Scheduling Colonoscopy Patients Under Uncertainty

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May 5, 2017
Research Team

• Systems Concepts for the Optimization and Personalization of Endoscopy Scheduling (SCOPES) Team:

  • Dr. Amy Cohn, Associate Professor, Industrial and Operations Engineering
  • Dr. Sameer Saini and Dr. Jacob Kurlander, The University of Michigan Health System (UMHS) and the Veterans Ann Arbor Healthcare System (VAAHS)
  • All CHEPS students who contribute to SCOPES
Presentation Outline

- Motivation and Overview
- The Offline Stochastic Colonoscopy Problem (OSSP)
- Solution Approach: Monte Carlo Optimization
- Numerical Experiment
- Conclusion and Future Directions
Presentation Outline

• Motivation and Overview
  • The Offline Stochastic Colonoscopy Problem (OSSP)
  • Solution Approach: Monte Carlo Optimization
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  • Conclusion and Future Directions
Endoscopy Clinic

- Outpatient Procedure Clinic

- Conducting screening and surveillance procedures for diseases affecting the digestive system
  - abdominal pain, colitis, constipation, etc.

- Encountering exponentially increasing demand in a resource-constrained setting
  - 7.25 million procedures in 2010\(^1\)

\(^1\) Peery, Anne F., et al, 2012
Colonoscopy Procedure

- The most common screening test for Colorectal Cancer (CRC)
  - 2nd leading cause of cancer-related death in the US²
  - 4.5 million age-eligible subjects in the US (≥50 years)³
Colonoscopy Procedure

• The most common screening test for Colorectal Cancer (CRC)
  – 2nd leading cause of cancer-related death in the US
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• Medical procedure, usually performed by a gastroenterologist, allows for direct visual examination of the entire colon and rectum
  – Spot existing cancer, prompting treatment
  – Prevent future cancer (*polyps*)
Colonoscopy Procedure

• The most common screening test for Colorectal Cancer (CRC)
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• Medical procedure, usually performed by a gastroenterologist, allows for direct visual examination of the entire colon and rectum
  – Spot existing cancer, prompting treatment
  – Prevent future cancer (polyps)

• Can help reduce CRC incidence by about 40% and mortality by about 50%¹
Colonoscopy Appointment Scheduling

- Rich literature about outpatient scheduling
Colonoscopy Appointment Scheduling

• Rich literature about outpatient scheduling
• However, one special characteristic of colonoscopy scheduling is the unique bimodal duration structure

<table>
<thead>
<tr>
<th>Type-Duration</th>
<th>Prep Quality</th>
<th>Health Conditions</th>
<th>No. of Polyps</th>
<th>Type of Sedation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easy-Short</td>
<td>Good</td>
<td>Good</td>
<td>Low</td>
<td>Conscious</td>
</tr>
<tr>
<td>Complex-Long</td>
<td>Poor</td>
<td>Poor</td>
<td>High</td>
<td>Anesthesia</td>
</tr>
</tbody>
</table>

* Note: also for some cases procedure is not performed
Colonoscopy Scheduling

• Rich literature about outpatient scheduling

• However, one special characteristic of colonoscopy scheduling is the unique bimodal duration structure

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• Extra time flushing out colon
Colonoscopy Scheduling

- Multiple conflicting criteria that affect the quality of schedule
  - Waiting, idling, overtime (this talk)
  - Appointment Time Preferences (future talk)
  - Procedure quality (future talk)
Research Goal

- Develop a **Decision Support Tool** to schedule colonoscopy patients while considering:
  
  1. The **unique and bimodal** duration structure
  2. Patient **absenteeism** and **unpunctuality**
  3. Multiple and **conflicting** criteria that affect the quality of schedule
     - Waiting, idling, and overtime
     - Patient outcomes and preferences (**future talk**)
Importance?

• On-time Schedule
  – Less provider fatigue
  – More efficient performance
Importance?

• On-time Schedule
  – Less provider fatigue
  – More efficient performance

• Less Waiting
  – Better experience
Importance?

- On-time Schedule
  - Less provider fatigue
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- Less Waiting
  - Better experience

- More appointments
Importance?

• On-time Schedule
  – Less provider fatigue
  – More efficient performance

• Less Waiting
  – Better experience

• More appointments

• Better Outcome!
Presentation Outline

- Motivation and Problem Overview
- **The Offline Stochastic Colonoscopy Problem (OSSP)**
- Solution Approach: Monte Carlo Optimization
- Numerical Experiment
- Conclusion and Future Directions
The Offline Stochastic Colonoscopy Scheduling Problem (OSSP)

Setup:

- A set of $P$ patients, each of a known type
- Colonoscopy duration, patient arrival, and patient attendance are random variables
  - Each has a known distribution, is independent of scheduled time and from other patients
  - Observed on the day of service after the appointment decisions are made
The Offline Stochastic Colonoscopy Scheduling Problem (OSSP)

• Setup:
  • A set of $P$ patients, each of a known type
  • Random characteristics
  • A single provider available during the clinic service hours ($P$ slots)

• Goal:
  • Find optimal scheduling decisions for this set of patients, that minimizes a convex combination of the expected total patients waiting time and expected total idle time, and expected overtime
### Table 1. OSSP sets, parameters, and variables

**Sets**

- \( \{1, \ldots, P\} \) : set of patients
- \( \{1, \ldots, P\} \) : set of appointment intervals
- \( \Omega \) : set of scenarios
### Table 1. OSSP sets, parameters, and variables

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#### Fixed (known) model parameters

- \(\mathcal{L}\) : planned length of the shift/day
- \(r\) : no-show rate
- \(\lambda_W\) : weight assigned to total patient waiting time
- \(\lambda_T\) : weight assigned to idle time
- \(\lambda_O\) : weight assigned to overtime
# OSSP Formulation

## Table 1. OSSP sets, parameters, and variables

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### Scenario dependent model parameters
- \( \tau^w_p \) : procedure duration of patient \( p \) in a scenario \( w \)
- \( u^w_p \) : unpunctuality of patient \( p \) in a scenario \( w \)
- \( \eta^w_p \) : no-show probability of patient \( p \) in a scenario \( w \)
- \( \phi(\omega) \) : probability of scenario \( \omega \)
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{1.....P} : set of patients
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Scenario dependent model parameters
\( \tau_p^w \) : procedure duration of patient \( p \) in a scenario \( w \)
\( u_p^w \) : unpunctuality of patient \( p \) in a scenario \( \omega \)
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\( \phi(\omega) \) : probability of scenario \( \omega \)

Sequencing and Scheduling Variables
\( x_{ip} \) : 1 if patient \( p \) is the \( i \)th patient; 0 otherwise
\( t_i \) : scheduled appointment time of the \( i \)th patient (continuous)
OSSP Formulation

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<td>y^w_i : idle time before the start of the (i)th scheduled patient under scenario (w)</td>
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<td>o^w_i : overtime under scenario (w)</td>
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OSSP: \[ v = \min f = \lambda^W E[\text{total waiting}] + \lambda^T E[\text{total idling}] + \lambda^O E[\text{overtime}] \]

s.t.

**Parameters**

- \(0 - L\): clinic service hours
- \(\tau_p\): patient \(p\) duration in \(\omega\)
- \(u_p^w\): patient \(p\) unpunctuality in \(\omega\)
- \(\phi(\omega)\): probability of scenario \(\omega\)

**Sequencing and scheduling variables**

- \(x_{ip}\): 1 if \(p\) is the \(i\)th patient; 0 otherwise
- \(t_i\): scheduled time of the \(i\)th patient

**Actual schedule variables**

- \(a_i^\omega\): arrival time of the \(i\)th patient under \(\omega\)
- \(s_i^\omega\): start time of \(i\)th patient under \(\omega\)
- \(g_i^\omega\): idle time before the start of the \(i\)th patient under \(\omega\)
- \(d_i^\omega\): idle time before the start of the \(i\)th patient under \(\omega\)
Each patient is assigned is assigned to one position in the sequence for \( p = 1, \ldots, P \)

for \( i = 1, \ldots, P \)
OSSP \[ v = \min f = \lambda W E[\text{total waiting}] + \lambda T E[\text{total idling}] + \lambda O E[\text{overtime}] \]

s.t.
\[ \sum_{i \in P} x_{ip} = 1 \]
\[ \sum_{p \in P} x_{ip} = 1 \]
\[ t_1 \geq 0 \]
\[ t_i \geq t_{i-1} \]
\[ t_i \leq L \]

for \( p = 1, ..., P \)

for \( i = 1, ..., P \)

Each patient is assigned to one position in the sequence

Appointment times:
- obey the sequence
- within the clinic service hours

Parameters
\[ 0 - L : \text{clinic service hours} \]
\[ \tau_{ip} : \text{patient } p \text{ duration in } \omega \]
\[ u_{ip} : \text{patient } p \text{ unpunctuality in } \omega \]
\[ \phi(\omega) : \text{probability of scenario } \omega \]

Sequencing and scheduling variables
\[ x_{ip} : 1 \text{ if } p \text{ is the } ith \text{ patient}; 0 \text{ otherwise} \]
\[ t_i : \text{scheduled time of the } ith \text{ patient} \]

Actual schedule variables
\[ a_i^\omega : \text{arrival time of the } ith \text{ patient under } \omega \]
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\]
for \( p = 1, \ldots, P \)
for \( i = 1, \ldots, P \)
Each patient is assigned to one position in the sequence

Appointment times:
- obey the sequence
- within the clinic service hours
for \( i = 2, \ldots, P \)
for \( i = 1, \ldots, P \)
for \( i = 1, \ldots, P, \omega \in \Omega \)
Arrival time = scheduled \pm unpunctuality

Parameters
- \( 0 - L \): clinic service hours
- \( \tau_p^\omega \): patient \( p \) duration in \( \omega \)
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\[ t_i \leq \mathcal{L} \]
\[ a_i^\omega = t_i + \sum_{p=1}^{P} u_{p}^\omega \cdot x_{ip} \]
\[ s_i^\omega \geq t_1 \]
\[ s_i^\omega \geq a_i^\omega \]
\[ s_i^\omega \geq s_{i-1}^\omega + \sum_{p=1}^{P} \tau_{p}^\omega \cdot x_{i-1p} \]

for \( p = 1, \ldots, P \)

for \( i = 1, \ldots, P \)

Each patient is assigned is assigned to one position in the sequence

for \( i = 2, \ldots, P \)

Appointment times:
- obey the sequence
- within the clinic service hours

Arrival time = scheduled ± unpunctuality

start time = max(arrival, completion of previous patient)

∀\( \omega \in \Omega \)

for \( i = 1, \ldots, P, \omega \in \Omega \)

\( s_i^\omega \geq s_{i-1}^\omega + \sum_{p=1}^{P} \tau_{p}^\omega \cdot x_{i-1p} \)

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- \( 0 - \mathcal{L} \) : clinic service hours
- \( \tau_{p}^\omega \) : patient \( p \) duration in \( \omega \)
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- \( \phi(\omega) \) : probability of scenario \( \omega \)

Sequencing and scheduling variables
- \( x_{ip} \) : 1 if \( p \) is the \( i \)th patient; 0 otherwise
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Actual schedule variables
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a_i^\omega = t_i + \sum_{p=1}^{P} u_p^\omega \cdot x_{ip}
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s_i^\omega \geq t_1
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s_i^\omega \geq a_i^\omega
\]
\[
s_i^\omega \geq s_{i-1}^\omega + \sum_{p=1}^{P} \tau_p^\omega \cdot x_{i-1p}
\]

for \( p = 1, \ldots, P \)

for \( i = 1, \ldots, P \)

for \( i = 1, \ldots, P, \omega \in \Omega \)

for \( \forall \omega \in \Omega \)

for \( \omega \in \Omega \)

Each patient is assigned is assigned to one position in the sequence

Appointment times:
- obey the sequence
- within the clinic service hours

Arrival time= scheduled± unpunctuality

start time=max(arrival, completion of previous patient)

Actual Schedule \((a, s) \neq (X, t)\) Planned Schedule

\[
\text{metrics :=} \quad \text{waiting (s - a) : failure to start and/or arrive as scheduled}
\]
\[
\text{overtime (o) : failure to complete all procedures within the planned time limit } \mathcal{L}
\]
\[
idling (g) : \text{free time between the completion of a procedure and arrival of the next scheduled patient}
\]

Parameters

\( 0 - \mathcal{L} : \text{clinic service hours} \)
\( \tau_p^\omega : \text{patient } p \text{ duration in } \omega \)
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\( x_{ip} : 1 \text{ if } p \text{ is the } i^{th} \text{ patient}; 0 \text{ otherwise} \)
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\( \delta^\omega : \text{idle time before the start of the } i^{th} \text{ patient under } \omega \)
OSSP \[ v = \min_{\omega \in \Omega} f = \sum_{\omega \in \Omega} \phi(\omega) \left[ \lambda^{W} \sum_{i=1}^{P} (s_{i}^{\omega} - a_{i}^{\omega}) + \lambda^{T} \sum_{i=1}^{P} g_{i}^{\omega} + \lambda^{O} o^{\omega} \right] \]

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\[ s_{i}^{\omega} \geq a_{i}^{\omega} \]
\[ s_{i}^{\omega} \geq s_{i-1}^{\omega} + \sum_{p=1}^{P} \tau_{p}^{\omega} \cdot x_{i-1p} \]
\[ g_{i}^{\omega} \geq a_{i}^{\omega} - t_{1} \]
\[ g_{i}^{\omega} \geq a_{i}^{\omega} - (s_{i-1}^{\omega} + \sum_{p=1}^{P} \tau_{p}^{\omega} \cdot x_{i-1p}) \]
\[ o^{\omega} \geq a_{P}^{\omega} + \sum_{j=1}^{P} \tau_{j}^{\omega} \cdot x_{pj} - \mathcal{L} \]
\[ g_{i}^{\omega}, o^{\omega} \geq 0 \]
\[ x_{ip} \in \{0, 1\} \]

Each patient is assigned to one position in the sequence

for \( p = 1, \ldots, P \)

Appointment times:

- obey the sequence
- within the clinic service hours

for \( i = 1, \ldots, P \)

Arrival time= scheduled± unpunctuality

for \( i = 1, \ldots, P, \omega \in \Omega \)

\[ \text{start time=}\max(\text{arrival, completion of previous patient}) \]

for \( i = 1, \ldots, P, \omega \in \Omega \)

Idle time before the \( i \)th patient

\( \forall \omega \in \Omega \)

\( \forall (i, p) \in \{1, \ldots, P\} \)

overtime

Sequential and scheduling variables

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<tr>
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<th>Sequencing and scheduling variables</th>
<th>Actual schedule variables</th>
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OSSP \[ \begin{align*}
    v = \min f &= \sum_{\omega \in \Omega} \phi(\omega) \left[ \lambda^W \sum_{i=1}^{W} (s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{T} g_i^\omega + \lambda^O \sigma^\omega \right] \quad \text{finite valued, piecewise linear, and convex on } (X,t) 
\end{align*} \]

OSSP(N) \[ \begin{align*}
    v_N &= \min \left\{ f_N(X,t) := N^{-1} \sum_{n=1}^{N} F(X, t, w^n) \right\} 
\end{align*} \]

\[ f_N(X,t) \to f_\Omega(X,t) \text{ w.p.1. as } N \to \infty \]
\[ v_N \to v^*_\Omega \text{ w.p.1. as } N \to \infty \]
\[ S_N := \left\{ (X,t) \in (X, T) : f_N(X,t) \leq v_N \right\} \xrightarrow{\text{w.p.1. as } N \to \infty} S_\Omega := \left\{ (X,t) \in (X, T) : f(X,t) \leq v^* \right\} \]

### Parameters

- 0 - \( \mathcal{L} \): clinic service hours
- \( \tau_p^\omega \): patient \( p \) duration in \( \omega \)
- \( u_p^\omega \): patient \( p \) unpunctuality in \( \omega \)
- \( \phi(\omega) \): probability of scenario \( \omega \)

### Sequencing and Scheduling Variables

- \( x_{ip} \): 1 if \( p \) is the \( i \)th patient; 0 otherwise

### Actual Schedule Variables

- \( a_i^\omega \): arrival time of the \( i \)th patient under \( \omega \)
- \( s_i^\omega \): start time of \( i \)th patient under \( \omega \)
- \( g_i^\omega \): idle time before the start of the \( i \)th patient under \( \omega \)
- \( \sigma^\omega \): idle time before the start of the \( i \)th patient under \( \omega \)
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- Conclusion and Future Directions
Solution Approach: Monte Carlo Optimization

\[ N \in \mathcal{N} := \{\text{Set of Candidates}\} \]
for $m=1:M$ do

**Step 1. Generate a sample of size $N$**
- Generate $N$ scenarios of $\eta_p(\eta^1_p, ..., \eta^N_p)$, $\forall p \in P$
- Generate $N$ scenarios of $\tau_p(\tau^1_p, ..., \tau^N_p)$, $\forall p \in P$
- Generate $N$ scenarios of $u_p(u^1_p, ..., u^N_p)$, $\forall p \in P$

**Step 2. Solve Sample Average Problem (OSSP($N$))**
Let $v^m_N$ be the corresponding optimal objective value

**Step 3. Evaluation of the true objective function using Monte Carlo simulation**

3.1 Generate $N$ scenarios, solve OSSP($N$), and obtain ($\hat{X}^m$, $\hat{t}^m$)
3.2 Generate $N' \gg N$ scenarios
3.2 Estimate the true value of the objective function ($\hat{v}^m_N$) using the schedule ($\hat{X}^m$, $\hat{t}^m$):

$$\hat{v}^m_N(\hat{X}^m, \hat{t}^m) = \frac{1}{N'} \sum_{n=1}^{N'} \left[ \sum_{i=1}^{P} \left( s^m_n(\hat{X}^m, \hat{t}^m) - a^m_i(\hat{X}^m, \hat{t}^m) \right) + g^m_i(\hat{X}^m, \hat{t}^m) + o^m(\hat{X}^m, \hat{t}^m) \right]$$

end

**Compute the average of $v^m_N$ and $\hat{v}^m_N$, overall replication:**

$$\bar{v}_N = \frac{1}{M} \sum_{m=1}^{M} v^m_N \quad \quad \bar{v}_{N'} = \frac{1}{M} \sum_{m=1}^{M} \hat{v}^m_N(\hat{X}^m, \hat{t}^m)$$

**Compute (Approximate) optimality indices**

$$AOI_1 = \frac{\min_{m \in M} (\hat{v}^m_{N'}) - \bar{v}_N}{\min_{m \in M} (\hat{v}^m_{N'})} \quad \quad AOI_2 = \frac{\bar{v}_{N'} - \bar{v}_N}{\bar{v}_{N'}}$$

**Algorithm 1: OSSP Monte Carlo Optimization (OSSP-MCO)**
for $m=1:M$ do

Step 1. Generate a sample of size $N$
- Generate $N$ scenarios of $\eta_p(\eta^1_p,...,\eta^N_p)$, $\forall p \in P$
- Generate $N$ scenarios of $\tau_p(\tau^1_p,...,\tau^N_p)$, $\forall p \in P$
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Step 2. Solve Sample Average Problem (OSSP($N$))
Let $v^m_N$ be the corresponding optimal objective value

Step 3. Evaluation of the true objective function using Monte Carlo simulation
3.1 Generate $N$ scenarios, solve OSSP($N$), and obtain $(\hat{X}^m, \hat{t}^m)$
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3.2 Estimate the true value of the objective function ($\hat{v}^m_N$) using the schedule $(\hat{X}^m, \hat{t}^m)$:

$$\hat{v}^m_N(\hat{X}^m, \hat{t}^m) = \frac{1}{N'} \sum_{n=1}^{N'} \left[ \sum_{i=1}^{P} \left( s^n(\hat{X}^m, \hat{t}^m) - a^n_1(\hat{X}^m, \hat{t}^m) \right) + g^n_i(\hat{X}^m, \hat{t}^m) + o^n(\hat{X}^m, \hat{t}^m) \right]$$

end

Compute the average of $v^m_N$ and $\hat{v}^m_N$, overall replication:

$$\bar{v}_N = \frac{1}{M} \sum_{m=1}^{M} v^m_N \quad \bar{v}_{N'} = \frac{1}{M} \sum_{m=1}^{M} \hat{v}^m_N(\hat{X}^m, \hat{t}^m)$$

Compute (Approximate) optimality indices

$$AOI_1 = \frac{\min_{m \in M} (\hat{v}^m_{N'}) - \bar{v}_N}{\min_{m \in M} (\hat{v}^m_{N'})} \quad AOI_2 = \frac{\bar{v}_{N'} - \bar{v}_N}{\bar{v}_{N'}}$$

Algorithm 1: OSSP Monte Carlo Optimization (OSSP-MCO)
for \( m = 1 : M \) do

Step 1. **Generate a sample of size** \( N \)
   - Generate \( N \) scenarios of \( \eta_p(\eta_p^1 \ldots \eta_p^N) \), \( \forall p \in P \)
   - Generate \( N \) scenarios of \( \tau_p(\tau_p^1 \ldots \tau_p^N) \), \( \forall p \in P \)
   - Generate \( N \) scenarios of \( u_p(u_p^1 \ldots u_p^N) \), \( \forall p \in P \)

Step 2. **Solve Sample Average Problem (OSSP(\( N \)))**
   Let \( \bar{v}^m_N \) be the corresponding optimal objective value

Step 3. **Evaluation of the true objective function using Monte Carlo simulation**

3.1 Generate \( N \) scenarios, solve OSSP(\( N \)), and obtain \( \hat{X}^m, \hat{t}^m \)

3.2 Generate \( N' >> N \) scenarios

3.2 Estimate the true value of the objective function (\( \hat{v}^m_{N'} \)) using the schedule (\( \hat{X}^m, \hat{t}^m \)):

\[
\hat{v}^m_{N'}(\hat{X}^m, \hat{t}^m) = \frac{1}{N'} \sum_{n=1}^{N'} \left[ \sum_{i=1}^{P} (s_i^n(\hat{X}^m, \hat{t}^m) - a_i^n(\hat{X}^m, \hat{t}^m)) + g_i^n(\hat{X}^m, \hat{t}^m) + o^n(\hat{X}^m, \hat{t}^m) \right]
\]

end

Compute the average of \( v^m_N \) and \( \hat{v}^m_{N'} \), overall replication:

\[
\bar{v}_N = \frac{1}{M} \sum_{m=1}^{M} v^m_N \quad \bar{\hat{v}}_{N'} = \frac{1}{M} \sum_{m=1}^{M} \hat{v}^m_{N'}(\hat{X}^m, \hat{t}^m)
\]

Compute (Approximate) optimality indices

\[
AOI_1 = \frac{\min_{m \in M} (\hat{v}^m_{N'}) - \bar{v}_N}{\min_{m \in M} (\hat{v}^m_{N'})} \quad AOI_2 = \frac{\bar{\hat{v}}_{N'} - \bar{v}_N}{\bar{\hat{v}}_{N'}}
\]

**Algorithm 1:** OSSP Monte Carlo Optimization (OSSP-MCO)
for $m=1:M$ do

**Step 1. Generate a sample of size $N$**
- Generate $N$ scenarios of $\eta_p(\eta^1_p, \ldots, \eta^N_p)$, $\forall p \in P$
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**Step 2. Solve Sample Average Problem (OSSP($N$))**

Let $v^m_N$ be the corresponding optimal objective value.

**Step 3. Evaluation of the true objective function using Monte Carlo simulation**

3.1 Generate $N$ scenarios, solve OSSP($N$), and obtain ($\hat{X}^m$, $\hat{t}^m$)

3.2 Generate $N' > N$ scenarios

3.2 Estimate the true value of the objective function ($\hat{v}^m_{N'}$) using the schedule ($\hat{X}^m$, $\hat{t}^m$):

$$\hat{v}^m_{N'}(\hat{X}^m, \hat{t}^m) = \frac{1}{N'} \sum_{n=1}^{N'} \left[ \sum_{i=1}^{P} \left( s_i^n(\hat{X}^m, \hat{t}^m) - a^n_i(\hat{X}^m, \hat{t}^m) \right) + g^n_i(\hat{X}^m, \hat{t}^m) + o^n(\hat{X}^m, \hat{t}^m) \right]$$

end

**Compute the average of $v^m_N$ and $\hat{v}^m_{N'}$ overall replication:**

$$\bar{v}_N = \frac{1}{M} \sum_{m=1}^{M} v^m_N, \quad \bar{v}_{N'} = \frac{1}{M} \sum_{m=1}^{M} \hat{v}^m_{N'}(\hat{X}^m, \hat{t}^m)$$

**Compute (Approximate) optimality indices**

$$AOI_1 = \frac{\min_{m \in M} (\hat{v}^m_{N'}) - \bar{v}_N}{\min_{m \in M} (\hat{v}^m_{N'})} \quad AOI_2 = \frac{\bar{v}_{N'} - \bar{v}_N}{\bar{v}_{N'}}$$

**Algorithm 1: OSSP Monte Carlo Optimization (OSSP-MCO)**
### OSSP Parameters: Type-based Duration Model

<table>
<thead>
<tr>
<th>Type</th>
<th>Prep Quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easy-Short</td>
<td>Good</td>
</tr>
<tr>
<td>Complex-Long</td>
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- **Type-duration likelihood 0.75**
OSSP Parameters: Type-based Duration Model

- Type-duration likelihood 0.75
- Discrete

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</tr>
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- Type-duration likelihood 0.75
- Discrete
- Continuous
  - e.g., If the predicted type is easy, then duration $\sim f(\cdot)^{short}$ with 0.75 probability or $\sim g(\cdot)^{long}$ with 0.25 probability
OSSP Parameters: Type-based Duration Model

<table>
<thead>
<tr>
<th>Type</th>
<th>Lowest conceivable value (LCV)</th>
<th>Mode</th>
<th>Highest conceivable value (HCV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easy-Short</td>
<td>20</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>Complex-Long</td>
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<td>90</td>
<td>120</td>
</tr>
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- **Type-duration likelihood 0.75**
- **Discrete**
- **Continuous**
  - e.g., If the predicted type is easy, then duration $\sim f(\cdot)^{short}$ with 0.75 probability or $\sim g(\cdot)^{long}$ with 0.25 probability

  \[ ||N(\mu_s(t), \sigma_{s(t)}^2)|| \]

  \[ \text{Three Sigma Rule} \quad \sigma_{s(t)} = \frac{HCV - LCV}{6} \]
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- Type-duration likelihood 0.75
- Discrete
- Continuous
  - e.g., If the predicted type is easy, then duration $\sim f(\cdot)^{short}$ with 0.75 probability or $\sim g(\cdot)^{long}$ with 0.25 probability

$$\quad f(\cdot)^{short} = [\lfloor N(30, 6.67) \rfloor]$$

$$\quad g(\cdot)^{long} = [\lfloor N(90, 10) \rfloor]$$
Presentation Outline

• Motivation and Problem Overview
• The Offline Stochastic Colonoscopy Problem (OSSP)
• Solution Approach: Monte Carlo Optimization
• Numerical Experiment
• Conclusion and Future Directions
Table 1: Characteristics and parameters of tested problem

<table>
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<th>Distribution</th>
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<td>(\alpha = 0.75)</td>
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<tr>
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<td>short(~</td>
</tr>
<tr>
<td></td>
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</table>
| Duration models parameters         | short \(\sim |\text{Normal}(30, 6.67)|\)
|                                    | long \(\sim |\text{Normal}(90, 10)|\)                      |
| Patient unpunctuality \((u)\)      | \text{Normal}(\mu_u = 0, -10, -20, -30, \sigma_u = 0, 15, 20) |
| No-show rate \((r)\)               | 18% (Berg et al. 2014)                           |
| No-show probability \((\eta)\)     | Uniform(0,1)                                     |
Sample Size
Sample Size: Discrete

\[ N \in \mathcal{N} := \{1, 5, 10, \ldots, 1000\} \quad m \in \mathcal{M} := 30 \text{ Replications} \]

\[ \bar{v}_{N'} = 191 \pm 0.19 \]

\[ \bar{v}_N = 182 \pm 0.32 \]

Figure 1. Convergence behavior of OSSP mean estimated objective (\( \bar{v}_{N'} \), red) and mean approximated objective (\( \bar{v}_N \), blue) as a function of number of scenarios \( N \) for the discrete duration model (DM).
Figure 1. Convergence behavior of OSSP mean estimated objective ($\bar{v}_N$, red) and mean approximated objective ($\bar{v}_N'$, blue) as a function of number of scenarios $N$ for the discrete duration model (DM)

\[ \bar{v}_N' = 191 \pm 0.19 \]

optimality indices \[\approx 0.01, 0.02\]

\[ \bar{v}_N = 182 \pm 0.32 \]
Figure 2. Convergence behavior of OSSP mean estimated objective ($\bar{v}_N'$, red) and mean approximated objective ($\bar{v}_N$, blue) as a function of number of scenarios $N$ for the continuous duration model (CM).
Figure 2. Convergence behavior of OSSP mean estimated objective ($\bar{v}_N'$, red) and mean approximated objective ($\bar{v}_N$, blue) as a function of number of scenarios $N$ for the continuous duration model (CM).
Analysis of the Properties of the "Approximated" Schedule/Policies
Sensitivity to Duration Uncertainty

Figure 3. Sensitivity of expected quality of the “approximated” OSSP schedule to the uncertainty of procedure duration

↑ ~1% increase
Sensitivity to Duration Uncertainty

- It would be possible to account for average bowel prep behavior but more challenging to account for variability in prep quality among patients.
Approximated Scheduling Policy as a Function of Patient Mix

Patient Mix

(8,0)

(#E, #C)

Appointment slot and slot length (minutes)
Approximated Scheduling Policy as a Function of Patient Mix

<table>
<thead>
<tr>
<th>Patient Mix</th>
<th>21</th>
<th>32</th>
<th>30</th>
<th>32</th>
<th>35</th>
<th>34</th>
<th>33</th>
<th>23</th>
</tr>
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<tbody>
<tr>
<td>(8,0)</td>
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<tr>
<td>(0, 8)</td>
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Appointment slot and sot length (minutes)
Approximated Scheduling Policy as a Function of Patient Mix

<table>
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<tr>
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<th>(#E, #C)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(8,0)</td>
<td>21 32 30 32 35 34 33 23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0,8)</td>
<td>72 80 86 82 91 90 91 123</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4,4)</td>
<td>25 33 86 82 37 62 80 75</td>
<td></td>
<td></td>
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<td>35</td>
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<td>36</td>
<td>74</td>
<td>42</td>
</tr>
<tr>
<td>(6, 2)</td>
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**Appointment slot and slot length (minutes)**
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<td>78</td>
<td>68</td>
<td>83</td>
<td>84</td>
<td>79</td>
<td>83</td>
<td>101</td>
</tr>
<tr>
<td>(2, 6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
</tbody>
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Appointment slot and slot length (minutes)
Approximated Scheduling Policy as a Function of Patient Mix

Patient Mix

≥ 2E: ≥ 1C
≥ 2E: ≥ 1C
≥ 2E: ≥ 1C

Appointment slot and slot length (minutes)

1 2 3 4 5 6 7 8
Sensitivity to Relative Importance Between Metrics

\[ OSSP(500) = \min \lambda \cdot \mathbb{E}[\text{Provider time}] + (1 - \lambda) \cdot \mathbb{E}[\text{Patients’ Time}] \]

Figure 3. Expected performances of OSSP(500) optimal schedule as function of the trade-off level \( \lambda \). As \( \lambda \) increases, the expected total waiting time is observed to increase while the expected provider time (overtime and idle time) is observed to decrease until all metrics start to stabilize near \( \lambda = 0.65 \).
Values of Stochastic Optimization in Colonoscopy Scheduling

Figure 6. Expected performance of OSSP and some traditional scheduling rules under different levels of no-show and arrivals uncertainties. The case of $r = 18\%$ and $u \sim N(0,0)$
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Values of Stochastic Optimization in Colonoscopy Scheduling

Tabel 3. The value of the stochastic solution for instances varying in uncertainty levels

<table>
<thead>
<tr>
<th>Type of Uncertainty</th>
<th>EMVP</th>
<th>OSSP(N)</th>
<th>VSS1 (% improvement from EMVP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration variability only</td>
<td>30.86</td>
<td>17.86</td>
<td>42.12%</td>
</tr>
<tr>
<td>Duration variability, $r = 18%$</td>
<td>66.04</td>
<td>25.78</td>
<td>60.96%</td>
</tr>
<tr>
<td>Duration variability, $r = 18%$,</td>
<td>80.57</td>
<td>54.62</td>
<td>32.21%</td>
</tr>
<tr>
<td>&amp; ($u \sim N(-13, 20)$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Duration variability, $r = 18%$,</td>
<td>89.31</td>
<td>48.83</td>
<td>45.32%</td>
</tr>
<tr>
<td>&amp; ($u \sim N(0, 20)$)</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
OSSP(N) = \min_{\boldsymbol{X}, t} f_N(\boldsymbol{X}, t, \omega^n) \\
MVP = \min_{\boldsymbol{X}, t} f(\boldsymbol{X}, t, \bar{\omega}) \\
VSS1 = E_{\omega} \left[ f(\bar{\boldsymbol{X}}(\bar{\omega}), \bar{t}(\bar{\omega}), \omega) \right] - v_N \\
= EMVP - v_N
\]
Values of Stochastic Optimization in Colonoscopy Scheduling

Table 3. The value of the stochastic solution for instances varying in uncertainty levels

<table>
<thead>
<tr>
<th>Type of Uncertainty</th>
<th>EMVP</th>
<th>OSSP(N)</th>
<th>VSS1 (% improvement from EMVP)</th>
<th>Average EMVP</th>
<th>Average OSSP(N)</th>
<th>VSS2 (% improv AvgEMVP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration variability only</td>
<td>30.86</td>
<td>17.86</td>
<td>42.12%</td>
<td>31.04</td>
<td>21.79</td>
<td>31.99%</td>
</tr>
<tr>
<td>Duration variability, r = 18%</td>
<td>66.04</td>
<td>25.78</td>
<td>60.96%</td>
<td>81.53</td>
<td>54.00</td>
<td>33.77%</td>
</tr>
<tr>
<td>Duration variability, r = 18%, ((u \sim N(-13, 20)))</td>
<td>80.57</td>
<td>54.62</td>
<td>32.21%</td>
<td>81.53</td>
<td>65.78</td>
<td>19.32%</td>
</tr>
<tr>
<td>4 Duration variability, r = 18%, ((u \sim N(0, 20)))</td>
<td>89.31</td>
<td>48.83</td>
<td>45.32%</td>
<td>90.32</td>
<td>69.51</td>
<td>23.04%</td>
</tr>
</tbody>
</table>

\(OSSP(N) = \min_{X,t} f_N(X,t,w^n)\)

\(MVP = \min_{X,t} f(X,t,\bar{\omega})\)

\(VSS1 = E_\omega[f(\bar{X}(\bar{\omega}), \bar{t}(\bar{\omega}), \omega)] - v_N\)

\(= EMVP - v_N\)

\(VSS2 = E_\omega[f(\bar{X}(\bar{\omega}), \bar{t}(\bar{\omega}), \omega)] - E_\omega[f(X_N^*, t_N^*, \omega)]\)

\(= EMVP - EOSSP(N)\)
Presentation Outline

• Motivation and Problem Overview
• Research Structure
• The Offline Stochastic Colonoscopy Problem (OSSP)
• Numerical Experiment
• Conclusion and Future Directions
Conclusion

• Colonoscopy scheduling have unique characteristics that are different in nature and potentially different from other OPC

• Scheduling polices depends on the uncertainty within these characteristics

• Properties of the “Approximated” schedules can be exploited in designing appointment scheduling tool (system)
Future directions

• Methodology:
  – Incorporate patient outcomes and preference
  – Probabilistic models for duration, absenteeism and arrivals
  – Incorporate interaction effect of uncertain parameters
  – Tractable and efficient solution methods

• Practice
  – Clinical observation
  – Bowel prep interventions
  – Universal template and scheduling rules
Acknowledgement

• My STAR Professor Amy Cohn

• Committee Members:
  – Professor Henry Lam
  – Professor Marina Epelman
  – Professor Ruiwei Jiang

• Center for Healthcare Engineering and Patient Safety, SCOPES family

• The Seth Bonder Foundation

• The IOE Family at the University of Michigan
No one can whistle a symphony. It takes a whole orchestra to play it
Questions and Discussions

THE QUESTION

MARK

IS IT ALWAYS SO UNCERTAIN?
I'M SO GLAD YOU ASKED.