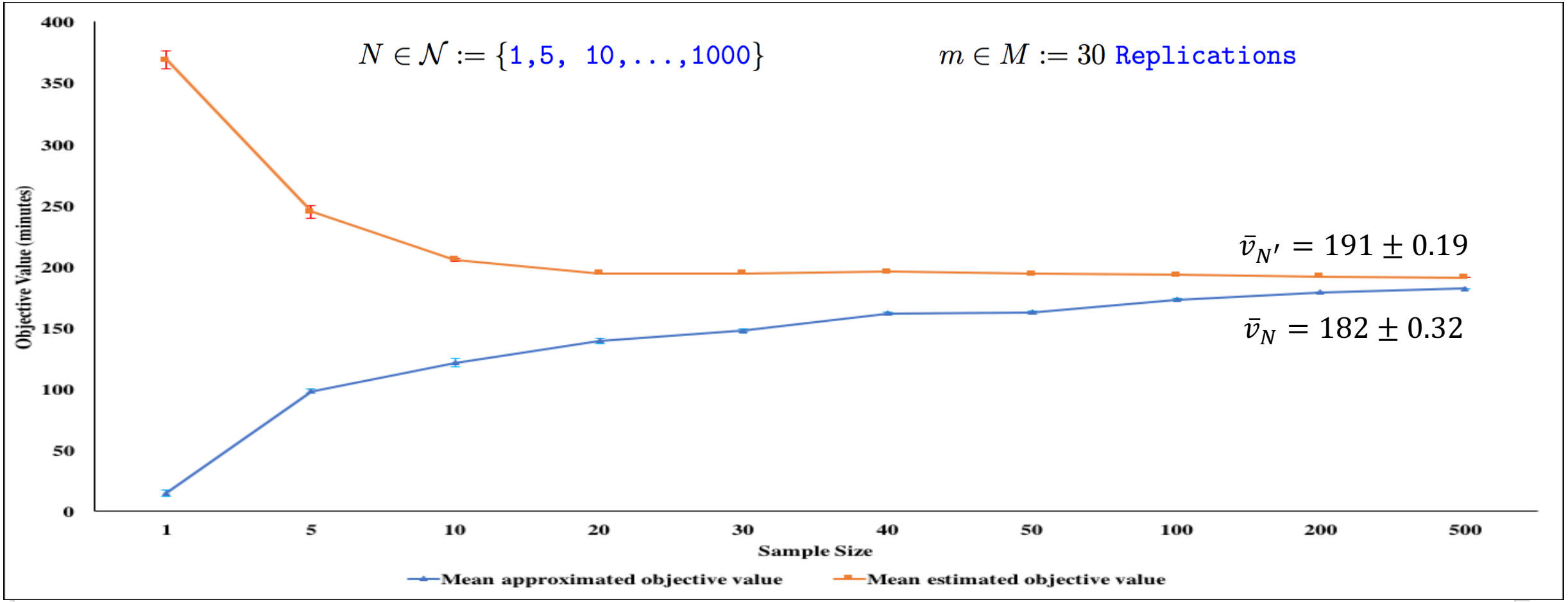
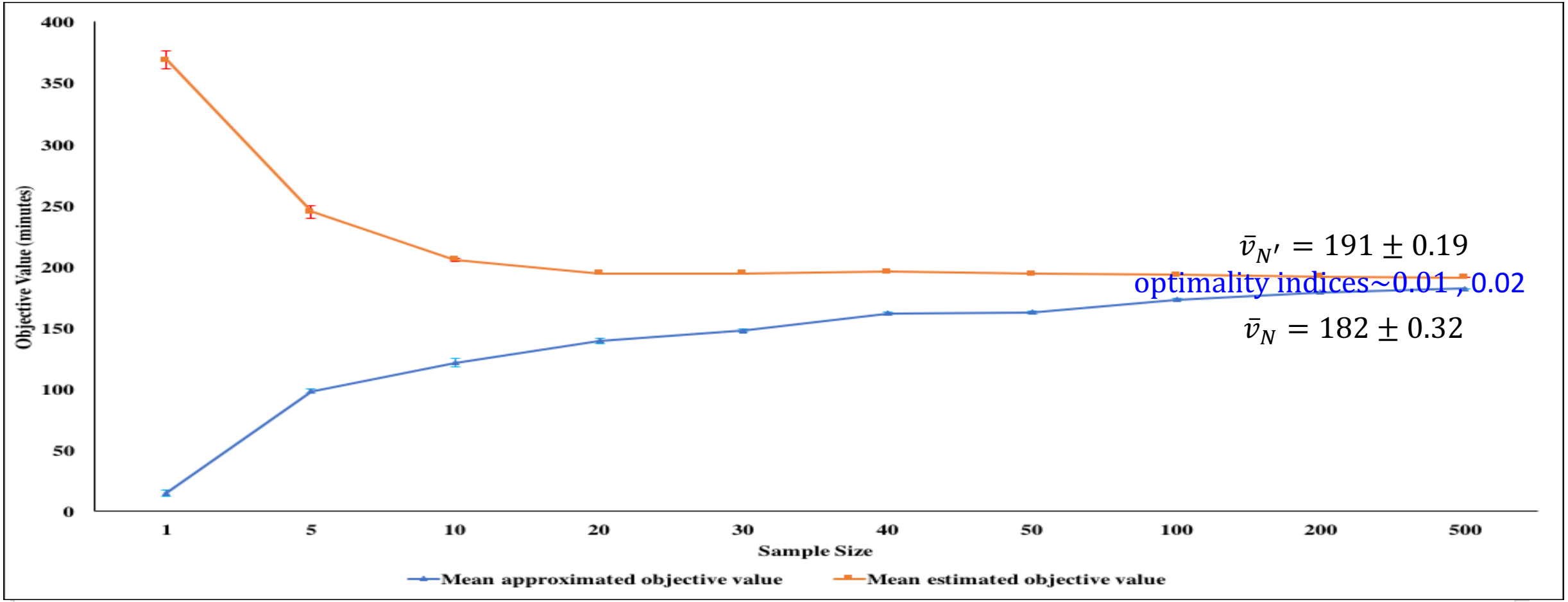


Figure 1. Convergence behavior of OSSP mean estimated objective ($\bar{v}_{N'}$, red) and mean approximated objective ($\bar{v}_N$, blue) as a function of number of scenarios N for the discrete duration model (DM)

Sample Size: Discrete



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Sample Size: Continuous

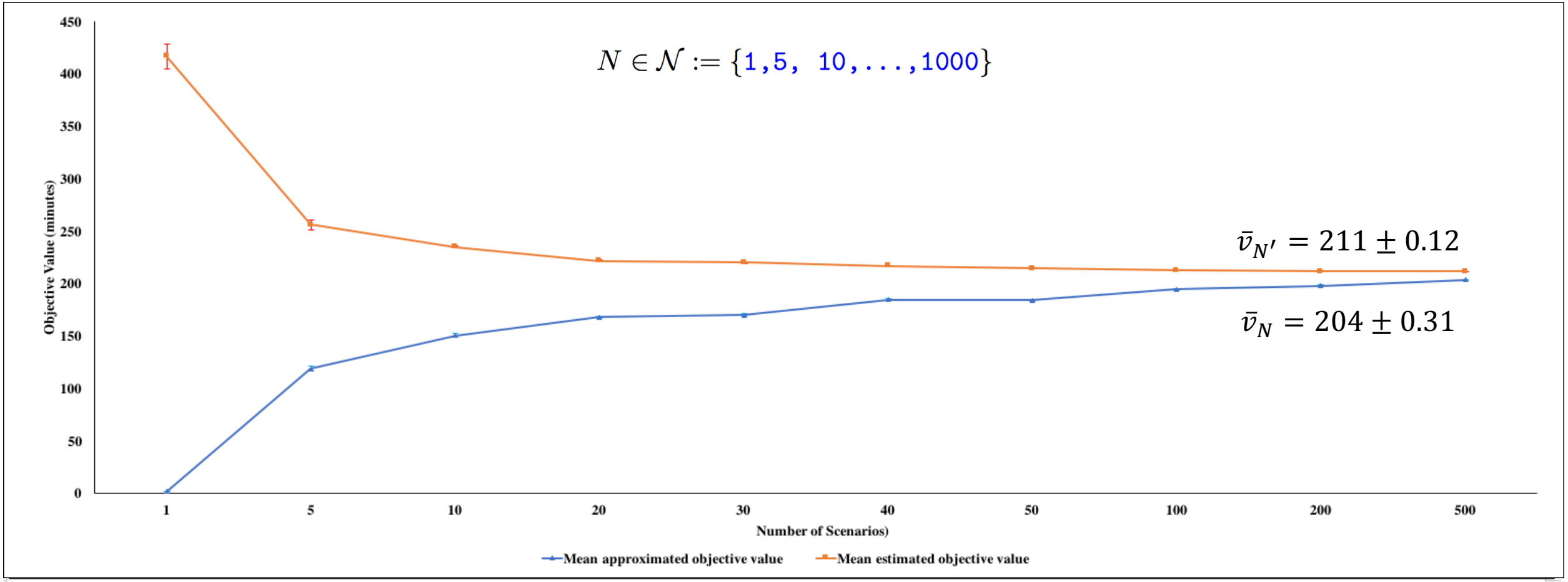
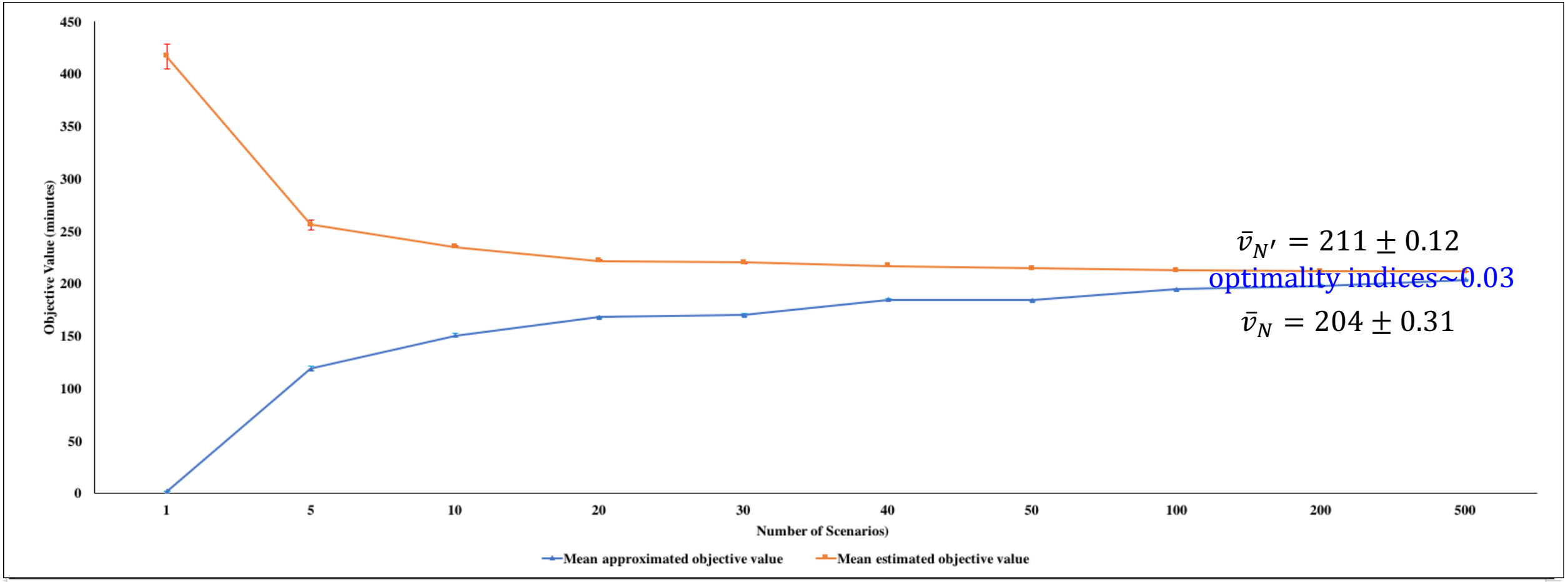


Figure 2. Convergence behavior of OSSP mean estimated objective ($\bar{v}_{N'}$, red) and mean approximated objective ($\bar{v}_N$, blue) as a function of number of scenarios N for the continuous duration model (CM)

Sample Size: Continuous



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Analysis of the Properties of the "Approximated" Schedule/Policies

Sensitivity to Duration Uncertainty

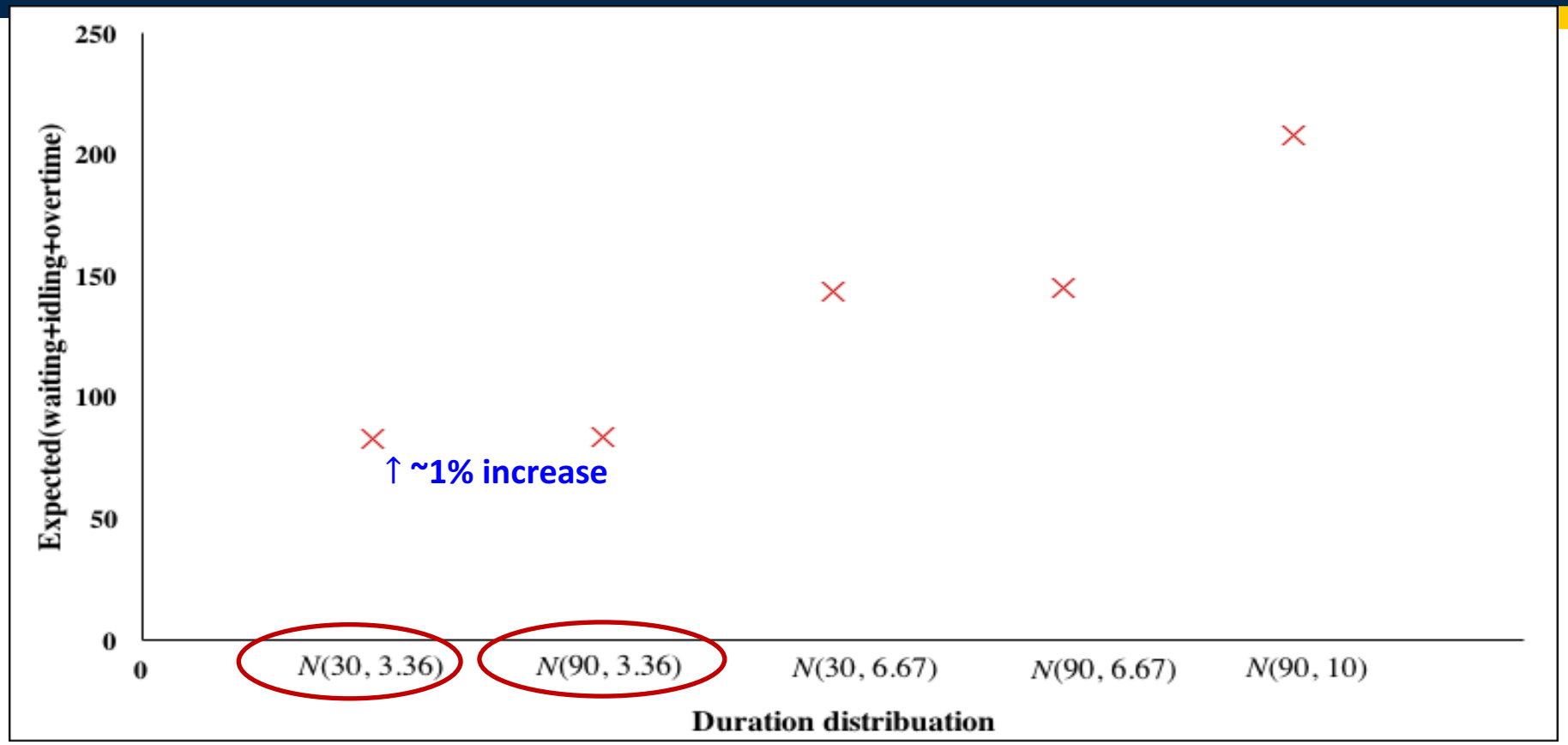


Figure 3. Sensitivity of expected quality of the "approximated" OSSP schedule to the uncertainty of procedure duration

Sensitivity to Duration Uncertainty

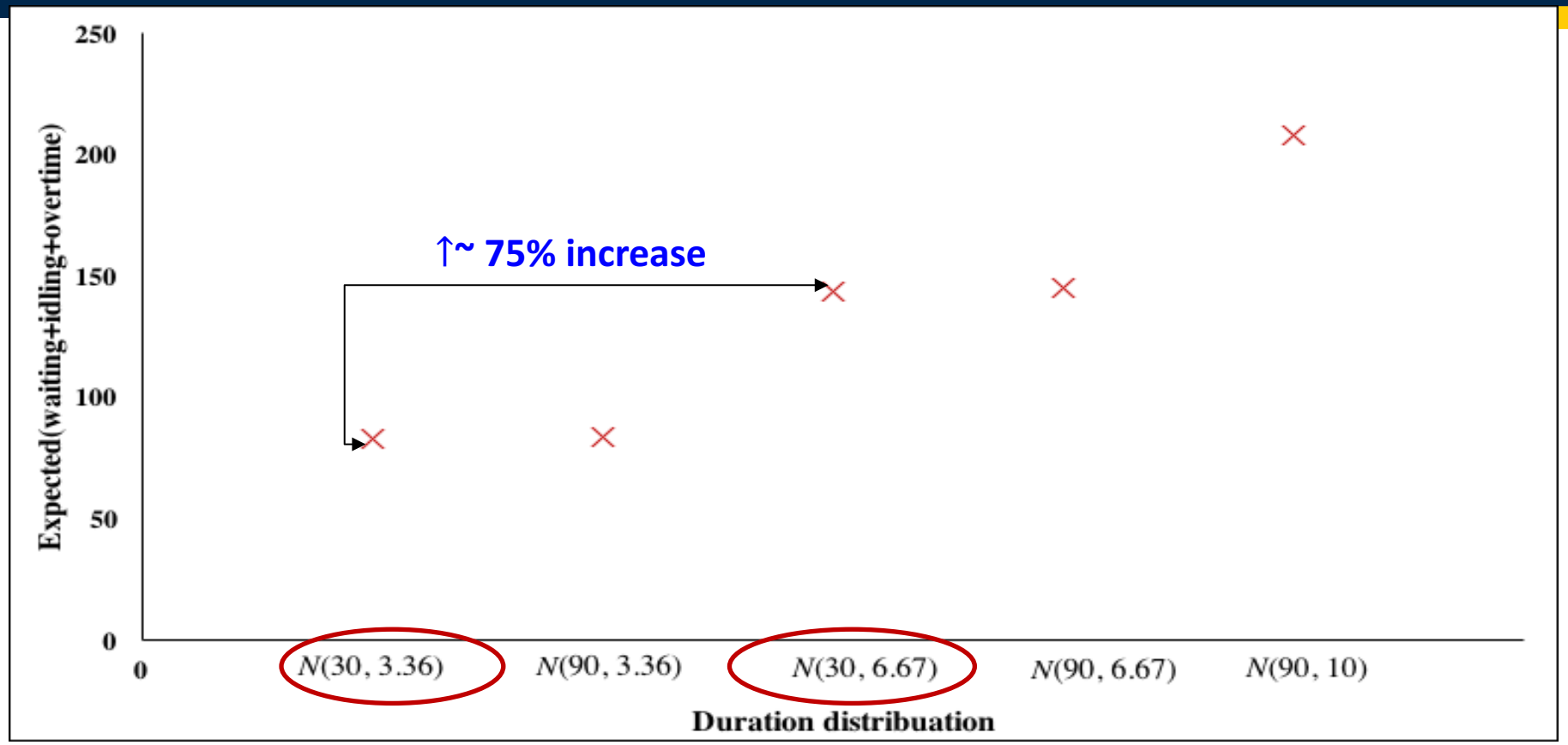
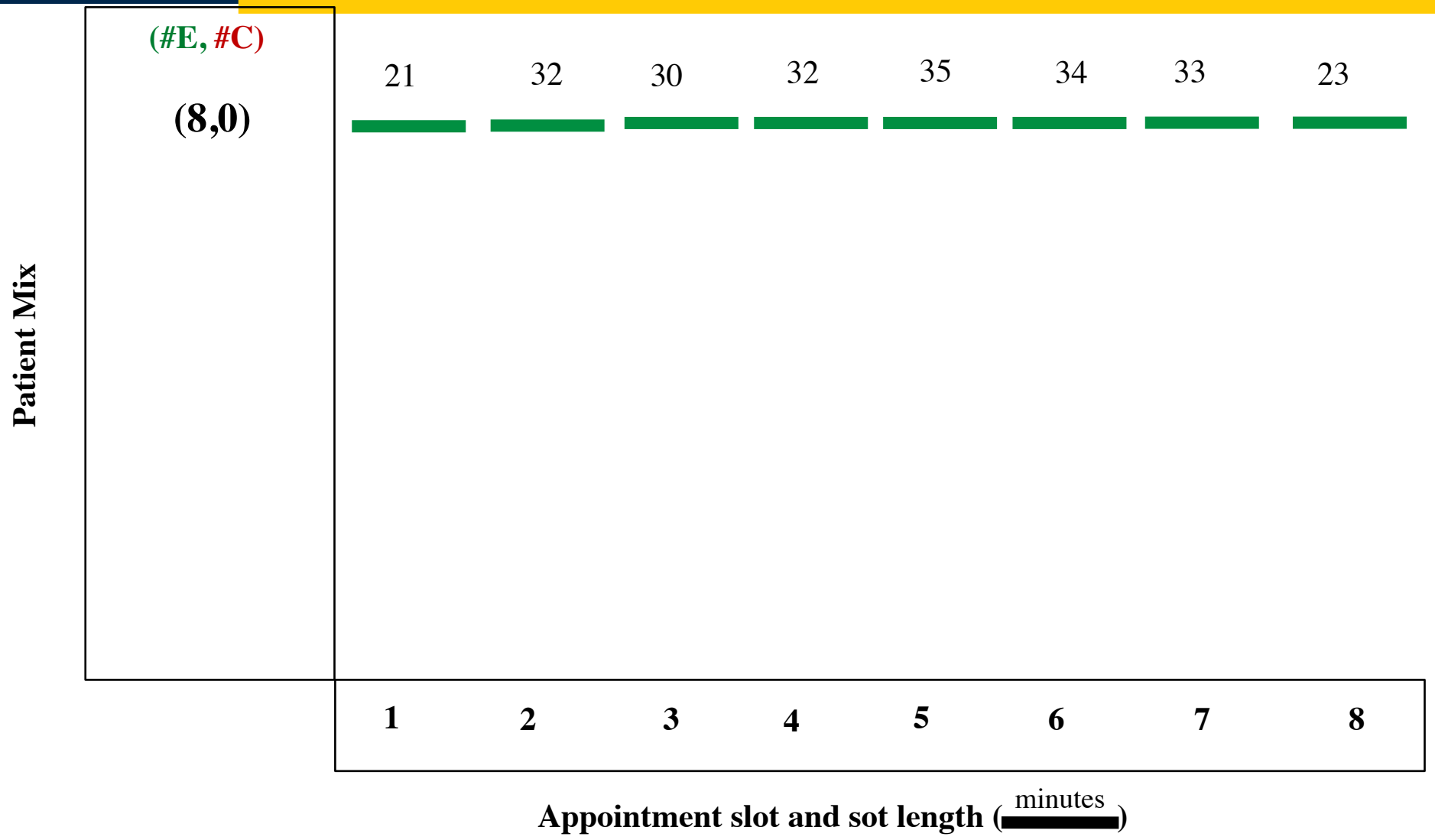


Figure 3. Sensitivity of expected quality of the "approximated" OSSP schedule to the uncertainty of procedure duration

- It would be possible to account for average bowel prep behavior but more challenging to account for variability in prep quality among patients

Approximated Scheduling Policy as a Function of Patient Mix



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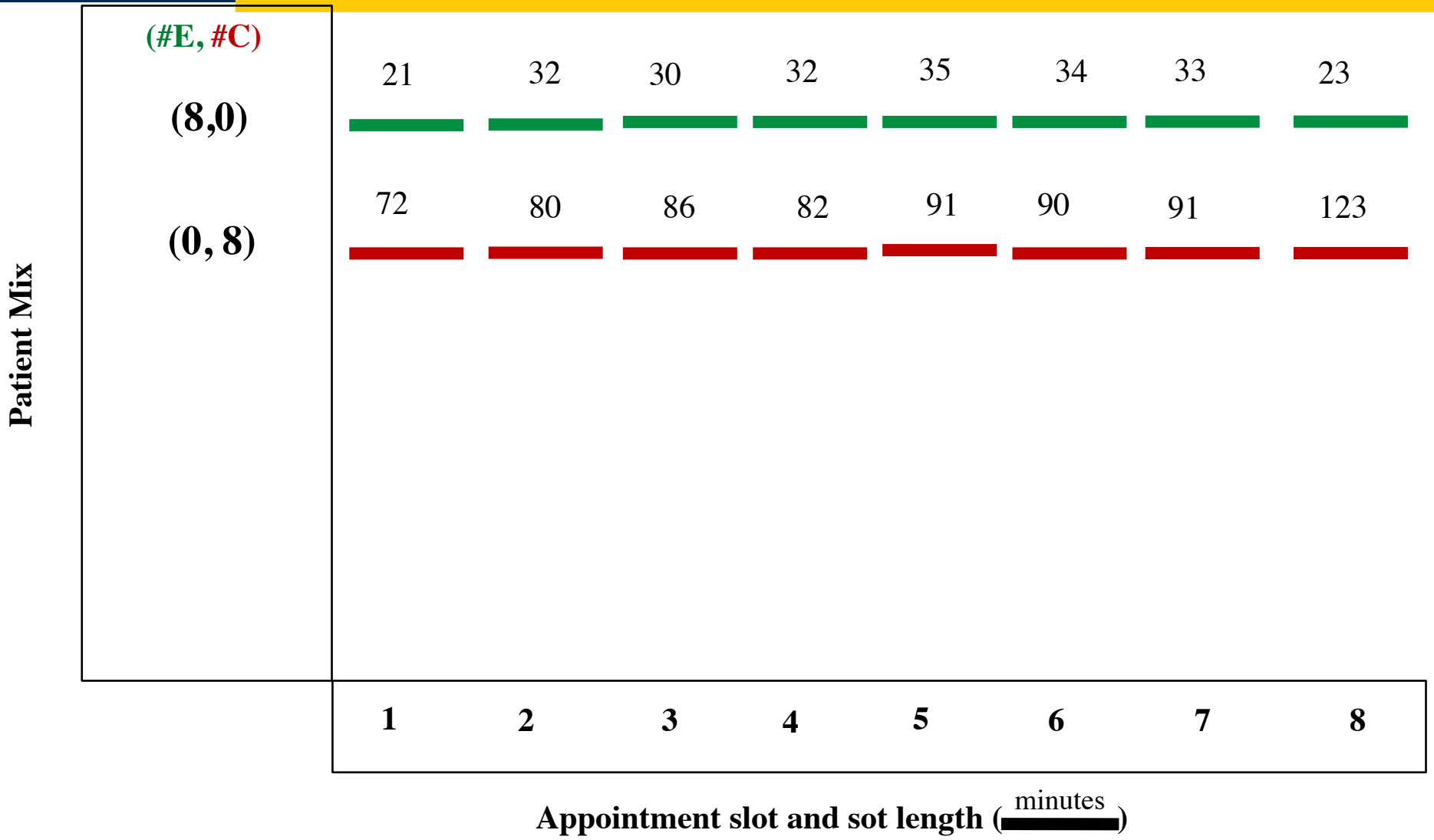

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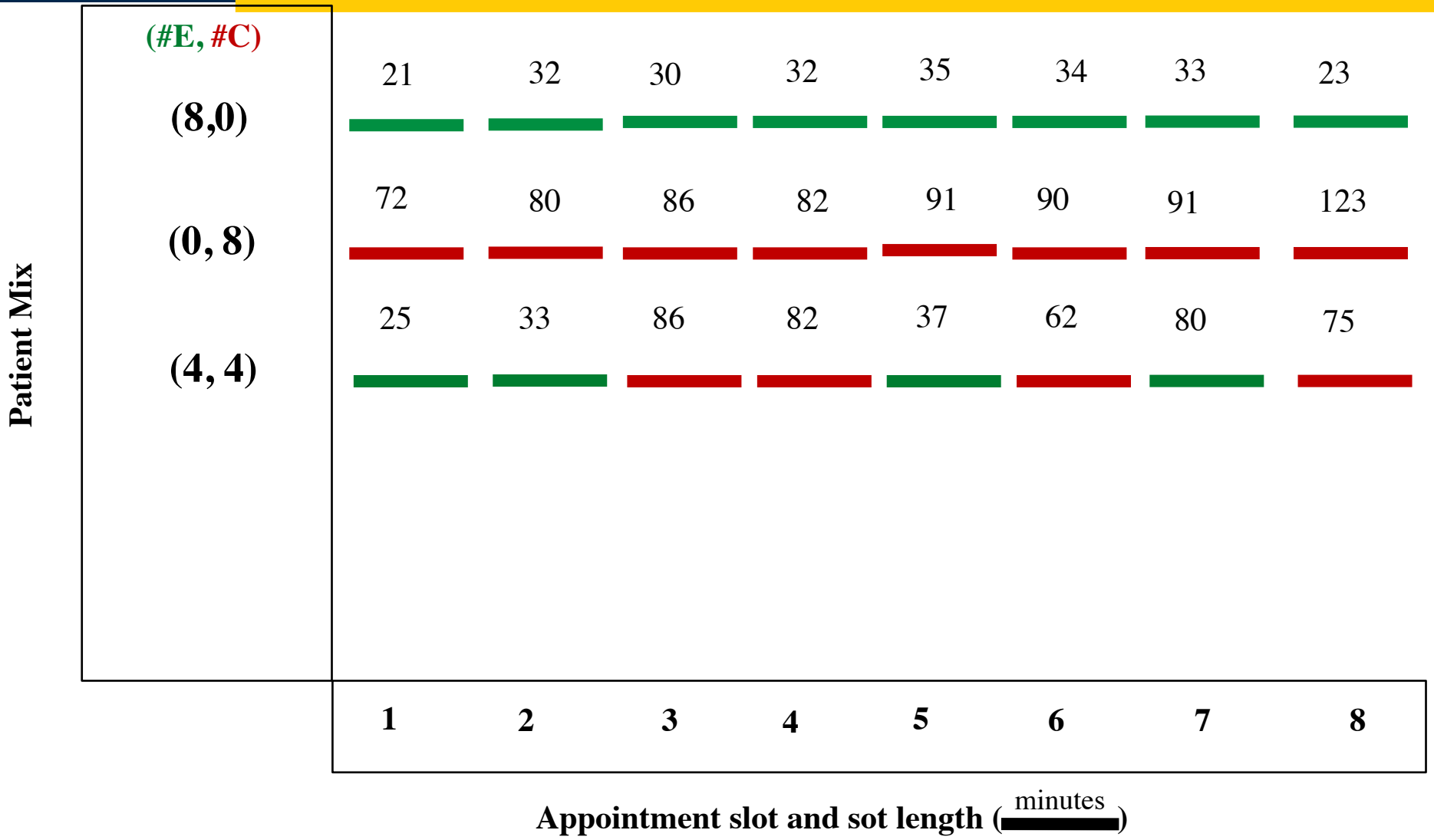

Approximated Scheduling Policy as a Function of Patient Mix

Patient Mix (#E, #C)	1	2	3	4	5	6	7	8
(8,0)	21	32	30	32	35	34	33	23
(0, 8)	72	80	86	82	91	90	91	123
(4, 4)	25	33	86	82	37	62	80	75
(6, 2)	25	33	35	83	36	36	74	42

Appointment slot and sot length (minutes)

Approximated Scheduling Policy as a Function of Patient Mix

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(8,0)	21	32	30	32	35	34	33	23
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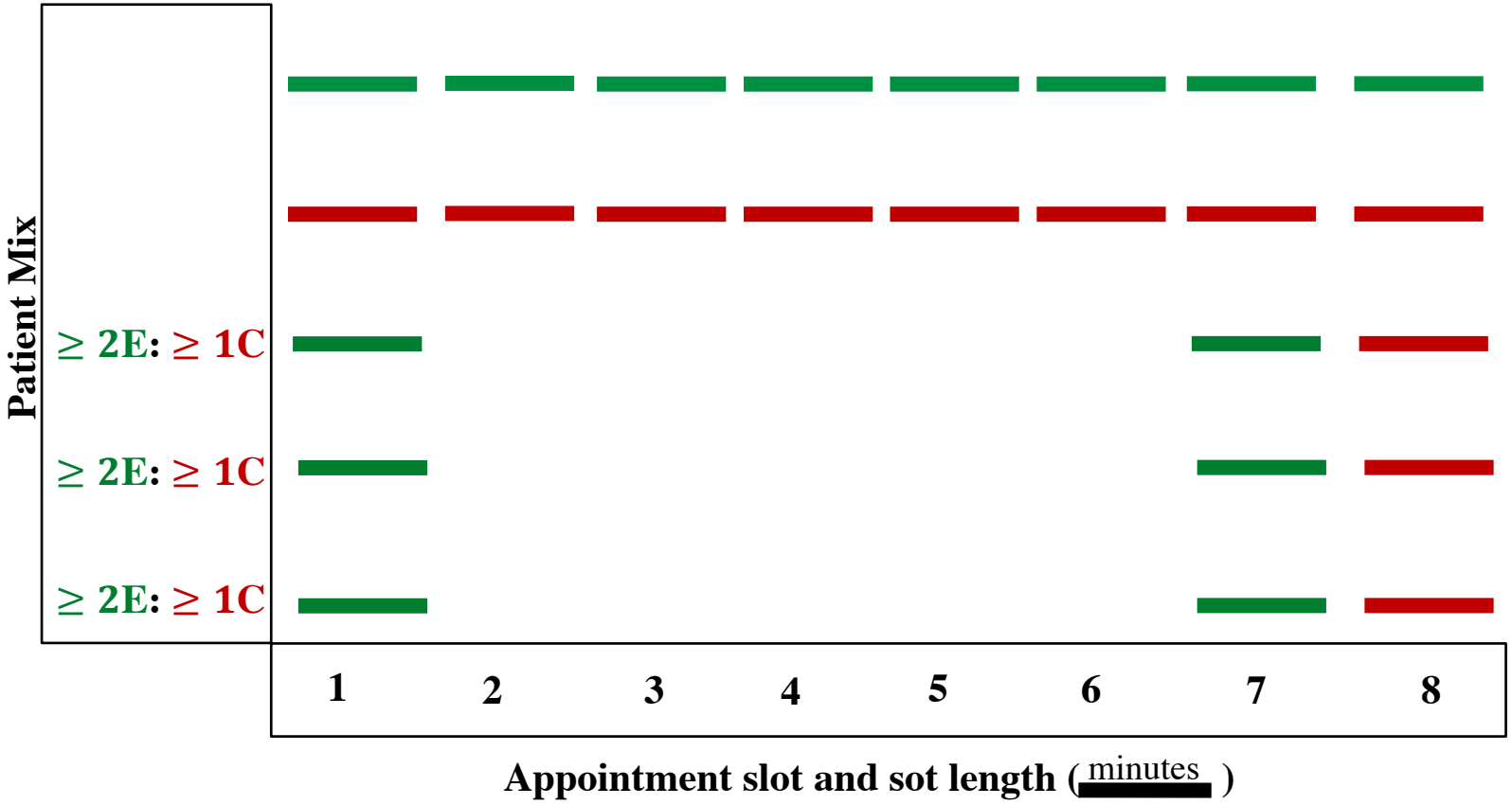
(4, 4)	25	33	86	82	37	62	80	75
(6, 2)	25	33	35	83	36	36	74	42
(2, 6)	24	78	68	83	84	79	83	101

Appointment slot and sot length (minutes)

Approximated Scheduling Policy as a Function of Patient Mix



Sensitivity to Relative Importance Between Metrics

$$OSSP(500) \qquad \min \quad \lambda \cdot \mathbb{E}[\texttt{Provider time}] + (1-\lambda) \cdot \mathbb{E}[\texttt{Patients' Time}]$$

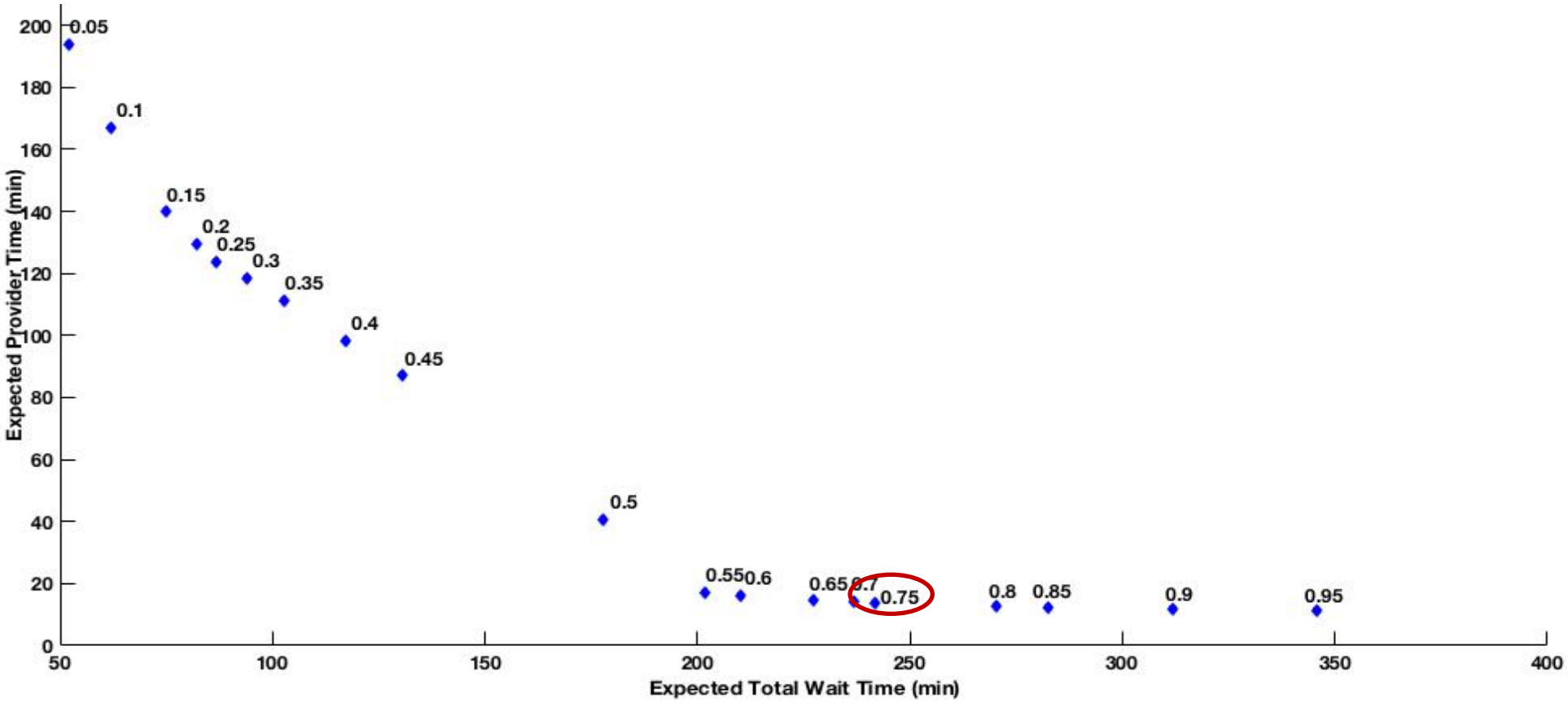


Figure 3. Expected performances of OSSP(500) optimal schedule as function of the trade-off level λ. As λ increases, the expected total waiting time is observed to increase while the expected provider time (overtime and idle time) is observed to decrease until all metrics start to stabilize near $\lambda = 0.65$

Values of Stochastic Optimization in Colonoscopy Scheduling

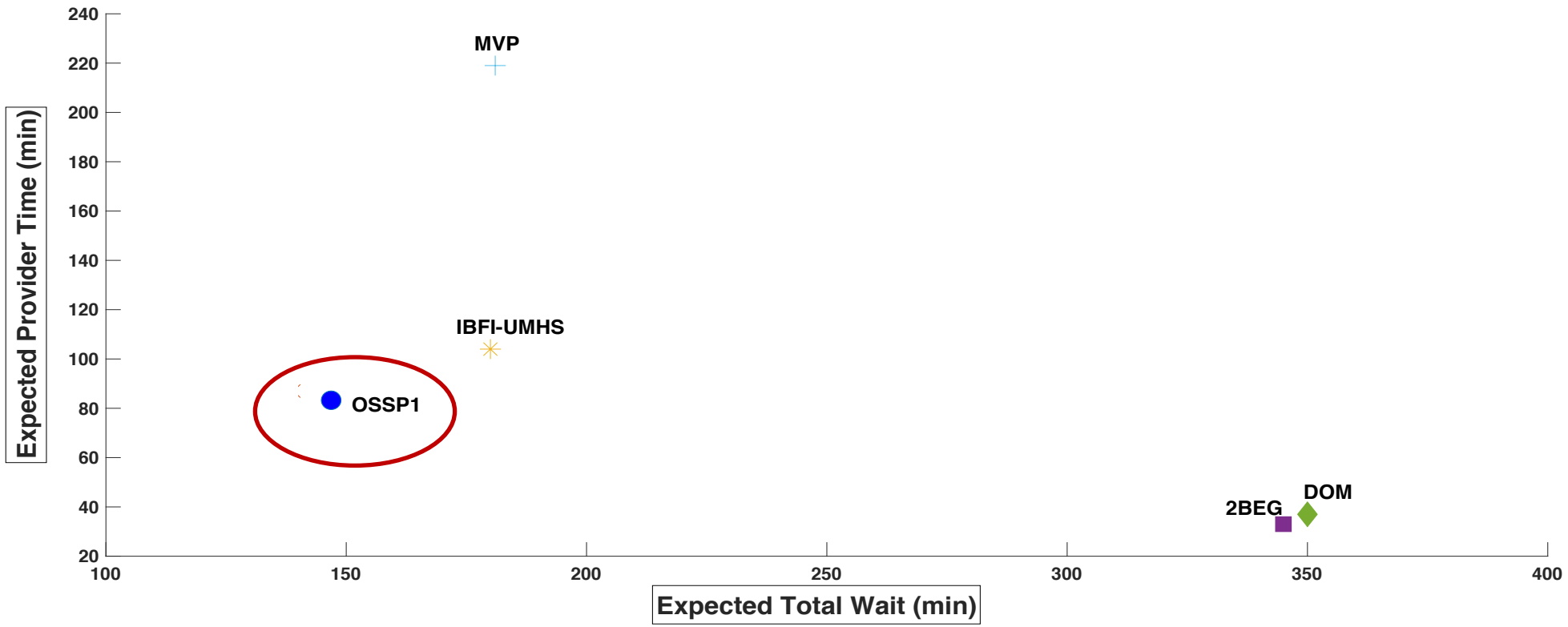
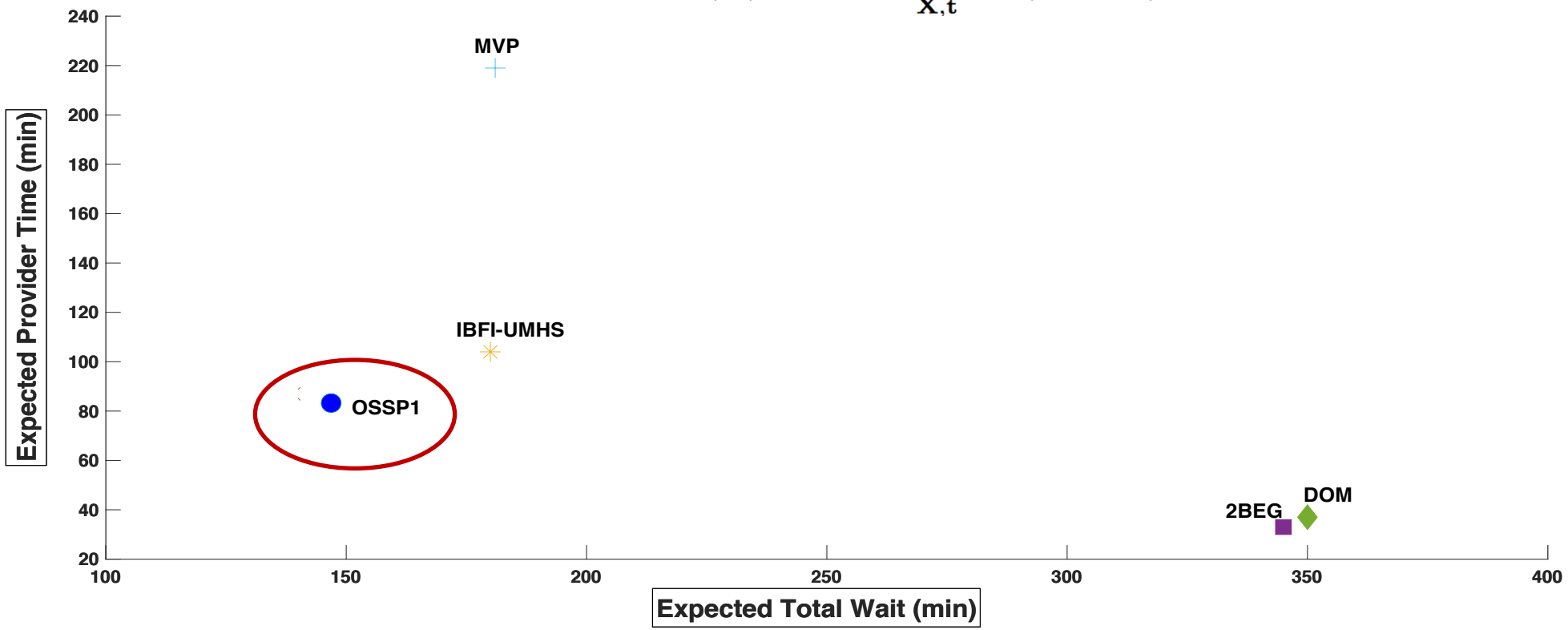


Figure 6. Expected performance of OSSP and some tradional scheduling rules under different levels of no-show and arrivals uncertainties. The case of $r = 18\%$ and $u \sim N(0, 0)$

Values of Stochastic Optimization in Colonoscopy Scheduling

$$OSSP(N) \qquad v_N = \min_{\mathbf{X},\mathbf{t}} \; f_N(\mathbf{X}, \mathbf{t}, w^n)$$



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Values of Stochastic Optimization in Colonoscopy Scheduling

Tabel 3. The value of the stochastic solution for instances varying in uncertainty levels

Type of Uncertainty	EMVP	OSSP(N)	VSS1 (%improvement from EMVP)
Duration variability only	30.86	17.86	42.12%
Duration variability, r $= 18\%$	66.04	25.78	60.96%
Duration variability, r $= 18\%$, &($\mathbf{u} \sim N(-13, 20)$)	80.57	54.62	32.21%
4 **Duration variability, r** $= 18\%$, & ($\mathbf{u} \sim N(0, 20)$)	89.31	48.83	45.32%

$$OSSP(N) \qquad v_N = \min_{\mathbf{X},\mathbf{t}} \; f_N(\mathbf{X}, \mathbf{t}, w^n)$$

$$\texttt{MVP} \qquad \min_{\mathbf{X},\mathbf{t}} \; f(\mathbf{X}, \mathbf{t}, \bar{\omega})$$

$$\begin{aligned} VSS1 &= \mathbf{E}_\omega\left[f\left(\bar{\mathbf{X}}(\bar{\omega}), \bar{t}(\bar{\omega}), \omega\right)\right] - v_N \\ &= EMVP - v_N \end{aligned}$$

Values of Stochastic Optimization in Colonoscopy Scheduling

Tabel 3. The value of the stochastic solution for instances varying in uncertainty levels

Type of Uncertainty	EMVP	OSSP(N)	VSS1 (%improvement from EMVP)	Average EMVP	Average OSSP(N)	VSS2 (%improv AvgEMVP)
Duration variability only	30.86	17.86	42.12%	31.04	21.79	31.99%
Duration variability, r $= 18\%$	66.04	25.78	60.96%	81.53	54.00	33.77%
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$$OSSP(N) \qquad v_N = \min_{\mathbf{X},\mathbf{t}} \; f_N(\mathbf{X}, \mathbf{t}, w^n)$$

$$\texttt{MVP} \qquad \min_{\mathbf{X},\mathbf{t}} \; f(\mathbf{X}, \mathbf{t}, \bar{\omega})$$

$$VSS1 = \mathbf{E}_\omega\left[f\left(\bar{\mathbf{X}}(\bar{\omega}), \bar{t}(\bar{\omega}), \omega\right)\right] - v_N$$
$$= EMVP - v_N$$

$$VSS2 = \mathbf{E}_\omega\left[f\left(\bar{\mathbf{X}}(\bar{\omega}), \bar{t}(\bar{\omega}), \omega\right)\right] - \mathbb{E}_\omega[f\left(\mathbf{X}_N^*, \mathbf{t}_N^*, \omega\right)]$$
$$= \texttt{EMVP-EOSSP(N)}$$

Presentation Outline

- Motivation and Problem Overview
- Research Structure
- The Offline Stochastic Colonoscopy Problem (OSSP)
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Conclusion

- Colonoscopy scheduling have unique characteristics that are different in nature and potentially different from other OPC

- Scheduling polices depends on the uncertainty within these characteristics

- Properties of the "Approximated" schedules can be exploited in designing appointment scheduling tool (system)

Future directions

- Methodology:
  - Incorporate patient outcomes and preference
  - Probabilistic models for duration, absenteeism and arrivals
  - Incorporate interaction effect of uncertain parameters
  - Tractable and efficient solution methods

- Practice
  - Clinical observation
  - Bowel prep interventions
  - Universal template and scheduling rules

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  - Professor Marina Epelman
  - Professor Ruiwei Jiang
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- The IOE Family at the University of Michigan

Thank You!

MICHIGAN ENGINEERING
UNIVERSITY OF MICHIGAN

No one can whistle a symphony. It takes a whole orchestra to play it

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Questions and Discussions