A Monte Carlo Optimization Framework for Solving the Colonoscopy Scheduling Problem under Uncertainty

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Research Team

- Systems Concepts for the Optimization and Personalization of Endoscopy Scheduling (SCOPES) Team:
  - Amy Cohn, Professor, Industrial and Operations Engineering
  - Dr. Sameer Saini and Dr. Jacob Kurlander, University of Michigan School of Medicine, University of Michigan Health System
  - CHEPS Students
Presentation Overview

1. Introduction and Motivation
2. The Offline Stochastic Colonoscopy Scheduling Problem (OSCSP)
3. Solution Approach: Monte Carlo Optimization
4. Numerical Example
5. Conclusion and Future Directions
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Colonoscopy Procedure

- The most common screening test for Colorectal Cancer (CRC)
  - 2nd leading cause of cancer-related death in the US\(^1\)
  - 4.5 million age-eligible subjects in the US (≥ 50 years)\(^2\)

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Colonoscopy Procedure

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- Medical procedure, usually performed by a gastroenterologist, allows for direct visual examination of the entire colon and rectum
  - **Spot** existing cancer, prompting treatment
  - **Prevent** future cancer (polyps)

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- Can help **reduce** CRC incidence by about **40%** and **mortality** by about **50%\(^1\)**

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Challenges to Daily Colonoscopy Schedule

- **Significant variability** in procedure duration due to quality of the pre-procedure bowel prep

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- **Multiple** and **conflicting** criteria that affect the quality of the schedule
  - Patient waiting, provider idle and over times
  - Patient access to screening and appointment time preferences (future talk)
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Research Goal

- Develop a decision support tool to optimize colonoscopy appointment scheduling
  - A list of daily appointment slots to offer for patients (template)
  - Instructions for scheduling patients (scheduling policies)
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Problem Statement

Setup:

- A set of $P$ patients, each of a \textit{known type}
- Colonoscopy duration, patient arrival, and patient attendance are \textit{random} variables
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  - Each has a *known distribution*, is *independent* of scheduled time and from other patients
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  - Observed on the **day of services** after the appointment decisions are made

**Goal:**

- Find **optimal scheduling decisions** for this set of patients, that minimizes a **convex combination of the expected** total patients **waiting** time expected total provider **idle** and **over** times
OSCSP Formulation

\[ v_\Omega = \min f_\Omega \]

\[ \text{s.t.} \]

\[ \sum_{\omega \in \Omega} \phi(\omega) \]
\[ \lambda WP \sum_{i=1} \left( s_{\omega i} - a_{\omega i} \right) + \lambda TP \sum_{i=1} g_{\omega i} + \lambda O o_{\omega} \]

\[ t_i \geq 0 \quad \forall i \in P \] (very patient is assigned to one appointment slot)

\[ \sum_{p=1} x_{ip} = 1 \quad \forall p \in P \] (one patient is assigned to every appointment slot)

\[ s_{\omega i} \geq a_{\omega i} \quad \forall i \in P, \omega \in \Omega \] (arrival time = scheduled ± unpunctuality)

\[ s_{\omega i} \geq s_{\omega i} - 1 + \sum_{p=1} \tau_{\omega p} \cdot x_{i-1,p} \quad \forall i = 2, \ldots, P, \omega \in \Omega \] (start = max (arrival time, completion time of prev procedure))

\[ g_{\omega i} \geq a_{\omega i} + 1 - \left( s_{\omega i} + \sum_{p=1} \tau_{\omega p} \cdot x_{i,p} \right) \quad \forall i < P, \omega \in \Omega \] (idle time after the \( i \)th patient)

\[ o_{\omega} \geq \left( s_{\omega P} + \sum_{j=1} \tau_{\omega j} \cdot x_{P,j} \right) - L \quad \forall \omega \in \Omega \] (overtime incurred to complete the last procedure)

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(arrival time = scheduled ± unpunctuality)
The Offline Stochastic Colonoscopy Scheduling (OSCSP)

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& \sum_{p=1}^{P} x_{ip} = 1 \quad \forall i \in P \quad \text{(one patient is assigned to every appointment slot)} \\
& t_i \geq 0 \quad \forall i \in P \\
& a^\omega_i = t_i + \sum_{p=1}^{P} u^\omega_p \cdot x_{ip} \quad \forall i \in P, \omega \in \Omega \quad \text{(arrival time= scheduled ± unpunctuality)} \\
& s^\omega_i \geq a^\omega_i \quad \forall i \in P, \omega \in \Omega \quad \text{(start = max (arrival time, completion time of prev procedure))} \\
& s^\omega_i \geq s^\omega_{i-1} + \sum_{p=1}^{P} \tau^\omega_p \cdot x_{i-1,p} \quad \forall i = 2, ..., P, \omega \in \Omega \quad \text{(start = max (arrival time, completion time of prev procedure))} \\
& g^\omega_i \geq a^\omega_{i+1} - (s^\omega_i + \sum_{p=1}^{P} \tau^\omega_p \cdot x_{i,p}) \quad \forall i < P, \omega \in \Omega \quad \text{(idle time after the } i\text{th patient)}
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**Input Parameters?**
Colonoscopy Duration Model

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- Clinical Observations at the University of Michigan Medical Procedure Unit
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  - e.g., if complex, then duration $\sim f(\cdot)^{long}$ with $\alpha$ probability or $\sim f(\cdot)^{short}$ with $1 - \alpha$
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$$\star \left| N(\mu_l(s), \sigma^2_l(s)) \right|$$
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\[
\star |N(\mu_l(s), \sigma^2_l(s))| \\
\star \sigma_l(s) = \frac{HOD-LOD}{6}
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\[
\begin{align*}
\star & |N(\mu_{l(s)}, \sigma_{l(s)}^2)| \\
\star & \sigma_{l(s)} = \frac{HOD - LOD}{6} \\
\star & f^{short} = |N(30, 6.67^2)|
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  $$\star | N(\mu_l(s), \sigma^2_{l(s)}) |$$

  $$\star \sigma_l(s) = \frac{HOD - LOD}{6}$$

  $$\star f^{short} = | N(30, 6.67^2) |$$

  $$\star f^{long} = | N(75, 10^2) |$$
So, we’ve got a HUGE number of scenarios

\[ v_\Omega = \min f_\Omega := \sum_{\omega \in \Omega} \phi(\omega) \left[ \lambda^W \sum_{i=1}^{P} (s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{P} g_i^\omega + \lambda^O o^\omega \right] \]
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OSCSP $v_\Omega = \min f_\Omega := \sum_{\omega \in \Omega} \phi(\omega) \left[ \lambda^W \sum_{i=1}^{P} (s^\omega_i - a^\omega_i) + \lambda^T \sum_{i=1}^{P} g^\omega_i + \lambda^O o^\omega \right]$ Computationally Expensive!!!
The Offline Stochastic Colonoscopy Scheduling (OSCSP)

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**Question**: Are there any qualifications for an efficient solution method?

**Practice**: a modest sample size to solve the model within a reasonable time that the clinic manager is happy with

**Theory**: a large number of scenarios to provide a good approximation to the true optimal schedule with reasonable accuracy that we, the “ORes”, are happy with

Sample Schedule converges w.p.1 as \( N \to \infty \) \( \rightarrow \) True Optimal Schedule
So, we’ve got a HUGE number of scenarios

OSCSP \[ v_\Omega = \min_{\omega \in \Omega} f_\omega := \sum_{\omega \in \Omega} \phi(\omega) \left[ \lambda^W \sum_{i=1}^{P} (s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{P} g_i^\omega + \lambda^O o^\omega \right] \quad \text{Computationally Expensive!!!} \]

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**Question**: Are there any qualifications for an efficient solution method?

**Short answer**: YES!

**Longer answer**: Sample Average Problem

OSCSP(N) & $v_N = \min f_N := \sum_{n=1}^{N} \frac{1}{N} \left[ \lambda^W \sum_{i=1}^{P} (s_i^n - a_i^n) + \lambda^T \sum_{i=1}^{P} g_i^n + \lambda^O o^n \right]$

Practice: A modest sample size to solve the model within a reasonable time that the clinic manager is happy with

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**Practice**: a modest sample size to solve the model within a reasonable time that the clinic manager is happy with
The Offline Stochastic Colonoscopy Scheduling (OSCSP)

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Practice: a modest sample size to solve the model within a reasonable time that the clinic manager is happy with

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Sample Schedule \( \xrightarrow{\text{converges w.p.1. as } N \to \infty} \) True Optimal Schedule
Solution Approach: Monte Carlo Optimization

Presentation Overview

1. Introduction and Motivation
2. The Offline Stochastic Colonoscopy Scheduling (OSCSP)
3. Solution Approach: Monte Carlo Optimization
4. Numerical Example
5. Conclusion and Future Directions
So, a sample of 1,000 scenarios is large enough?
So, a sample of 1,000 scenarios is large enough?

Here is a simple recipe:

STEP 1. Generate 1,000 scenarios
STEP 2. Solve the sample average problem
\[ \text{OSCSP}^{1000} = 1000 \sum_{n=1}^{N} \left[ \lambda WP \sum_{i=1}^{I} (s_{ni} - a_{ni}) + \lambda TP \sum_{i=1}^{I} g_{ni} + \lambda O \right] \]
(s. t. (X, t) (planned schedule) (a, s, g, o) (actual schedule))
STEP 3. Test the performance of the optimized schedule (X, t) for 10,000 days
3.1 Each day record waiting, idle and over times
3.2 Calculate \[ v_{10,000} \] the average of the resulting 10,000 metrics values
If \[ v_{10,000} \] and \[ v_{1000} \] are relatively close, then YES; otherwise seek a larger sample.

That's all there is to Naive Monte Carlo Optimization!
So, a sample of 1,000 scenarios is large enough?

Here is a simple recipe:

**STEP 1.** Generate 1,000 scenarios

\[
\text{STEP 2.} \quad \text{Solve the sample average problem}
\]

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\text{OSCSP}^\text{v_{1000}} = \frac{1}{1000} \sum_{n=1}^{1000} \left[ \lambda \text{WP} \sum_{i=1}^{n} (s_{ni} - a_{ni}) + \lambda \text{TP} \sum_{i=1}^{n} g_{ni} + \lambda \text{o} \text{o}_n \right]
\]

(a Lower bound on Expected Performance)

\[
\text{STEP 3.} \quad \text{Test the performance of the optimized schedule } (X, t) \text{ for 10,000 days}
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s.t. \((X, t)\) (planned schedule)

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That’s all there is to Naive Monte Carlo Optimization!
A Smarter Monte Carlo Optimization Recipe

for each candidate sample size $N \in \{1, 5, \ldots\}$ do

Step 1. Generate a sample of size $N$ - for each patient $p$, generate $N$ scenarios of no-show probability, duration, and punctuality

Step 2. Solve Sample Average Problem (OSCSP($N$)) with the generated sample in step 1
Let $v_m N$ be the corresponding optimal objective value

Step 3. Evaluation of the true objective function using Monte Carlo simulation
3.1 Generate another $N$ scenarios of $P$ patients, solve OSCSP($N$), and obtain an optimal schedule ($X_m, t_m$)
3.2 Generate $N' \gg N$ scenarios of $P$ patients
3.2 Estimate $\hat{v}_m N'$, the true value of the objective function ($\hat{v}_m N'$) using the schedule ($X_m, t_m$)

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OSCSP($N$) Solution Quality:
Compute a lower and upper bound on the full problem, with all scenarios, objective function by taking the average of $v N$ and $\hat{v}_N'$, respectively, overall replications

$v N = \frac{1}{M} \sum_{m=1}^{M} v_m N$

$v N' = \frac{1}{M} \sum_{m=1}^{M} \hat{v}_m N'$

Compute (Approximate) optimality index

$AOI = \frac{v N'}{v N}$

Stopping Rule:
$N$ is large enough to obtain a high quality schedule
If $AOI \leq$ threshold, BREAK;
A Smarter Monte Carlo Optimization Recipe

for each candidate sample size $N \in \{1, 5, \ldots\}$ do
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    \end{itemize}
  end
end

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Compute a lower and upper bound on the full problem, with all scenarios, objective function by taking the average of $v_{N}$ and $\hat{v}_{N}'$, respectively, overall replications

\[ v_{N} = \frac{1}{M} \sum_{m=1}^{M} v_{N} \]
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      Let $v^m_N$ be the corresponding optimal objective value
  end
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OSCSP($N$) Solution Quality:
Compute a lower and upper bound on the full problem, with all scenarios, objective function by taking the average of $v^m_N$ and $\hat{v}^m_{N'}$, respectively, overall replications

$v_N = \frac{1}{M} \sum_{m=1}^{M} v^m_N$
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    \item \textbf{Step 3.} Evaluation of the true objective function using Monte Carlo simulation
      \begin{itemize}
      \item 3.1 Generate a another $N$ scenarios of $P$ patients, solve OSCSP(N), and obtain an optimal schedule $(X^m, t^m)$
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      \item 3.2 Estimate $\hat{v}^m_{N'}$, the true value of the objective function $(\hat{v}^m_{N'})$ using the schedule $(X^m, t^m)$
      \end{itemize}
    \end{itemize}
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end

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Compute a lower and upper bound on the full problem, with all scenarios, objective function by taking the average of $v^N_N$ and $\hat{v}^N_{N'}$, respectively, overall replications

$$v^N_N = \frac{1}{M} \sum_{m=1}^{M} v^m_N$$
$$\hat{v}^N_{N'} = \frac{1}{M} \sum_{m=1}^{M} \hat{v}^m_{N'}$$

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$$AOI = \frac{\hat{v}^N_{N'}}{v^N_N}$$

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  end
end

OSCSP($N$) Solution Quality:

$\text{AOI} = \frac{\hat{v}_{N'} - v_N}{v_N} \leq \text{threshold}$, BREAK;
A Smarter Monte Carlo Optimization Recipe

for each candidate sample size \( N \in \{1, 5, \ldots\} \) do

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end

OSCSP(N) Solution Quality:

Compute a lower and upper bound on the full problem, with all scenarios, objective function by taking the average of \( v^m_N \) and \( \hat{v}^m_{N'} \), respectively, overall replications

\[
\bar{v}_N = \frac{1}{M} \sum_{m=1}^{M} v^m_N \quad \bar{v}_{N'} = \frac{1}{M} \sum_{m=1}^{M} \hat{v}^m_{N'}
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$$v_N = \frac{1}{M} \sum_{m=1}^{M} v^m_N \quad \quad \quad \quad \quad \hat{v}_{N'} = \frac{1}{M} \sum_{m=1}^{M} \hat{v}^m_{N'}$$

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$$AOI = \frac{\hat{v}_{N'} - v_N}{\hat{v}_{N'}}$$

end
Solution Approach: Monte Carlo Optimization

A Smarter Monte Carlo Optimization Recipe

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\overline{v}_N = \frac{1}{M} \sum_{m=1}^{M} v^m_N \quad \overline{v}_{N'} = \frac{1}{M} \sum_{m=1}^{M} \hat{v}^m_{N'}
\]

Compute (Approximate) optimality index

\[
AOI = \frac{\overline{v}_{N'} - \overline{v}_N}{\overline{v}_N}
\]

Stopping Rule: \( N \) is large enough to obtain a high quality schedule

\( If \ AOI \leq \ threshold, \ BREAK; \)
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Table 1: Characteristics of tested instance

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Details</th>
</tr>
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<tbody>
<tr>
<td>Number of Patients ((P))</td>
<td>P Patients; (easy-short, complex-long)=((#E, #C))</td>
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<tr>
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# Tested Instance

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</tr>
<tr>
<td>Type-duration probability ((\alpha))</td>
<td>(\alpha = 0.75)</td>
</tr>
<tr>
<td>Duration models parameters</td>
<td>short(\sim</td>
</tr>
<tr>
<td></td>
<td>long(\sim</td>
</tr>
<tr>
<td>Patient unpunctuality ((u))</td>
<td>late on-average i.e., (u \sim N(0, 20^2))</td>
</tr>
<tr>
<td>No-show rate ((r))</td>
<td>18% (Berg et al.)</td>
</tr>
<tr>
<td>No-show probability ((\eta))</td>
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Before all else, we need a set of scenarios (a sample)

- Let us try the Monte Carlo Optimization recipe
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Hypotheses

1. $M = 30$ replications are enough to remove the effect of the large variance between the expected performance of the sample based schedules
Before all else, we need a set of scenarios (a sample)

Let us try the Monte Carlo Optimization recipe

Hypotheses

1. \( M = 30 \) replications are enough to remove the effect of the large variance between the expected performance of the sample based schedules

2. \( N \in \mathcal{N} := \{1, 5, 10, 30, 40, 50, 100, 300, 500, 1000\} \) is large enough for solving the problem with 95\% approximation accuracy (\( AOI = 0.05 \))
Numerical Example

Sample Size: \((4:4)\), 0\% no-show rate, punctual

\[ f_{\text{short}} = |N(30, 6.67^2)| \]
\[ f_{\text{long}} = |N(75, 10^2)| \]
Sample Size: \((4:4)\), 0% no-show rate, punctual

\[ f_{\text{short}} = |N(30, 6.67^2)| \]
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Figure 1: Convergence behavior of OSCSP mean approximated performance \((\bar{v}_N, \text{blue})\) and mean estimated performance \((\bar{v}_{N'}, \text{red})\) under duration uncertainty.
Sample Size: (4:4), 0% no-show rate, punctual

So, 500 scenarios?

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Figure 1: Convergence behavior of OSCSP mean approximated performance (\(\overline{v}_N\), blue) and mean estimated performance (\(\overline{v}_N'\), red) under duration uncertainty.
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- So, 500 scenarios?
  - Expected Performance of the “true” optimal schedule to the problem with all scenarios:

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\[
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  - Expected Performance of the “true” optimal schedule to the problem with all scenarios:
    - Lower bound = 194 ± 0.7
    - Upper Bound = 202 ± 0.20
    - Tight Confidence Intervals, i.e., small variances M=30 replicates ✓

\[ f_{\text{short}} = |N(30, 6.67^2)| \]
\[ f_{\text{long}} = |N(75, 10^2)| \]

Figure 1: Convergence behavior of OSCSP mean approximated performance \((\bar{v}_N, \text{blue})\) and mean estimated performance \((\bar{v}_N', \text{red})\) under duration uncertainty.
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A sample of $N = 500$ scenarios is large enough to approximate the optimal schedule

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\begin{align*}
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\end{align*}
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**Figure 1:** Convergence behavior of OSCSP mean approximated performance ($\bar{v}_N$, blue) and mean estimated performance ($\bar{v}_{N'}$, red) under duration uncertainty.
Sample Size: (4:4), 18% no-show rate, 20 min late

$$f_{\text{short}} = |N(30, 6.67^2)|$$
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Numerical Example

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\[ f_{\text{short}} = |N(30, 6.67^2)| \]
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**Figure 2:** Convergence behavior of OSCSP mean approximated performance ($\bar{v}_N$, blue) and mean estimated performance ($\bar{v}_{N'}$, red) under duration, no-show and arrival uncertainties.
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**Figure 2:** Convergence behavior of OSCSP mean approximated performance ($\bar{v}_N$, blue) and mean estimated performance ($\bar{v}_{N'}$, red) under duration, no-show and arrival uncertainties
So, we’ve got a representative sample for the observed uncertainty what to do next?
Provider Time versus Patient Time

Expected idle & over \[500\] + (1 - \[\lambda\]) \cdot \text{Expected total waiting} \[500\]

Gives clinic managers several schedules from which they may select based on their goals.
Provider Time versus Patient Time

\[
\min \lambda \cdot \text{Expected}_{500}[\text{Idle & Over}] + (1 - \lambda) \cdot \text{Expected}_{500}[\text{Total Waiting}]
\]
**Provider Time versus Patient Time**

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\min \lambda \cdot \text{Expected}_{500}\text{[Idle \& Over]} + (1 - \lambda) \cdot \text{Expected}_{500}\text{[Total Waiting]}
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★ Gives clinic managers several schedules from which they may select based on their goals.
Numerical Example

Provider Time versus Patient Time

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\]

★ Gives clinic managers several schedules from which they may select based on their goals

Figure 3: Expected performance of Pareto optimal schedules as function of the trade-off level \( \lambda \).
Scheduling Policies for Different Case Mix

One Type: Appointments Lengths

- "Dome-Shape"

Balanced Mix: (e.g., 4E:4C)

Sequence: "Alternating Bucket" of Easy → Complex

Appointments Lengths: "Bucket-based-Dome"

Unbalanced Mix (e.g., 6E:2C)

Sequence: "Alternating Bucket" of Easy → Complex

Appointments Lengths: "Bucket-based-Dome"

\(^3\text{a.k.a Time allowance between two consecutive patients}\)
One Type: Appointments Lengths$^3$ \sim "Dome-Shape"

---

$^3$ Time allowance between two consecutive appointments
Scheduling Policies for Different Case Mix

- **One Type:** Appointments Lengths\(^3\) \sim \text{"Dome-Shape"}

- **Balanced Mix:** (e.g., 4E:4C)

\(^3\text{a.k.a Time allowance between two consecutive patients}\)
**Scheduling Policies for Different Case Mix**

- **One Type:** Appointments Lengths\(^3\) ~ “Dome-Shape”
- **Balanced Mix:** (e.g., 4E:4C)

![Graph showing optimal interval length vs appointment interval]

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Scheduling Policies for Different Case Mix

- **One Type:** Appointments Lengths$^3$ \(\sim\) “Dome-Shape"

- **Balanced Mix:** (e.g., 4E:4C)
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Scheduling Policies for Different Case Mix

Natural Question: how to leverage small schedules properties in solving the more challenging versions of the problem?
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<table>
<thead>
<tr>
<th>Patient Mix</th>
<th>Easy</th>
<th>Easy</th>
<th>Complex</th>
</tr>
</thead>
<tbody>
<tr>
<td>≥ 2 Easy: ≥ 1 Complex</td>
<td>1 2 3</td>
<td>… … … … … … … … P-1</td>
<td>P</td>
</tr>
</tbody>
</table>

Appointment slot
Scheduling Policies for Different Case Mix

Natural Question: how to leverage small schedules properties in solving the more challenging versions of the problem?

<table>
<thead>
<tr>
<th>Patient Mix</th>
<th>Intractable 10 assignment and 10 appointment times decisions problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>5Easy: 5Complex</td>
<td>1 2 3 ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... 9 10</td>
</tr>
</tbody>
</table>

Appointment slot
Natural Question: how to leverage small schedules properties in solving the more challenging versions of the problem?

Patient Mix

<table>
<thead>
<tr>
<th>Easy</th>
<th>Easier to solve problem~10 minutes</th>
<th>Easy</th>
<th>Complex</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

Appointment slot

Paired $t$-test

$f(S^{\text{Fixed}}) - f(S^{0\text{pt}}) \in [-0.75 \pm 1.87]$, i.e., No “statistically” significant differences
So, Worth the Computational Efforts?
So, Worth the Computational Efforts?

4Easy:4Complex
18\% noShow
±20min
So, Worth the Computational Efforts?

4Easy:4Complex
18\% noShow
\pm 20\text{min}
1,000\text{Days}
Numerical Example

So, Worth the Computational Efforts?

Figure 4: Expected performance from 1,000 simulation runs of OSCSP schedule and some traditional scheduling rules. The case of 18% no-show rate and 20 min early/late.
Numerical Example

So, Worth the Computational Efforts?

Figure 4: Expected performance from 1,000 simulation runs of OSCSP schedule and some traditional scheduling rules. The case of 18% no-show rate and 20 min early/late.
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Figure 4: Expected performance from 1,000 simulation runs of OSCSP schedule and some traditional scheduling rules. The case of 18% no-show rate and 20 min early/late.
1 Introduction and Motivation

2 The Offline Stochastic Colonoscopy Scheduling (OSCSP)

3 Solution Approach: Monte Carlo Optimization

4 Numerical Example

5 Conclusion and Future Directions
Conclusion

- Colonoscopy scheduling have unique characteristics that are different in nature and potentially different from other OPC.

- Better scheduling policies can be developed by approximating the uncertainty within these characteristics.

- Properties of the “approximated” schedules can be exploited in designing a fast and easy to use schedule optimization tool.
Acknowledgment

- My Star Professor Amy Cohn
- The Center for Healthcare Engineering and Patient Safety
- The Seth Bonder Foundation
- The IOE Family at the University of Michigan
"No one can whistle a symphony. It takes a whole orchestra to play it"
Q&A

THE QUESTION MARK

IS IT ALWAYS SO UNCERTAIN?
I'M SO GLAD YOU ASKED.

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