A Monte Carlo Optimization Framework for Solving the Colonoscopy Scheduling Problem under Uncertainty

Karmel Shehadeh, PhD Candidate
Professor Amy Cohn

Department of Industrial and Operations Engineering
Center for Healthcare Engineering and Patient Safety
University of Michigan, Ann Arbor

July 20, 2017
IFORS 2017 | Quebec City

Research Team

- Systems Concepts for the Optimization and Personalization of Endoscopy Scheduling (SCOPES) Team:
  - Amy Cohn, Professor, Industrial and Operations Engineering
  - Dr. Sameer Saini and Dr. Jacob Kurlander, University of Michigan School of Medicine, University of Michigan Health System
  - CHEPS Students

Presentation Overview

1. Introduction and Motivation
2. The Offline Stochastic Colonoscopy Scheduling Problem (OSCSP)
3. Solution Approach: Monte Carlo Optimization
4. Numerical Example
5. Conclusion and Future Directions

Presentation Overview

Colonoscopy Procedure

- The most common screening test for Colorectal Cancer (CRC)
  - 2nd leading cause of cancer-related death in the US[1]
  - 4.5 million age-eligible subjects in the US ($\geq$ 50 years)[2]

[1] American Cancer Society]
[2] Jiang et al. (2015)

Colonoscopy Procedure

- The most common screening test for Colorectal Cancer (CRC)
  - 2nd leading cause of cancer-related death in the US[1]
  - 4.5 million age-eligible subjects in the US ($\geq$ 50 years)[2]
- Medical procedure, usually performed by a gastroenterologist, allows for direct visual examination of the entire colon and rectum
  - Spot existing cancer, prompting treatment
  - Prevent future cancer (polyps)

[1] American Cancer Society]
[2] Jiang et al. (2015)

Colonoscopy Procedure

- The most common screening test for Colorectal Cancer (CRC)
  - 2nd leading cause of cancer-related death in the US[1]
  - 4.5 million age-eligible subjects in the US ($\geq$ 50 years)[2]
- Medical procedure, usually performed by a gastroenterologist, allows for direct visual examination of the entire colon and rectum
  - Spot existing cancer, prompting treatment
  - Prevent future cancer (polyps)
- Can help reduce CRC incidence by about 40% and mortality by about 50%[1]

[1] American Cancer Society]
[2] Jiang et al. (2015)

Challenges to Daily Colonoscopy Schedule

- Significant variability in procedure duration due to quality of the pre -procedure bowel prep

Prep Quality	Type-Duration
Good	Easy-Short
Poor	Complex-Long

Challenges to Daily Colonoscopy Schedule

- Significant variability in procedure duration due to quality of the pre -procedure bowel prep

Prep Quality	Type-Duration
Good	Easy-Short
Poor	Complex-Long

- Patient absenteeism, lack of punctuality, and cancellations

Challenges to Daily Colonoscopy Schedule

- Significant variability in procedure duration due to quality of the pre -procedure bowel prep

Prep Quality	Type-Duration
Good	Easy-Short
Poor	Complex-Long

- Patient absenteeism, lack of punctuality, and cancellations
- Multiple and conflicting criteria that affect the quality of the schedule
  - Patient waiting, provider idle and over times
  - Patient access to screening and appointment time preferences (future talk)

Challenges to Daily Colonoscopy Schedule

- Significant variability in procedure duration due to quality of the pre -procedure bowel prep

Prep Quality	Type-Duration
Good	Easy-Short
Poor	Complex-Long

- Patient absenteeism, lack of punctuality, and cancellations
- Multiple and conflicting criteria that affect the quality of the schedule
  - Patient waiting, provider idle and over times
  - Patient access to screening and appointment time preferences (future talk)

Research Goal

- Develop a decision support tool to optimize colonoscopy appointment scheduling
  - A list of daily appointment slots to offer for patients (template)
  - Instructions for scheduling patients (scheduling policies)

Presentation Overview

Problem Statement

- **Setup:**
  - A set of P patients, each of a known type
  - Colonoscopy duration, patient arrival, and patient attendance are random variables

Problem Statement

- **Setup:**
  - A set of P patients, each of a known type
  - Colonoscopy duration, patient arrival, and patient attendance are random variables
    - Each has a known distribution, is independent of scheduled time and from other patients
    - Observed on the day of services after the appointment decisions are made

Problem Statement

- **Setup:**
  - A set of P patients, each of a known type
  - Colonoscopy duration, patient arrival, and patient attendance are random variables
    - Each has a known distribution, is independent of scheduled time and from other patients
    - Observed on the day of services after the appointment decisions are made
- **Goal:**
  - Find optimal scheduling decisions for this set of patients, that minimizes a convex combination of the expected total patients waiting time expected total provider idle and over times

OSCSP Formulation

OSCSP $\quad v_\Omega = \min\ f_\Omega$

s.t.

OSCSP Formulation

OSCSP $\quad v_\Omega = \min f_\Omega$

$$\text{s.t.} \sum_{i=1}^{P} x_{ip} = 1 \qquad \forall p \in P \qquad \text{(very patient is assigned to one appointment slot)}$$

$$\sum_{p=1}^{P} x_{ip} = 1 \qquad \forall i \in P \qquad \text{(one patient is assigned to every appointment slot)}$$

OSCSP Formulation

OSCSP $\quad v_\Omega = \min f_\Omega$

$$
\begin{aligned}
\text{s.t.} \quad & \sum_{i=1}^{P} x_{ip} = 1 && \forall p \in P && \text{(very patient is assigned to one appointment slot)} \\
& \sum_{p=1}^{P} x_{ip} = 1 && \forall i \in P && \text{(one patient is assigned to every appointment slot)} \\
& t_i \geq 0 && \forall i \in P && \\
& a_i^\omega = t_i + \sum_{p=1}^{P} u_p^\omega \cdot x_{ip} && \forall i \in P, \omega \in \Omega && \text{(arrival time= scheduled} \pm \text{unpunctuality)}
\end{aligned}
$$

OSCSP Formulation

OSCSP $\quad v_\Omega = \min f_\Omega$

$$
\begin{aligned}
\text{s.t.} \quad & \sum_{i=1}^{P} x_{ip} = 1 && \forall p \in P && \text{(very patient is assigned to one appointment slot)} \\
& \sum_{p=1}^{P} x_{ip} = 1 && \forall i \in P && \text{(one patient is assigned to every appointment slot)} \\
& t_i \geq 0 && \forall i \in P && \\
& a_i^\omega = t_i + \sum_{p=1}^{P} u_p^\omega \cdot x_{ip} && \forall i \in P, \omega \in \Omega && \text{(arrival time= scheduled} \pm \text{unpunctuality)} \\
& s_i^\omega \geq a_i^\omega && \forall i \in P, \omega \in \Omega && \text{(start} = \max\text{(arrival time, completion time of prev procedure))} \\
& s_i^\omega \geq s_{i-1}^\omega + \sum_{p=1}^{P} \tau_p^\omega \cdot x_{i-1,p} && \forall i = 2, ..., P, \omega \in \Omega && \text{(start} = \max\text{(arrival time, completion time of prev procedure))}
\end{aligned}
$$

OSCSP Formulation

OSCSP $\quad v_\Omega = \min f_\Omega$

$$
\begin{aligned}
\text{s.t.} \quad & \sum_{i=1}^{P} x_{ip} = 1 && \forall p \in P && \text{(very patient is assigned to one appointment slot)} \\
& \sum_{p=1}^{P} x_{ip} = 1 && \forall i \in P && \text{(one patient is assigned to every appointment slot)} \\
& t_i \geq 0 && \forall i \in P && \\
& a_i^\omega = t_i + \sum_{p=1}^{P} u_p^\omega \cdot x_{ip} && \forall i \in P, \omega \in \Omega && \text{(arrival time= scheduled} \pm \text{unpunctuality)} \\
& s_i^\omega \geq a_i^\omega && \forall i \in P, \omega \in \Omega && \text{(start} = \max\text{(arrival time, completion time of prev procedure))} \\
& s_i^\omega \geq s_{i-1}^\omega + \sum_{p=1}^{P} \tau_p^\omega \cdot x_{i-1,p} && \forall i = 2, ..., P, \omega \in \Omega && \text{(start} = \max\text{(arrival time, completion time of prev procedure))} \\
& g_i^\omega \geq a_{i+1}^\omega - (s_i^\omega + \sum_{p=1}^{P} \tau_p^\omega \cdot x_{i,p}) && \forall i < P, \omega \in \Omega && \text{(idle time after the } ith \text{ patient)}
\end{aligned}
$$

OSCSP Formulation

OSCSP $\quad v_\Omega = \min f_\Omega$

$$
\begin{aligned}
\text{s.t.} \quad & \sum_{i=1}^{P} x_{ip} = 1 && \forall p \in P && \text{(very patient is assigned to one appointment slot)} \\
& \sum_{p=1}^{P} x_{ip} = 1 && \forall i \in P && \text{(one patient is assigned to every appointment slot)} \\
& t_i \geq 0 && \forall i \in P && \\
& a_i^\omega = t_i + \sum_{p=1}^{P} u_p^\omega \cdot x_{ip} && \forall i \in P, \omega \in \Omega && \text{(arrival time= scheduled} \pm \text{unpunctuality)} \\
& s_i^\omega \geq a_i^\omega && \forall i \in P, \omega \in \Omega && \text{(start} = \max\text{(arrival time, completion time of prev procedure))} \\
& s_i^\omega \geq s_{i-1}^\omega + \sum_{p=1}^{P} \tau_p^\omega \cdot x_{i-1,p} && \forall i = 2, ..., P, \omega \in \Omega && \text{(start} = \max\text{(arrival time, completion time of prev procedure))} \\
& g_i^\omega \geq a_{i+1}^\omega - (s_i^\omega + \sum_{p=1}^{P} \tau_p^\omega \cdot x_{i,p}) && \forall i < P, \omega \in \Omega && \text{(idle time after the } ith \text{ patient)} \\
& o^\omega \geq (s_P^\omega + \sum_{j=1}^{P} \tau_j^\omega \cdot x_{P,j}) - \mathcal{L} && \forall \omega \in \Omega && \text{(overtime incured to complete the last procedure)}
\end{aligned}
$$

OSCSP Formulation

OSCSP $\quad v_\Omega = \min f_\Omega$

$$
\begin{aligned}
\text{s.t.} \quad & \sum_{i=1}^{P} x_{ip} = 1 && \forall p \in P && \text{(very patient is assigned to one appointment slot)} \\
& \sum_{p=1}^{P} x_{ip} = 1 && \forall i \in P && \text{(one patient is assigned to every appointment slot)} \\
& t_i \geq 0 && \forall i \in P && \\
& a_i^\omega = t_i + \sum_{p=1}^{P} u_p^\omega \cdot x_{ip} && \forall i \in P, \omega \in \Omega && \text{(arrival time= scheduled} \pm \text{unpunctuality)} \\
& s_i^\omega \geq a_i^\omega && \forall i \in P, \omega \in \Omega && \text{(start} = \max\text{(arrival time, completion time of prev procedure))} \\
& s_i^\omega \geq s_{i-1}^\omega + \sum_{p=1}^{P} \tau_p^\omega \cdot x_{i-1,p} && \forall i = 2, ..., P, \omega \in \Omega && \text{(start} = \max\text{(arrival time, completion time of prev procedure))} \\
& g_i^\omega \geq a_{i+1}^\omega - (s_i^\omega + \sum_{p=1}^{P} \tau_p^\omega \cdot x_{i,p}) && \forall i < P, \omega \in \Omega && \text{(idle time after the } ith \text{ patient)} \\
& o^\omega \geq (s_P^\omega + \sum_{j=1}^{P} \tau_j^\omega \cdot x_{P,j}) - \mathcal{L} && \forall \omega \in \Omega && \text{(overtime incured to complete the last procedure)} \\
& g_i^\omega \geq 0 && \forall i \in P, \omega \in \Omega && \\
& s_i^\omega \geq 0 && \forall i \in P, \omega \in \Omega && \\
& o^\omega \geq 0 && \forall \omega \in \Omega && \\
& x_{i,p} \in \{0, 1\} && &&
\end{aligned}
$$

OSCSP Formulation

OSCSP $\quad v_\Omega = \min\ f_\Omega := \sum_{\omega\in\Omega} \phi(\omega)\Big[\lambda^W \sum_{i=1}^{P}(s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{P} g_i^\omega + \lambda^O o^\omega\Big]$

s.t. $\sum_{i=1}^{P} x_{ip} = 1$	$\forall p \in P$	(very patient is assigned to one appointment slot)
$\sum_{p=1}^{P} x_{ip} = 1$	$\forall i \in P$	(one patient is assigned to every appointment slot)
$t_i \geq 0$	$\forall i \in P$	
$a_i^\omega = t_i + \sum_{p=1}^{P} u_p^\omega \cdot x_{ip}$	$\forall i \in P, \omega \in \Omega$	(arrival time= scheduled $\pm$ unpunctuality)
$s_i^\omega \geq a_i^\omega$	$\forall i \in P, \omega \in \Omega$	(start $=$ max (arrival time, completion time of prev procedure))
$s_i^\omega \geq s_{i-1}^\omega + \sum_{p=1}^{P} \tau_p^\omega \cdot x_{i-1,p}$	$\forall i = 2, ..., P, \omega \in \Omega$	(start $=$ max (arrival time, completion time of prev procedure))
$g_i^\omega \geq a_{i+1}^\omega - (s_i^\omega + \sum_{p=1}^{P} \tau_p^\omega \cdot x_{i,p})$	$\forall i < P, \omega \in \Omega$	(idle time after the *ith* patient)
$o^\omega \geq (s_P^\omega + \sum_{j=1}^{P} \tau_j^\omega \cdot x_{P,j}) - \mathcal{L}$	$\forall \omega \in \Omega$	(overtime incured to complete the last procedure)
$g_i^\omega \geq 0$	$\forall i \in P, \omega \in \Omega$	
$s_i^\omega \geq 0$	$\forall i \in P, \omega \in \Omega$	
$o^\omega \geq 0$	$\forall \omega \in \Omega$	
$x_{i,p} \in \{0, 1\}$		

OSCSP Formulation

$$\text{OSCSP} \qquad v_\Omega = \min\ f_\Omega := \sum_{\omega\in\Omega} \phi(\omega)\Big[\lambda^W \sum_{i=1}^{P}(s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{P} g_i^\omega + \lambda^O o^\omega\Big]$$

$$\text{s.t.} \quad (\mathbf{X}, \mathbf{t}) \qquad \text{(planned schedule)}$$

$$(a^\omega, s^\omega, g^\omega, o^\omega) \qquad \text{(actual schedule)}$$

OSCSP Formulation

OSCSP $\quad v_\Omega = \min\ f_\Omega := \sum_{\omega \in \Omega} \phi(\omega) \Big[\lambda^W \sum_{i=1}^{P} (s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{P} g_i^\omega + \lambda^O o^\omega \Big]$

$$\begin{aligned} \text{s.t.} \quad & (\mathbf{X}, \mathbf{t}) && \text{(planned schedule)} \\ & (a^\omega, s^\omega, g^\omega, o^\omega) && \text{(actual schedule)} \end{aligned}$$

Input Parameters?

Colonoscopy Duration Model

Prep Quality	Type-Duration
Good	Easy-Short
Poor	Complex-Long

- Clinical Observations at the University of Michigan Medical Procedure Unit

Colonoscopy Duration Model

Prep Quality	Type-Duration
Good	Easy-Short
Poor	Complex-Long

- Clinical Observations at the University of Michigan Medical Procedure Unit
- Type-Duration likelihood α

Colonoscopy Duration Model

Prep Quality	Type-Duration
Good	Easy-Short
Poor	Complex-Long

- Clinical Observations
- Type-duration likelihood α
- **Continuous**

Colonoscopy Duration Model

Prep Quality	Type-Duration
Good	Easy-Short
Poor	Complex-Long

- Clinical Observations
- Type-duration likelhood α
- **Continuous**
  - e.g., if **complex**, then duration $\sim f(\cdot)^{long}$ with α probability or $\sim f(\cdot)^{short}$ with $1 - \alpha$

Colonoscopy Duration Model (example)

Type	Lowest Observed Duration (LOD)	Frequently Observed Duration	Highest Observed Duration (HOD)
Short		30	
Good		75	

- Clinical Observations
- Type-duration likelihood α
- **Continuous**
  - e.g., if complex, then duration $\sim f(\cdot)^{long}$ with α probability or $\sim f(\cdot)^{short}$ with $1-\alpha$

Colonoscopy Duration Model (example)

Type	Lowest Observed Duration (LOD)	Frequently Observed Duration	Highest Observed Duration (HOD)
Short	20	30	60
Good	60	75	120

- Clinical Observations
- Type-duration likelihood α
- **Continuous**
  - e.g., if complex, then duration $\sim f(\cdot)^{long}$ with α probability or $\sim f(\cdot)^{short}$ with $1-\alpha$

Colonoscopy Duration Model (example)

Type	Lowest Observed Duration (LOD)	Frequently Observed Duration	Highest Observed Duration (HOD)
Short	20	30	60
Good	60	75	120

- Clinical Observations
- Type-duration likelihood α
- **Continuous**
  - e.g., if complex, then duration $\sim f(\cdot)^{long}$ with α probability or $\sim f(\cdot)^{short}$ with $1-\alpha$

    $\star$ $|N(\mu_{l(s)}, \sigma^2_{l(s)})|$

Colonoscopy Duration Model (example)

Type	Lowest Observed Duration (LOD)	Frequently Observed Duration	Highest Observed Duration (HOD)
Short	20	30	60
Good	60	75	120

- Clinical Observations
- Type-duration likelihood α
- **Continuous**
  - e.g., if complex, then duration $\sim f(\cdot)^{long}$ with α probability or $\sim f(\cdot)^{short}$ with $1-\alpha$

    $\star$ $|N(\mu_{l(s)}, \sigma^2_{l(s)})|$

    $\star$ $\sigma_{l(s)} = \frac{HOD-LOD}{6}$

Colonoscopy Duration Model (example)

Type	Lowest Observed Duration (LOD)	Frequently Observed Duration	Highest Observed Duration (HOD)
Short	20	30	60
Good	60	75	120

- Clinical Observations
- Type-duration likelihood α
- **Continuous**
  - e.g., if complex, then duration $\sim f(\cdot)^{long}$ with α probability or $\sim f(\cdot)^{short}$ with $1-\alpha$

    $\star$ $|N(\mu_{l(s)}, \sigma^2_{l(s)})|$

    $\star$ $\sigma_{l(s)} = \frac{HOD-LOD}{6}$

    $\star$ $f^{\text{short}} = |N(30, 6.67^2)|$

Colonoscopy Duration Model (example)

Type	Lowest Observed Duration (LOD)	Frequently Observed Duration	Highest Observed Duration (HOD)
Short	20	30	60
Good	60	75	120

- Clinical Observations
- Type-duration likelihood α
- **Continuous**
  - e.g., if complex, then duration $\sim f(\cdot)^{long}$ with α probability or $\sim f(\cdot)^{short}$ with $1-\alpha$
    - $\star$ $|N(\mu_{l(s)}, \sigma^2_{l(s)})|$
    - $\star$ $\sigma_{l(s)} = \frac{HOD-LOD}{6}$
    - $\star$ $f^{\text{short}} = |N(30, 6.67^2)|$
    - $\star$ $f^{\text{long}} = |N(75, 10^2)|$

So, we've got a HUGE number of scenarios

OSCSP $$v_\Omega = \min f_\Omega := \sum_{\omega \in \Omega} \phi(\omega) \Big[\lambda^W \sum_{i=1}^{P} (s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{P} g_i^\omega + \lambda^O o^\omega \Big]$$

So, we've got a HUGE number of scenarios

OSCSP $\quad v_\Omega = \min f_\Omega := \sum_{\omega \in \Omega} \phi(\omega) \Big[\lambda^W \sum_{i=1}^{P} (s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{P} g_i^\omega + \lambda^O o^\omega \Big]$ **Computationally Expensive!!!**

So, we've got a HUGE number of scenarios

OSCSP $\quad v_\Omega = \min f_\Omega := \sum_{\omega \in \Omega} \phi(\omega) \Big[\lambda^W \sum_{i=1}^{P} (s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{P} g_i^\omega + \lambda^O o^\omega \Big]$ **Computationally Expensive!!!**

Question : Are there any qualifications for an efficient solution method?

So, we've got a HUGE number of scenarios

OSCSP $\quad v_\Omega = \min f_\Omega := \sum_{\omega \in \Omega} \phi(\omega) \Big[\lambda^W \sum_{i=1}^{P} (s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{P} g_i^\omega + \lambda^O o^\omega \Big]$ **Computationally Expensive!!!**

Question : Are there any qualifications for an efficient solution method?

Short answer: YES!

So, we've got a HUGE number of scenarios

OSCSP $\quad v_\Omega = \min f_\Omega := \sum_{\omega \in \Omega} \phi(\omega) \Big[\lambda^W \sum_{i=1}^{P} (s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{P} g_i^\omega + \lambda^O o^\omega \Big]$ **Computationally Expensive!!!**

Question : Are there any qualifications for an efficient solution method?

Short answer: YES!

Longer answer: Sample Average Problem

OSCSP(N) $\quad v_N = \min f_N := \sum_{n=1}^{N} \frac{1}{N} \Big[\lambda^W \sum_{i=1}^{P} (s_i^n - a_i^n) + \lambda^T \sum_{i=1}^{P} g_i^n + \lambda^O o^n \Big]$

So, we've got a HUGE number of scenarios

OSCSP $\quad v_\Omega = \min f_\Omega := \sum_{\omega \in \Omega} \phi(\omega) \Big[\lambda^W \sum_{i=1}^{P} (s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{P} g_i^\omega + \lambda^O o^\omega \Big]$ **Computationally Expensive!!!**

Question : Are there any qualifications for an efficient solution method?

Short answer: YES!

Longer answer: Sample Average Problem

OSCSP(N) $\quad v_N = \min f_N := \sum_{n=1}^{N} \frac{1}{N} \Big[\lambda^W \sum_{i=1}^{P} (s_i^n - a_i^n) + \lambda^T \sum_{i=1}^{P} g_i^n + \lambda^O o^n \Big]$

Practice: a modest sample size to solve the model within a reasonable time that the clinic manager is happy with

So, we've got a HUGE number of scenarios

OSCSP $\quad v_\Omega = \min f_\Omega := \sum_{\omega \in \Omega} \phi(\omega) \Big[\lambda^W \sum_{i=1}^{P} (s_i^\omega - a_i^\omega) + \lambda^T \sum_{i=1}^{P} g_i^\omega + \lambda^O o^\omega \Big]$ **Computationally Expensive!!!**

Question : Are there any qualifications for an efficient solution method?

Short answer: YES!

Longer answer: Sample Average Problem

OSCSP(N) $\quad v_N = \min f_N := \sum_{n=1}^{N} \frac{1}{N} \Big[\lambda^W \sum_{i=1}^{P} (s_i^n - a_i^n) + \lambda^T \sum_{i=1}^{P} g_i^n + \lambda^O o^n \Big]$

Practice: a modest sample size to solve the model within a reasonable time that the clinic manager is happy with

Theory: a large number of scenarios to provide a good approximation to the true optimal schedule with reasonable accuracy that we, the "ORes", are happy with

$$\text{Sample Schedule} \xrightarrow{\text{converges w.p.1. as } N \to \infty} \text{True Optimal Schedule}$$

Presentation Overview

So, a sample of 1,000 scenarios is large enough?

So, a sample of 1,000 scenarios is large enough?

Here is a simple recipe:

So, a sample of 1,000 scenarios is large enough?

Here is a simple recipe:

STEP 1. Generate 1,000 scenarios

So, a sample of 1,000 scenarios is large enough?

Here is a simple recipe:

STEP 1. Generate 1,000 scenarios

STEP 2. Solve the sample average problem

$$\text{OSCSP} \qquad v_{1000} = := \sum_{n=1}^{1000} \frac{1}{1000}\Big[\lambda^W \sum_{i=1}^{P}(s_i^n - a_i^n) + \lambda^T \sum_{i=1}^{P} g_i^n + \lambda^O o^n\Big] \quad \text{(a Lower bound on Expected Performance)}$$

$$\text{s.t.} \quad (\mathbf{X}, \mathbf{t}) \qquad \text{(planned schedule)}$$

$$(a^n, s^n, g^n, o^n) \qquad \text{(actual schedule)}$$

So, a sample of 1,000 scenarios is large enough?

Here is a simple recipe:

STEP 1. Generate 1,000 scenarios

STEP 2. Solve the sample average problem

$$\text{OSCSP} \quad v_{1000} = := \sum_{n=1}^{1000} \frac{1}{1000}\Big[\lambda^W \sum_{i=1}^{P}(s_i^n - a_i^n) + \lambda^T \sum_{i=1}^{P} g_i^n + \lambda^O o^n\Big] \quad \text{(a Lower bound on Expected Performance)}$$

$$\text{s.t.} \quad (\mathbf{X}, \mathbf{t}) \qquad \text{(planned schedule)}$$

$$(a^n, s^n, g^n, o^n) \qquad \text{(actual schedule)}$$

STEP 3. Test the performance of the optimized schedule ($\mathbf{X}$,t) for 10,000 days

3.1 Each day record waiting, idle and over times

3.2 Calculate $v_{10,00}$ the average of the resulting 10,000 metrics values

So, a sample of 1,000 scenarios is large enough?

Here is a simple recipe:

STEP 1. Generate 1,000 scenarios

STEP 2. Solve the sample average problem

$$\text{OSCSP} \quad v_{1000} = := \sum_{n=1}^{1000} \frac{1}{1000}\Big[\lambda^W \sum_{i=1}^{P}(s_i^n - a_i^n) + \lambda^T \sum_{i=1}^{P} g_i^n + \lambda^O o^n\Big] \quad \text{(a Lower bound on Expected Performance)}$$

$$\text{s.t.} \quad (\mathbf{X}, \mathbf{t}) \qquad \text{(planned schedule)}$$

$$(a^n, s^n, g^n, o^n) \qquad \text{(actual schedule)}$$

STEP 3. Test the performance of the optimized schedule (**X**,t) for 10,000 days

3.1 Each day record waiting, idle and over times

3.2 Calculate $v_{10,00}$ the average of the resulting 10,000 metrics values

If $v_{10,000}$ and v_{1000} are relatively close, then **YES**; otherwise seek a larger sample

So, a sample of 1,000 scenarios is large enough?

Here is a simple recipe:

STEP 1. Generate 1,000 scenarios

STEP 2. Solve the sample average problem

$$\text{OSCSP} \qquad v_{1000} = := \sum_{n=1}^{1000} \frac{1}{1000}\Big[\lambda^W \sum_{i=1}^{P}(s_i^n - a_i^n) + \lambda^T \sum_{i=1}^{P} g_i^n + \lambda^O o^n\Big] \quad \text{(a Lower bound on Expected Performance)}$$

$$\text{s.t.} \quad (\mathbf{X}, \mathbf{t}) \qquad \text{(planned schedule)}$$

$$(a^n, s^n, g^n, o^n) \qquad \text{(actual schedule)}$$

STEP 3. Test the performance of the optimized schedule ($\mathbf{X}$,t) for 10,000 days

3.1 Each day record waiting, idle and over times

3.2 Calculate $v_{10,00}$ the average of the resulting 10,000 metrics values

If $v_{10,000}$ and v_{1000} are relatively close, then **YES**; otherwise seek a larger sample

That's all there is to Naive Monte Carlo Optimization!

A Smarter Monte Carlo Optimization Recipe

for *each candidate sample size* $N \in \{1, 5, \ldots.\}$ **do**

end

A Smarter Monte Carlo Optimization Recipe

```
for each candidate sample size N ∈ {1, 5, ....} do
    for m=1:M do

    end

end
```

A Smarter Monte Carlo Optimization Recipe

for *each candidate sample size* $N \in \{1, 5, \ldots.\}$ **do**

  for *m=1:M* **do**

    Step 1. ***Generate a sample of size*** N

      - for each patient p, generate N scenarios of no-show probability, duration, and punctuality

  end

end

A Smarter Monte Carlo Optimization Recipe

for *each candidate sample size* $N \in \{1, 5, \ldots.\}$ **do**

  for *m=1:M* **do**

    Step 1. ***Generate a sample of size*** N

      - for each patient p, generate N scenarios of no-show probability, duration, and punctuality

    Step 2. ***Solve Sample Average Problem (OSCSP(N)) with the generated sample in step 1***

      Let v_N^m be the corresponding optimal objective value

  end

end

A Smarter Monte Carlo Optimization Recipe

for *each candidate sample size* $N \in \{1, 5, \ldots.\}$ **do**

  for *m=1:M* **do**

    Step 1. ***Generate a sample of size*** N

      - for each patient p, generate N scenarios of no-show probability, duration, and punctuality

    Step 2. ***Solve Sample Average Problem (OSCSP(N)) with the generated sample in step 1***

      Let v_N^m be the corresponding optimal objective value

    Step 3. ***Evaluation of the true objective function using Monte Carlo simulation***

      3.1 Generate a another N scenarios of P patients, solve *OSCSP(N)*, and obtain an optimal schedule $(\mathbf{X}^m, \mathbf{t}^m)$

      3.2 Generate $N' >> N$ scenarios of P patients

      3.2 Estimate $\hat{v}_{N'}^m$, the true value of the objective function $(\hat{v}_{N'}^m)$ using the schedule $(\mathbf{X}^m, \mathbf{t}^m)$

  end

end

A Smarter Monte Carlo Optimization Recipe

for *each candidate sample size* $N \in \{1, 5, \ldots.\}$ **do**

  for *m=1:M* **do**

    Step 1. ***Generate a sample of size*** N

      - for each patient p, generate N scenarios of no-show probability, duration, and punctuality

    Step 2. ***Solve Sample Average Problem (OSCSP(N)) with the generated sample in step 1***

      Let v_N^m be the corresponding optimal objective value

    Step 3. ***Evaluation of the true objective function using Monte Carlo simulation***

      3.1 Generate a another N scenarios of P patients, solve *OSCSP(N)*, and obtain an optimal schedule $(\mathbf{X}^m, \mathbf{t}^m)$

      3.2 Generate $N' >> N$ scenarios of P patients

      3.2 Estimate $\hat{v}_{N'}^m$, the true value of the objective function $(\hat{v}_{N'}^m)$ using the schedule $(\mathbf{X}^m, \mathbf{t}^m)$

  end

  OSCSP(N) Solution Quality:

end

A Smarter Monte Carlo Optimization Recipe

for *each candidate sample size* $N \in \{1, 5, \ldots.\}$ **do**

for *m=1:M* **do**

Step 1. ***Generate a sample of size*** N

- for each patient p, generate N scenarios of no-show probability, duration, and punctuality

Step 2. ***Solve Sample Average Problem (OSCSP(N)) with the generated sample in step 1***

Let v_N^m be the corresponding optimal objective value

Step 3. ***Evaluation of the true objective function using Monte Carlo simulation***

3.1 Generate a another N scenarios of P patients, solve *OSCSP(N)*, and obtain an optimal schedule $(\mathbf{X}^m, \mathbf{t}^m)$

3.2 Generate $N' >> N$ scenarios of P patients

3.2 Estimate $\hat{v}_{N'}^m$, the true value of the objective function $(\hat{v}_{N'}^m)$ using the schedule $(\mathbf{X}^m, \mathbf{t}^m)$

end

OSCSP(N) Solution Quality:

Compute a lower and upper bound on the full problem, with all scenarios, objective function by taking the average of v_N^m *and* $\hat{v}_{N'}^m$*, respectively, overall replications*

$$\overline{v}_N = \frac{1}{M}\sum_{m=1}^{M} v_N^m \qquad \overline{v}_{N'} = \frac{1}{M}\sum_{m=1}^{M} \hat{v}_{N'}^m$$

end

A Smarter Monte Carlo Optimization Recipe

for *each candidate sample size* $N \in \{1, 5, \ldots.\}$ **do**

for *m=1:M* **do**

Step 1. ***Generate a sample of size*** N

- for each patient p, generate N scenarios of no-show probability, duration, and punctuality

Step 2. ***Solve Sample Average Problem (OSCSP(N)) with the generated sample in step 1***

Let v_N^m be the corresponding optimal objective value

Step 3. ***Evaluation of the true objective function using Monte Carlo simulation***

3.1 Generate a another N scenarios of P patients, solve *OSCSP(N)*, and obtain an optimal schedule $(\mathbf{X}^m, \mathbf{t}^m)$

3.2 Generate $N' >> N$ scenarios of P patients

3.2 Estimate $\hat{v}_{N'}^m$, the true value of the objective function $(\hat{v}_{N'}^m)$ using the schedule $(\mathbf{X}^m, \mathbf{t}^m)$

end

OSCSP(N) Solution Quality:

Compute a lower and upper bound on the full problem, with all scenarios, objective function by taking the average of v_N^m *and* $\hat{v}_{N'}^m$*, respectively, overall replications*

$$\overline{v}_N = \frac{1}{M}\sum_{m=1}^{M} v_N^m \qquad \overline{v}_{N'} = \frac{1}{M}\sum_{m=1}^{M} \hat{v}_{N'}^m$$

Compute (Approximate) optimality indix

$$AOI = \frac{\overline{v}_{N'} - \overline{v}_N}{\overline{v}_{N'}}$$

end

A Smarter Monte Carlo Optimization Recipe

for *each candidate sample size* $N \in \{1, 5, \ldots.\}$ **do**

for *m=1:M* **do**

Step 1. ***Generate a sample of size*** N

- for each patient p, generate N scenarios of no-show probability, duration, and punctuality

Step 2. ***Solve Sample Average Problem (OSCSP(N)) with the generated sample in step 1***

Let v_N^m be the corresponding optimal objective value

Step 3. ***Evaluation of the true objective function using Monte Carlo simulation***

3.1 Generate a another N scenarios of P patients, solve *OSCSP(N)*, and obtain an optimal schedule $(\mathbf{X}^m, \mathbf{t}^m)$

3.2 Generate $N' >> N$ scenarios of P patients

3.2 Estimate $\hat{v}_{N'}^m$, the true value of the objective function $(\hat{v}_{N'}^m)$ using the schedule $(\mathbf{X}^m, \mathbf{t}^m)$

end

OSCSP(N) Solution Quality:

Compute a lower and upper bound on the full problem, with all scenarios, objective function by taking the average of v_N^m *and* $\hat{v}_{N'}^m$*, respectively, overall replications*

$$\overline{v}_N = \frac{1}{M}\sum_{m=1}^{M} v_N^m \qquad \overline{v}_{N'} = \frac{1}{M}\sum_{m=1}^{M} \hat{v}_{N'}^m$$

Compute (Approximate) optimality indix

$$AOI = \frac{\overline{v}_{N'} - \overline{v}_N}{\overline{v}_{N'}}$$

Stopping Rule: N is large enough to obtain a high quality schedule

If $AOI \leq$ **threshold, BREAK;**

end

Presentation Overview

Tested Instance

Table 1: Characteristics of tested instance

Parameters	**Details**
Number of Patients (P)	P Patients; (easy-short, complex-long)=($\#E$, $\#C$)
Clinic working hours ($0 - \mathcal{L}$)	0-expected duration for ($\#E$, $\#C$)

Tested Instance

Table 1: Characteristics of tested instance

Parameters	**Details**
Number of Patients (P)	P Patients; (easy-short, complex-long)=($\#E$, $\#C$)
Clinic working hours ($0 - \mathcal{L}$)	0-expected duration for ($\#E$, $\#C$)
Type-duration probability	$\alpha = 0.75$
Duration models parameters	short$\sim \lvert Normal(30, 6.67^2)\rvert$
	long$\sim \lvert Normal(75, 10^2)\rvert$
Patient unpunctuality (u)	late on-average i.e., $u \sim N(0, 20^2)$
No-show rate (r)	18% (Berg et al.)
No-show probability (η)	Uniform(0,1)

Before all else, we need a set of scenarios (a sample)

⋆ Let us try the Monte Carlo Optimization recipe

Before all else, we need a set of scenarios (a sample)

⋆ Let us try the Monte Carlo Optimization recipe

Hypotheses

1. $M = 30$ replications are enough to remove the effect of the large variance between the expected performance of the sample based schedules

Before all else, we need a set of scenarios (a sample)

⋆ Let us try the Monte Carlo Optimization recipe

Hypotheses

1. $M = 30$ replications are enough to remove the effect of the large variance between the expected performance of the sample based schedules

2. $N \in \mathcal{N} := \{1, 5, 10, 30, 40, 50, 100, 300, 500, 1000\}$ is large enough for solving the problem with 95% approximation accuracy ($AOI = 0.05$)

Sample Size: (4:4), 0% no-show rate, punctual

$f^{\text{short}} = |N(30, 6.67^2)|$
$f^{\text{long}} = |N(75, 10^2)|$

Sample Size: (4:4), 0% no-show rate, punctual

$f^{\text{short}} = |N(30, 6.67^2)|$
$f^{\text{long}} = |N(75, 10^2)|$

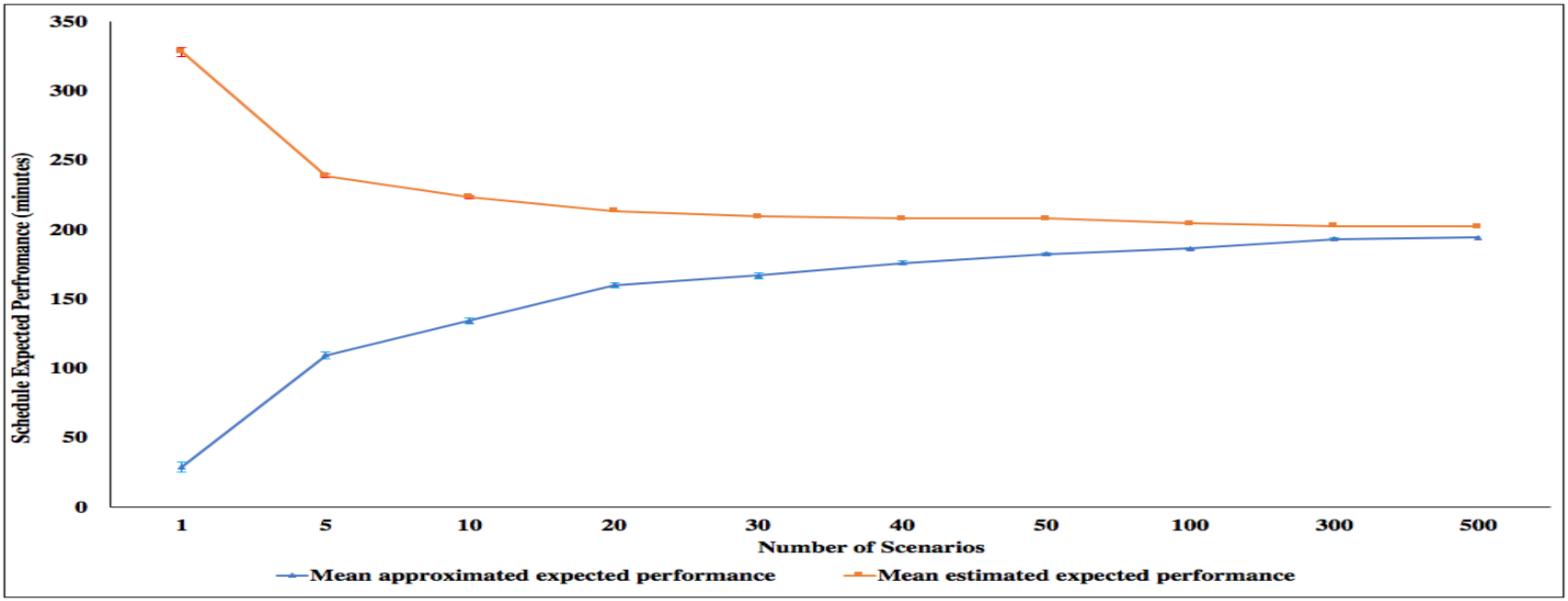


Figure 1: Convergence behavior of OSCSP mean approximated performance ($\overline{v}_N$, blue) and mean estimated performance ($\overline{v}_{N'}$, red) under duration uncertainty

Sample Size: (4:4), 0% no-show rate, punctual

- So, 500 scenarios?

$f^{\text{short}} = |N(30, 6.67^2)|$
$f^{\text{long}} = |N(75, 10^2)|$

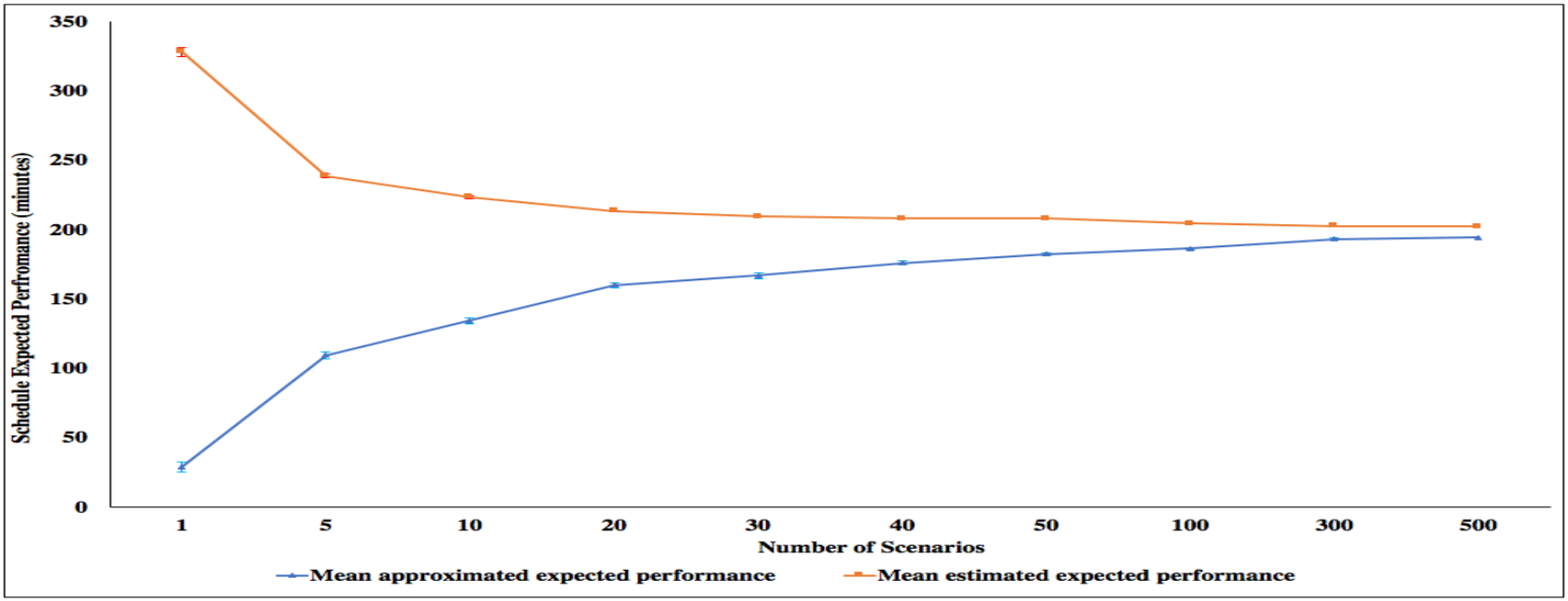


Figure 1: Convergence behavior of OSCSP mean approximated performance ($\overline{v}_N$, blue) and mean estimated performance ($\overline{v}_{N'}$, red) under duration uncertainty

Sample Size: (4:4), 0% no-show rate, punctual

$f^{\text{short}} = |N(30, 6.67^2)|$
$f^{\text{long}} = |N(75, 10^2)|$

- So, 500 scenarios?
  - Expected Performance of the "true" optimal schedule to the problem with all scenarios:

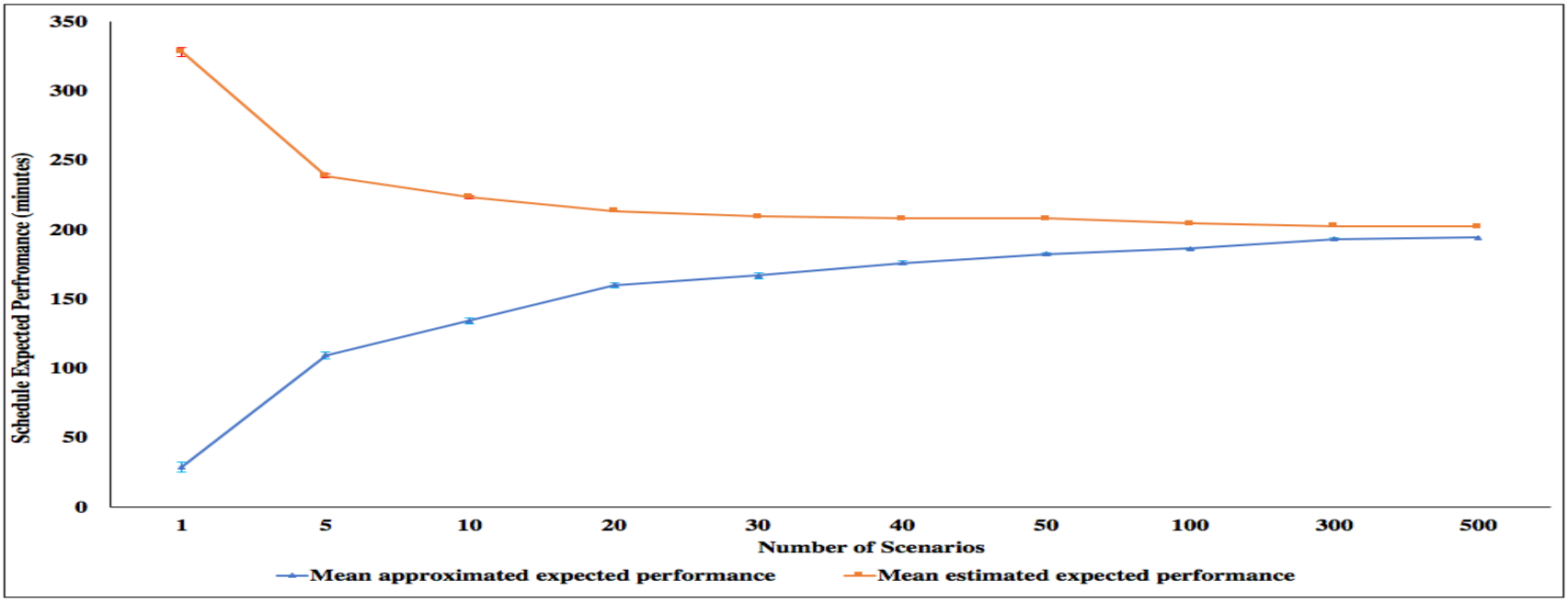


Figure 1: Convergence behavior of OSCSP mean approximated performance ($\overline{v}_N$, blue) and mean estimated performance ($\overline{v}_{N'}$, red) under duration uncertainty

Sample Size: (4:4), 0% no-show rate, punctual

$f^{\text{short}} = |N(30, 6.67^2)|$
$f^{\text{long}} = |N(75, 10^2)|$

- So, 500 scenarios?
  - Expected Performance of the "true" optimal schedule to the problem with all scenarios:
    - Lower bound=194 $\pm$ 0.7 Upper Bound=202 $\pm$ 0.20

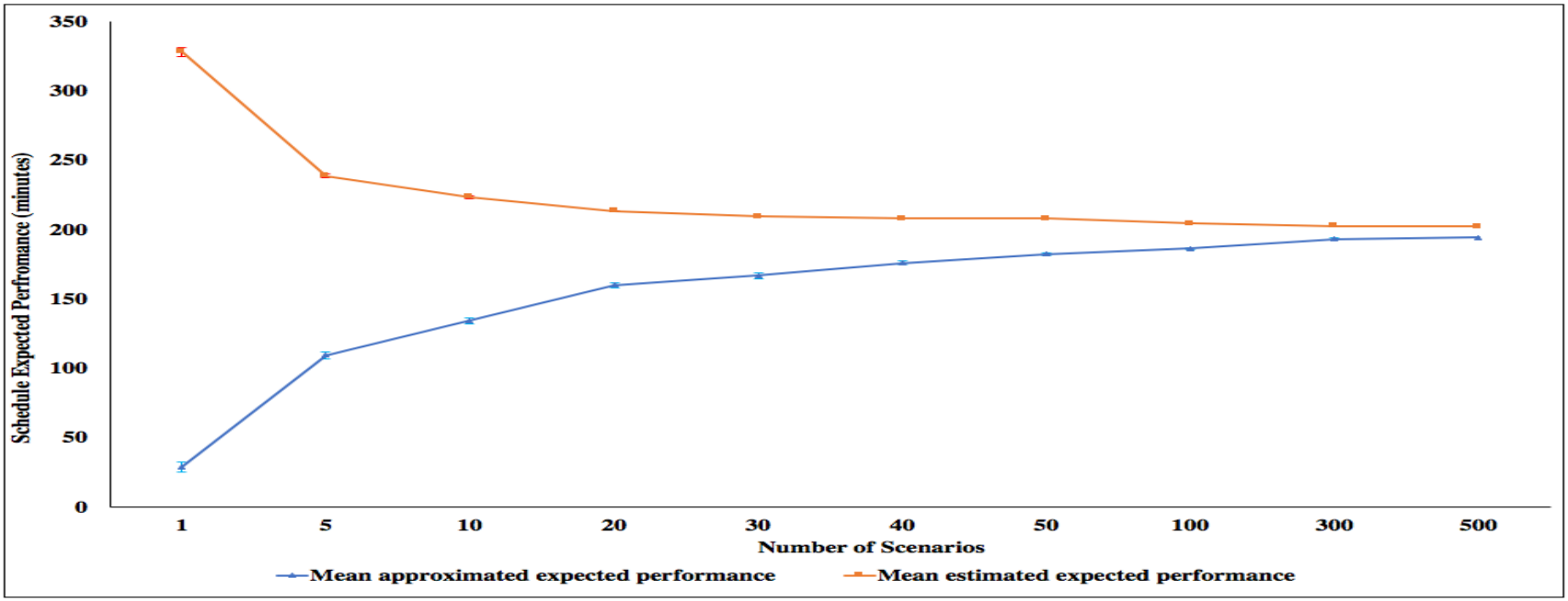


Figure 1: Convergence behavior of OSCSP mean approximated performance ($\overline{v}_N$, blue) and mean estimated performance ($\overline{v}_{N'}$, red) under duration uncertainty

Sample Size: (4:4), 0% no-show rate, punctual

$f^{\text{short}} = |N(30, 6.67^2)|$
$f^{\text{long}} = |N(75, 10^2)|$

- So, 500 scenarios?
  - Expected Performance of the "true" optimal schedule to the problem with all scenarios:
    - Lower bound=194 $\pm$ 0.7 Upper Bound=202 $\pm$ 0.20

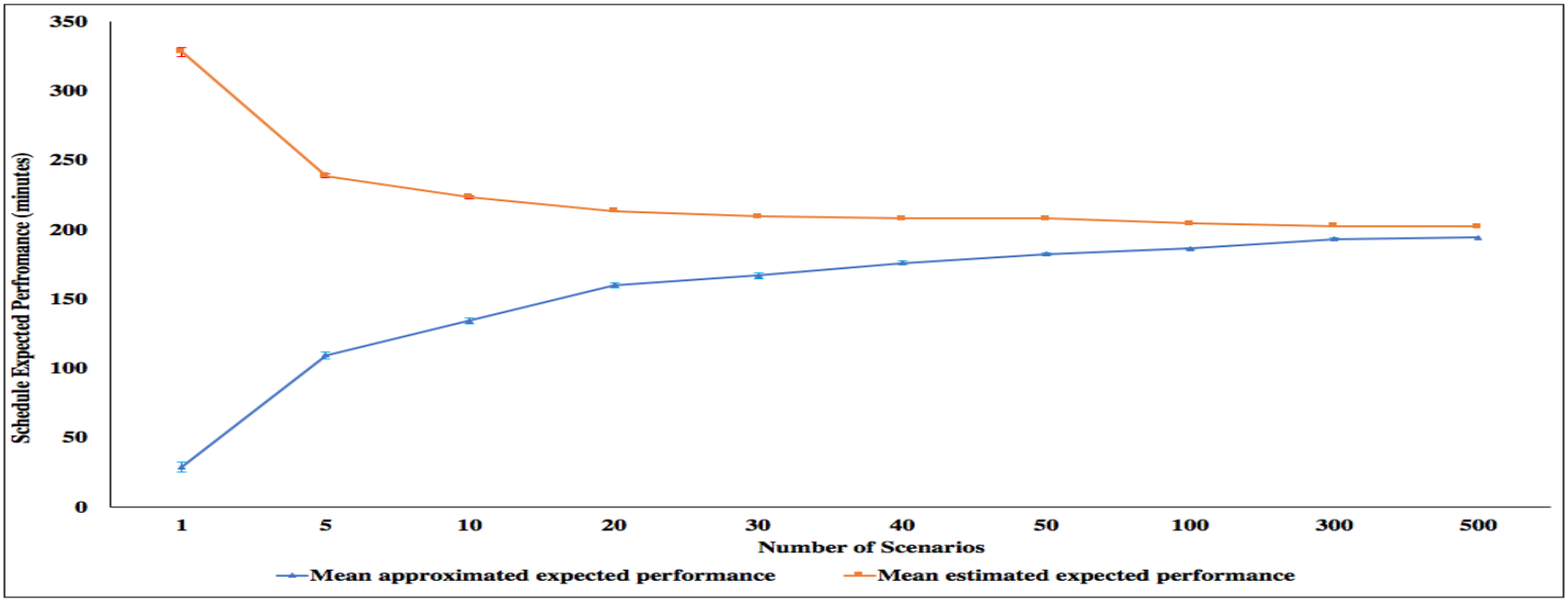


Figure 1: Convergence behavior of OSCSP mean approximated performance ($\overline{v}_N$, blue) and mean estimated performance ($\overline{v}_{N'}$, red) under duration uncertainty

Sample Size: (4:4), 0% no-show rate, punctual

$f^{\text{short}} = |N(30, 6.67^2)|$
$f^{\text{long}} = |N(75, 10^2)|$

- So, 500 scenarios?
  - Expected Performance of the "true" optimal schedule to the problem with all scenarios:
    - Lower bound=194 ± 0.7 Upper Bound=202 ± 0.20
    - Tight Confidence Intervals, i.e., small variances M=30 replicates ✓

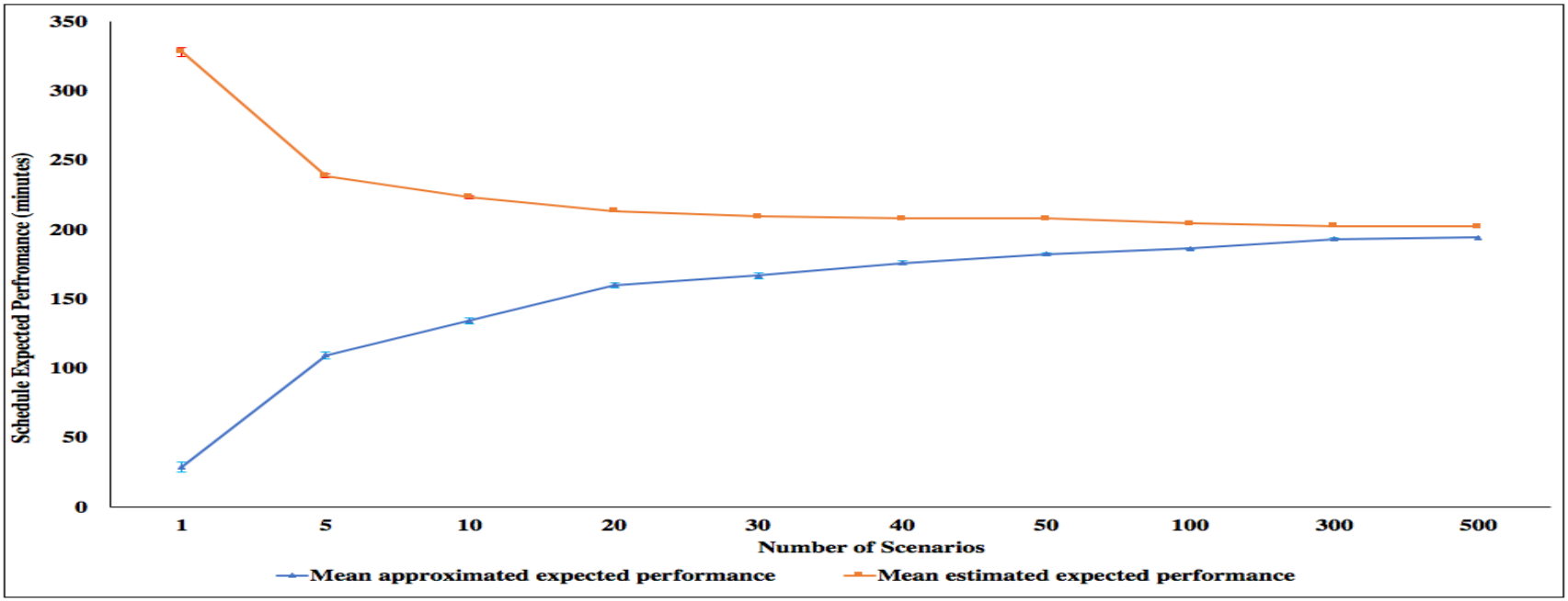


Figure 1: Convergence behavior of OSCSP mean approximated performance ($\overline{v}_N$, blue) and mean estimated performance ($\overline{v}_{N'}$, red) under duration uncertainty

Sample Size: (4:4), 0% no-show rate, punctual

$f^{\text{short}} = |N(30, 6.67^2)|$
$f^{\text{long}} = |N(75, 10^2)|$

- So, 500 scenarios?
  - Expected Performance of the "true" optimal schedule to the problem with all scenarios:
    - Lower bound=194 $\pm$ 0.7 Upper Bound=202 $\pm$ 0.20
    - Tight Confidence Intervals, i.e., small variances M=30 replicates ✓
  - Approximate Optimality Indix: 0.02 ✓

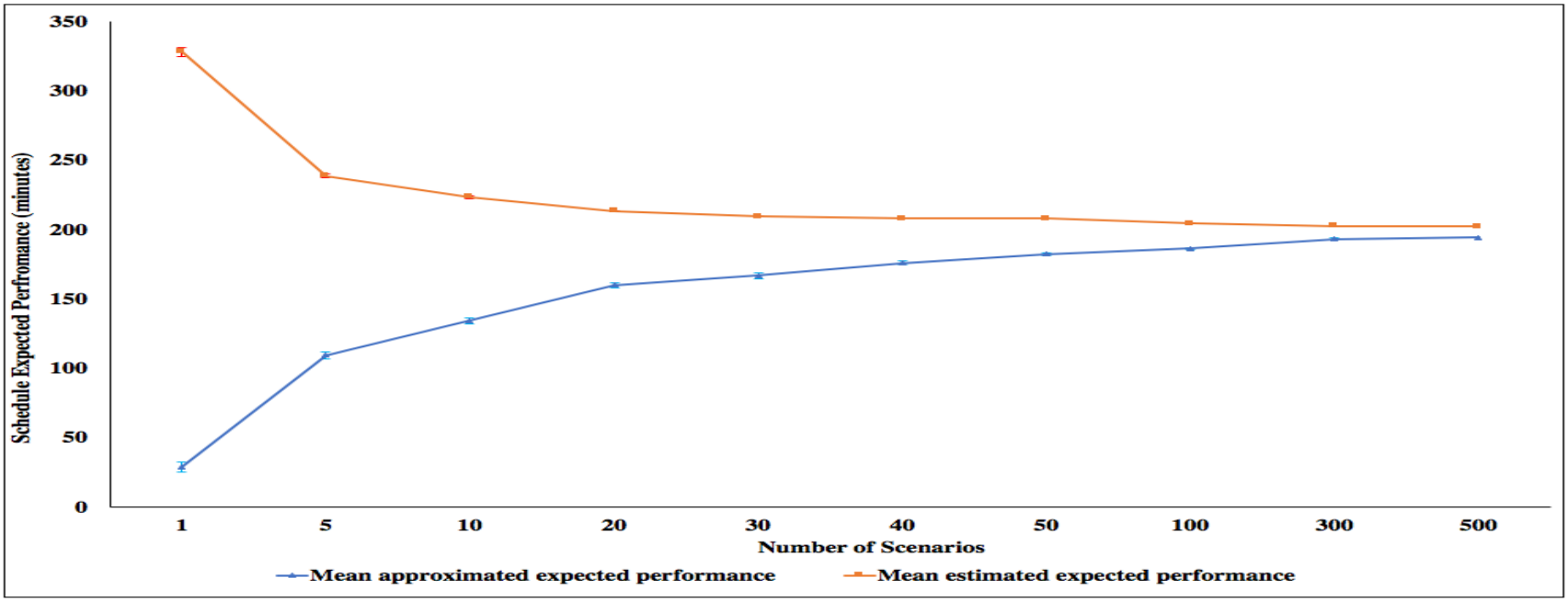


Figure 1: Convergence behavior of OSCSP mean approximated performance ($\overline{v}_N$, blue) and mean estimated performance ($\overline{v}_{N'}$, red) under duration uncertainty

Sample Size: (4:4), 0% no-show rate, punctual

$f^{\text{short}} = |N(30, 6.67^2)|$
$f^{\text{long}} = |N(75, 10^2)|$

- So, 500 scenarios?
  - Expected Performance of the "true" optimal schedule to the problem with all scenarios:
    - Lower bound=194 $\pm$ 0.7 Upper Bound=202 $\pm$ 0.20
    - Tight Confidence Intervals, i.e., small variances M=30 replicates ✓
  - Approximate Optimality Indix: 0.02 ✓
- A sample of $N = 500$ scenarios is large enough to approximate the optimal schedule

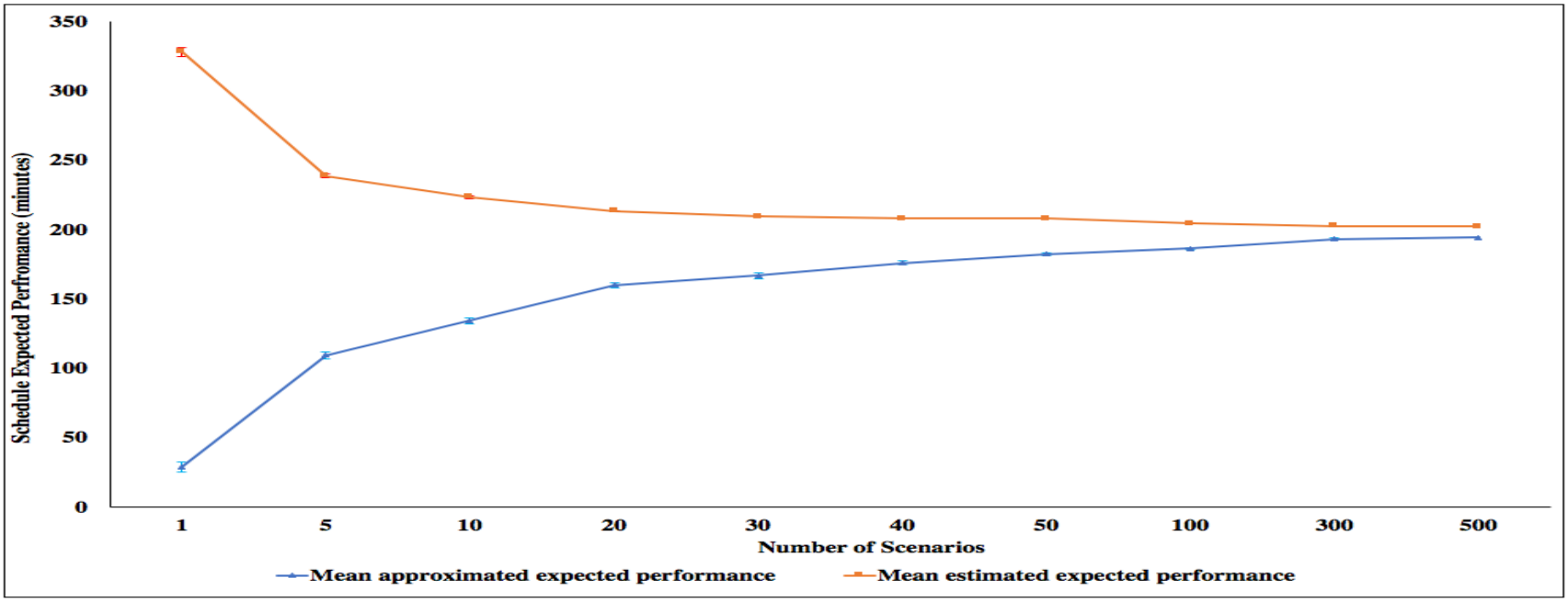


Figure 1: Convergence behavior of OSCSP mean approximated performance ($\overline{v}_N$, blue) and mean estimated performance ($\overline{v}_{N'}$, red) under duration uncertainty

Sample Size: (4:4), **18% no-show rate**, 20 **min late**

$f^{\text{short}} = |N(30, 6.67^2)|$
$f^{\text{long}} = |N(75, 10^2)|$

Sample Size: (4:4), 18% no-show rate, 20 min late

$f^{\text{short}} = |N(30, 6.67^2)|$
$f^{\text{long}} = |N(75, 10^2)|$

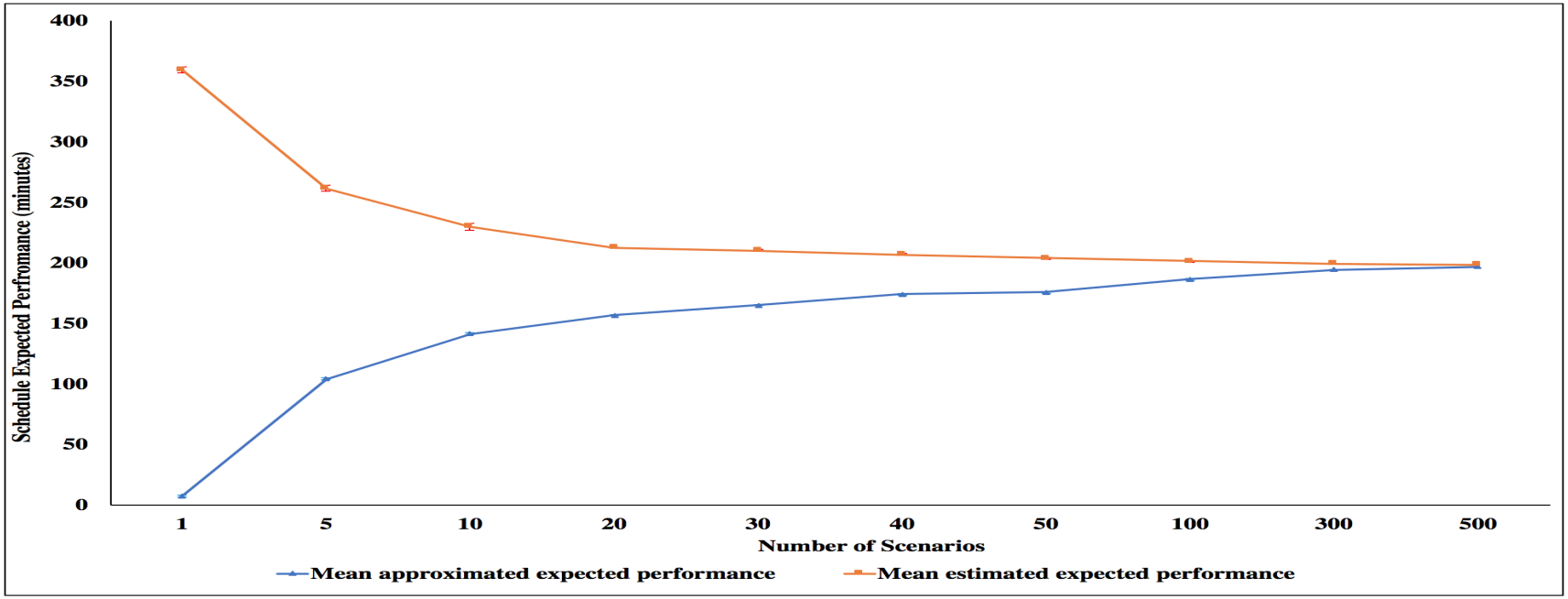


Figure 2: Convergence behavior of OSCSP mean approximated performance ($\overline{v}_N$, blue) and mean estimated performance ($\overline{v}_{N'}$, red) under duration, no-show and arrival uncertainties

Sample Size: (4:4), 18% no-show rate, 20 min late

$f^{\text{short}} = |N(30, 6.67^2)|$
$f^{\text{long}} = |N(75, 10^2)|$

- A sample of $N = 500$ scenarios is large enough to approximate the optimal schedule

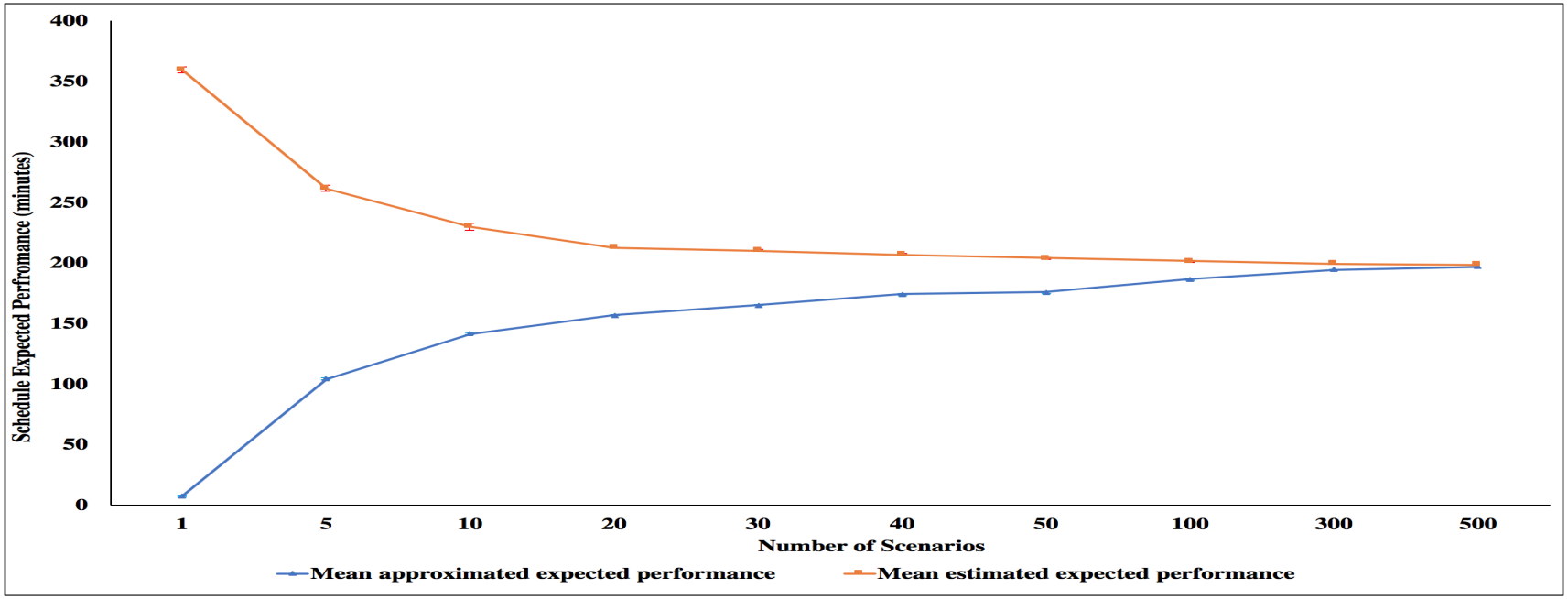


Figure 2: Convergence behavior of OSCSP mean approximated performance ($\overline{v}_N$, blue) and mean estimated performance ($\overline{v}_{N'}$, red) under duration, no-show and arrival uncertainties

So, we've got a representative sample for the observed uncertainty what to do next?

Provider Time versus Patient Time

Provider Time versus Patient Time

$$\min \ \lambda \cdot \texttt{Expected}_{500}[\texttt{Idle \& Over}] + (1-\lambda) \cdot \texttt{Expected}_{500}[\texttt{Total Waiting}]$$

Provider Time versus Patient Time

$$\min \ \lambda \cdot \texttt{Expected}_{500}[\texttt{Idle \& Over}] + (1-\lambda) \cdot \texttt{Expected}_{500}[\texttt{Total Waiting}]$$

⋆ Gives clinic managers several schedules from which they may select based on their goals

Provider Time versus Patient Time

$$\min\ \lambda \cdot \texttt{Expected}_{500}[\texttt{Idle \& Over}] + (1-\lambda) \cdot \texttt{Expected}_{500}[\texttt{Total Waiting}]$$

⋆ Gives clinic managers several schedules from which they may select based on their goals

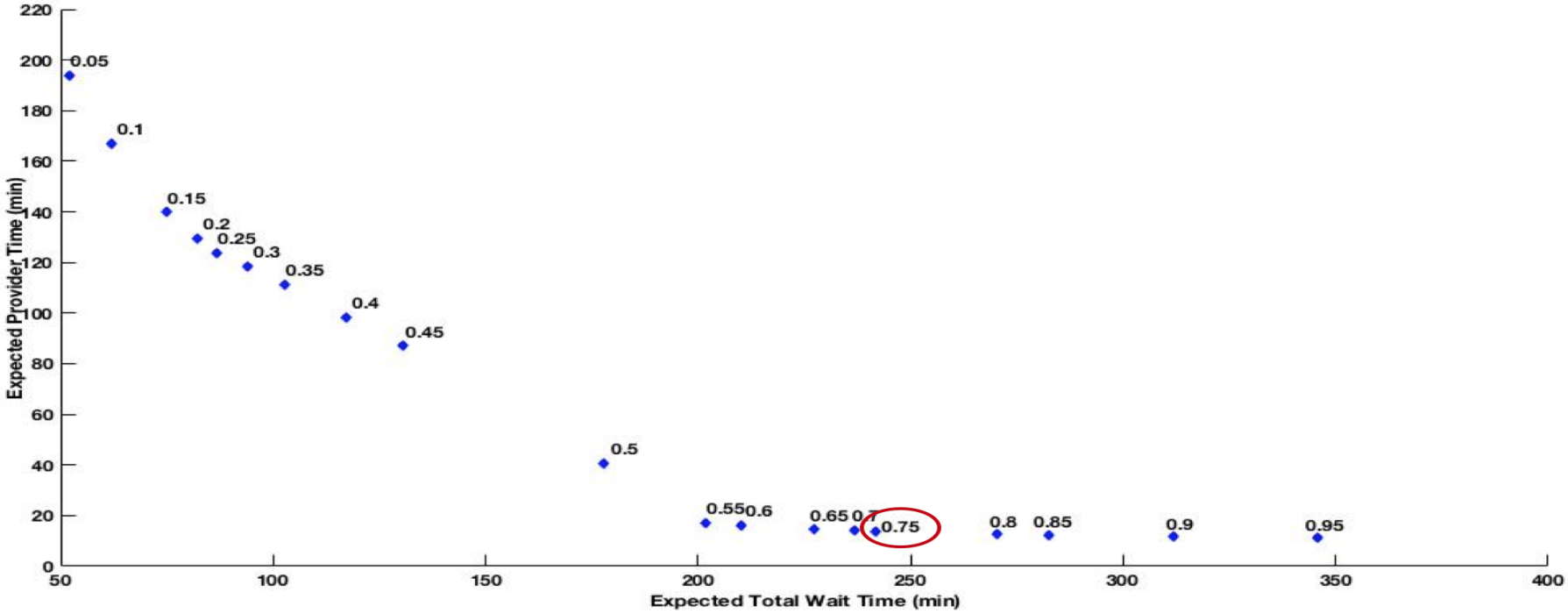


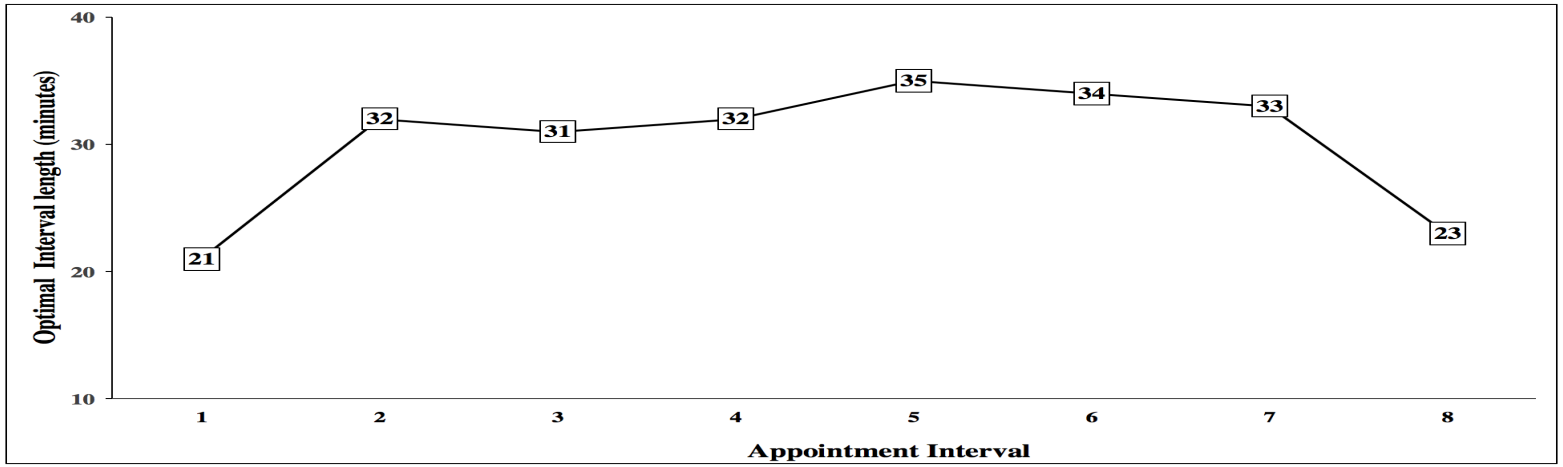
Figure 3: Expected performance of Pareto optimal schedules as function of the trade-off level λ.

Scheduling Polices for Different Case Mix

[3] a.k.a Time allowance between two consecutive patients

Scheduling Polices for Different Case Mix

- **One Type:** Appointments Lengths[3] $\sim$ *"Dome-Shape"*



[3]

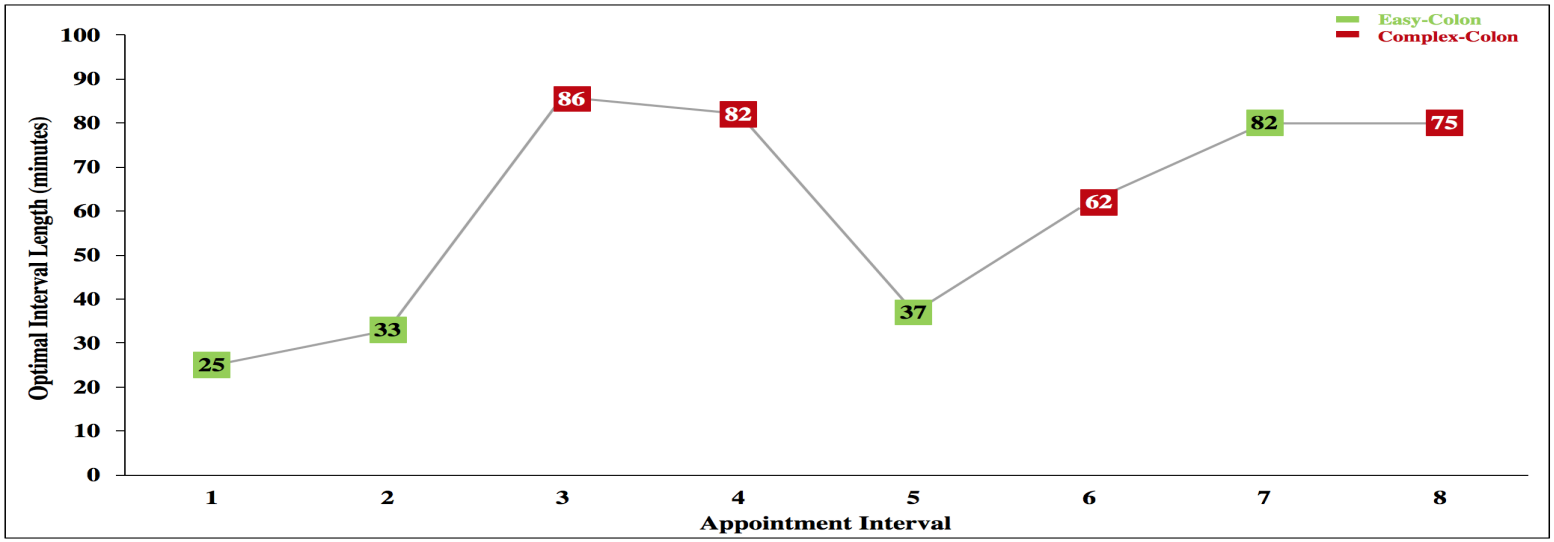
Scheduling Polices for Different Case Mix

- **One Type:** Appointments Lengths[3] $\sim$ *"Dome-Shape"*
- **Balanced Mix:** (e.g., 4E:4C)

[3] a.k.a Time allowance between two consecutive patients

Scheduling Polices for Different Case Mix

- **One Type:** Appointments Lengths[3] $\sim$ *"Dome-Shape"*
- **Balanced Mix:** (e.g., 4E:4C)



Scheduling Polices for Different Case Mix

- **One Type:** Appointments Lengths[3] $\sim$ *"Dome-Shape"*
- **Balanced Mix:** (e.g., 4E:4C)
  - **Sequence:** "*Alternating Bucket*" of *Easy* $\rightarrow$ *Complex*
  - **Appointments Lengths:** *"Bucket-based-Dome"*

[3] a.k.a Time allowance between two consecutive patients

Scheduling Polices for Different Case Mix

- **One Type:** Appointments Lengths[3] $\sim$ *"Dome-Shape"*
- **Balanced Mix:** (e.g., 4E:4C)
  - **Sequence:** "*Alternating Bucket*" of *Easy* $\rightarrow$ *Complex*
  - **Appointments Lengths:** *"Bucket-based-Dome"*
- **Unbalanced Mix** (e.g., 6E:2C)
  - **Sequence:** "*Alternating Bucket*" of *Easy* $\rightarrow$ *Complex*
  - **Appointments Lengths:** *"Bucket-based-Dome"*

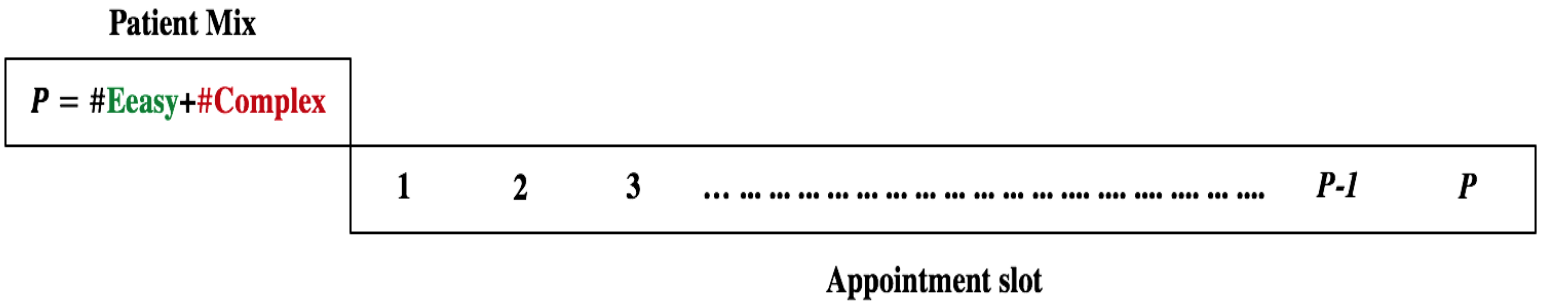
[3] a.k.a Time allowance between two consecutive patients

Scheduling Polices for Different Case Mix

- Natural Question: how to leverage small schedules properties in solving the more challenging versions of the problem?

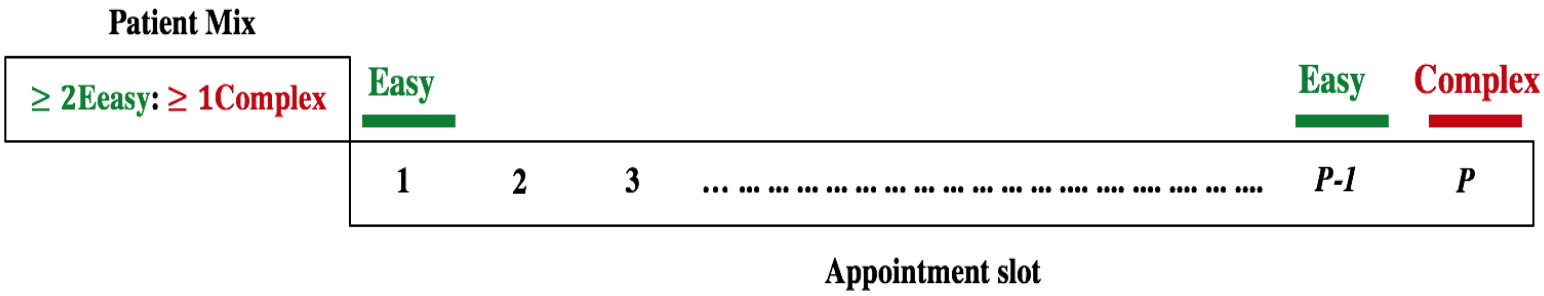
Scheduling Polices for Different Case Mix

- Natural Question: how to leverage small schedules properties in solving the more challenging versions of the problem?



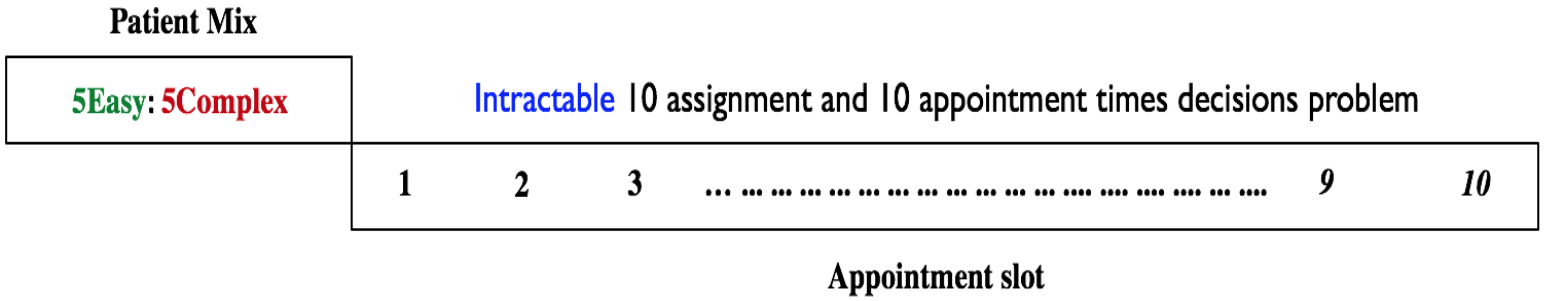
Scheduling Polices for Different Case Mix

- Natural Question: how to leverage small schedules properties in solving the more challenging versions of the problem?



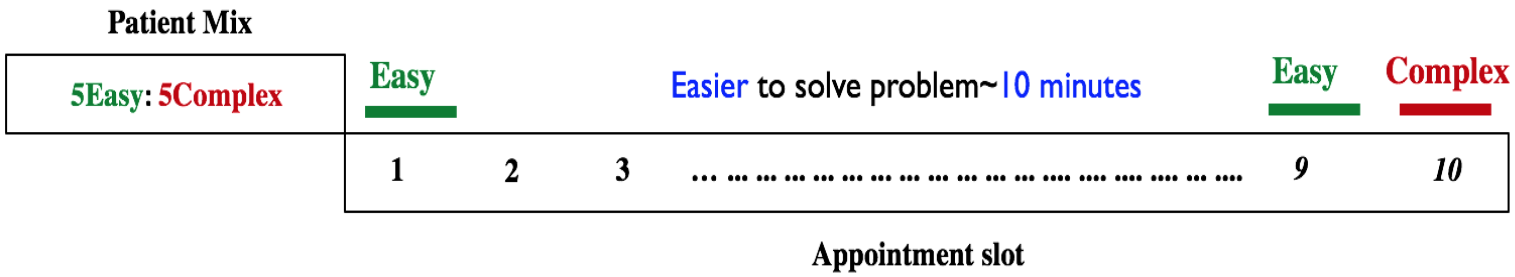
Scheduling Polices for Different Case Mix

- Natural Question: how to leverage small schedules properties in solving the more challenging versions of the problem?



Scheduling Polices for Different Case Mix

- Natural Question: how to leverage small schedules properties in solving the more challenging versions of the problem?



- Paired t-test

  $f(\mathcal{S}^{\texttt{Fixed}})$- $f(\mathcal{S}^{\texttt{Opt}}) \in$[-0.75±1.87], i.e., No "statistically" significant differences

So, Worth the Computational Efforts?

So, Worth the Computational Efforts?

4Easy:4Complex
18%noShow
±20min

So, Worth the Computational Efforts?

4Easy:4Complex
18%noShow
±20min
1,000Days

So, Worth the Computational Efforts?

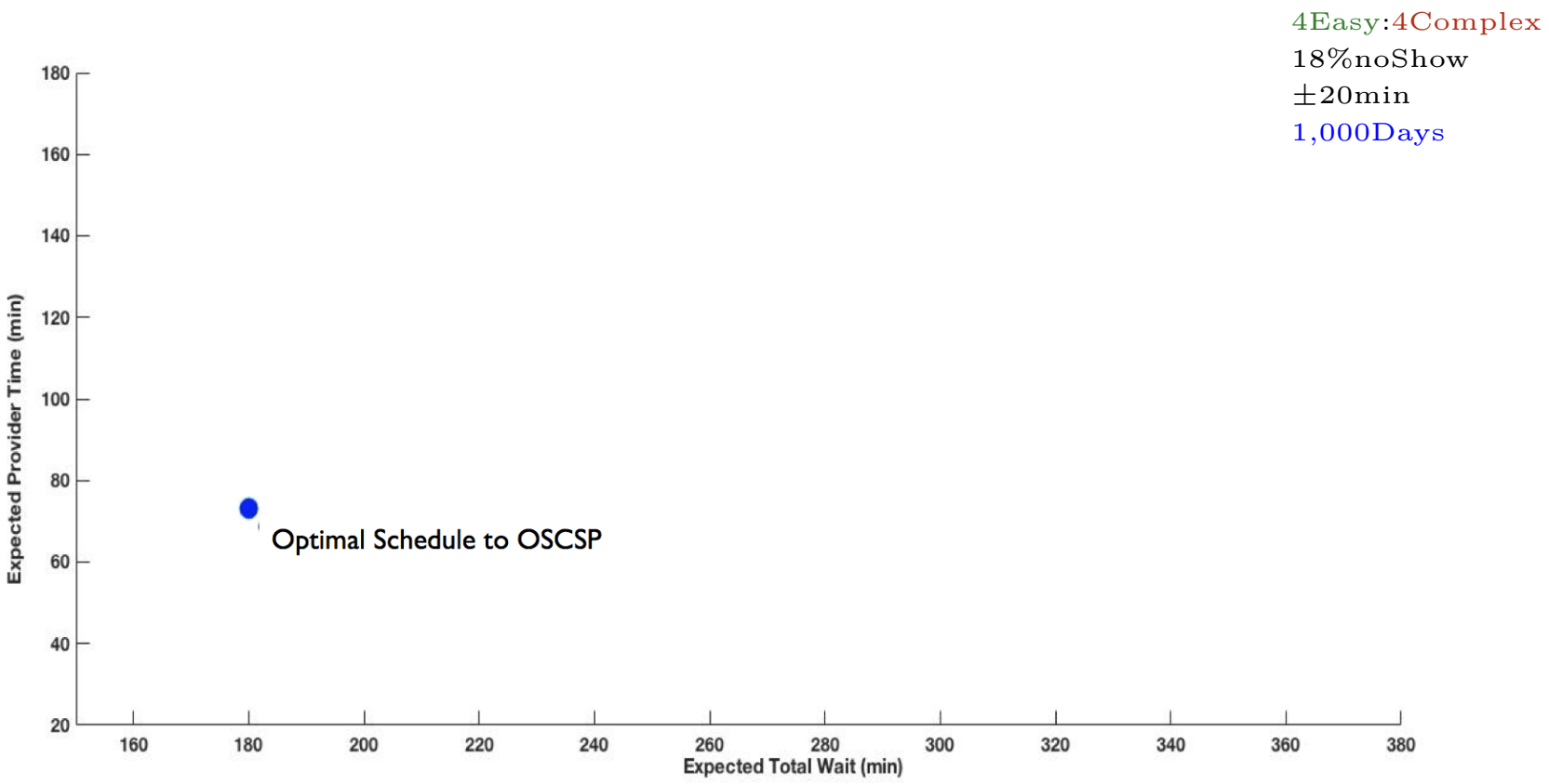


Figure 4: Expected performance form 1,000 simulation runs of OSCSP schedule and some traditional scheduling rules.The case of 18% no-show rate and 20 min early/late

So, Worth the Computational Efforts?

4Easy:4Complex
18%noShow
±20min
1,000Days

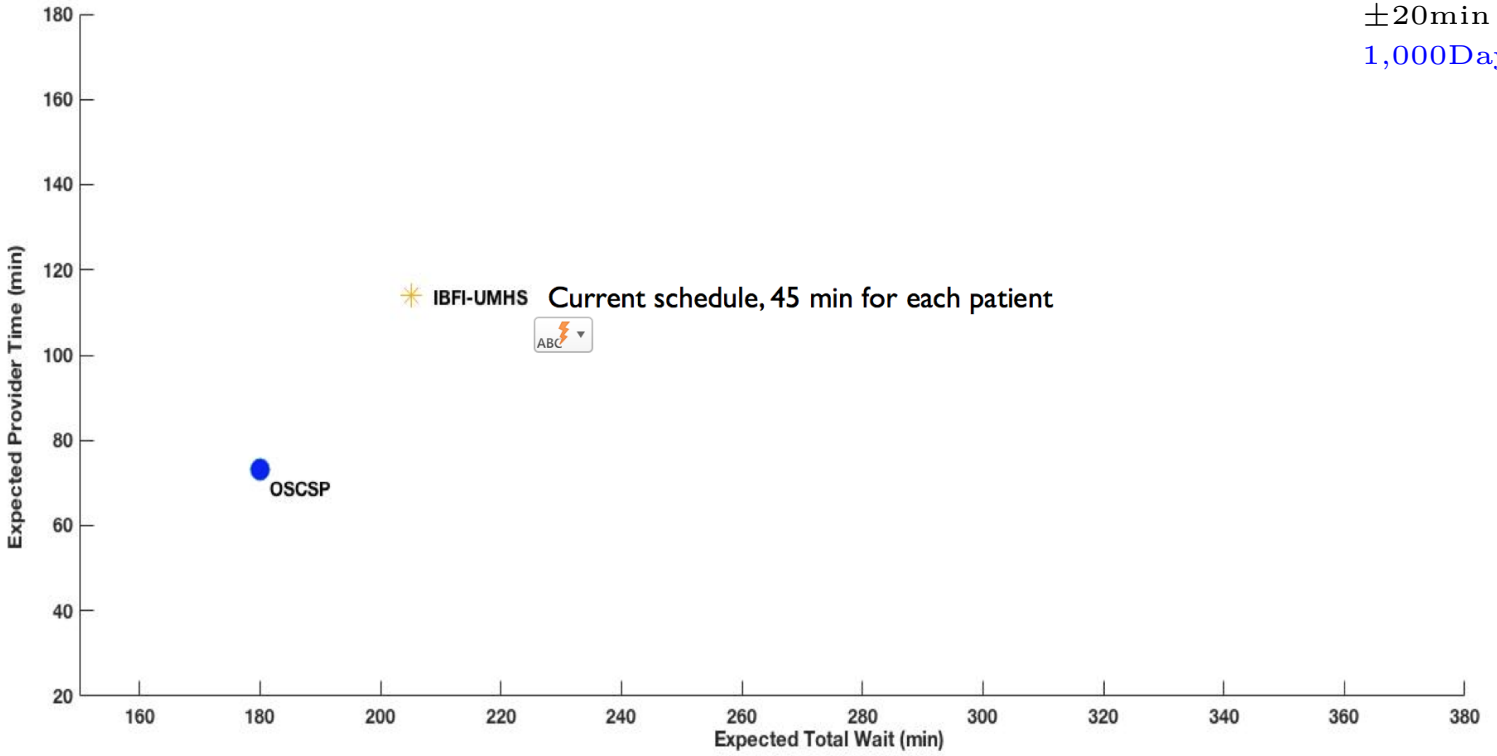


Figure 4: Expected performance form 1,000 simulation runs of OSCSP schedule and some traditional scheduling rules.The case of 18% no-show rate and 20 min early/late

So, Worth the Computational Efforts?

4Easy:4Complex
18%noShow
±20min
1,000Days

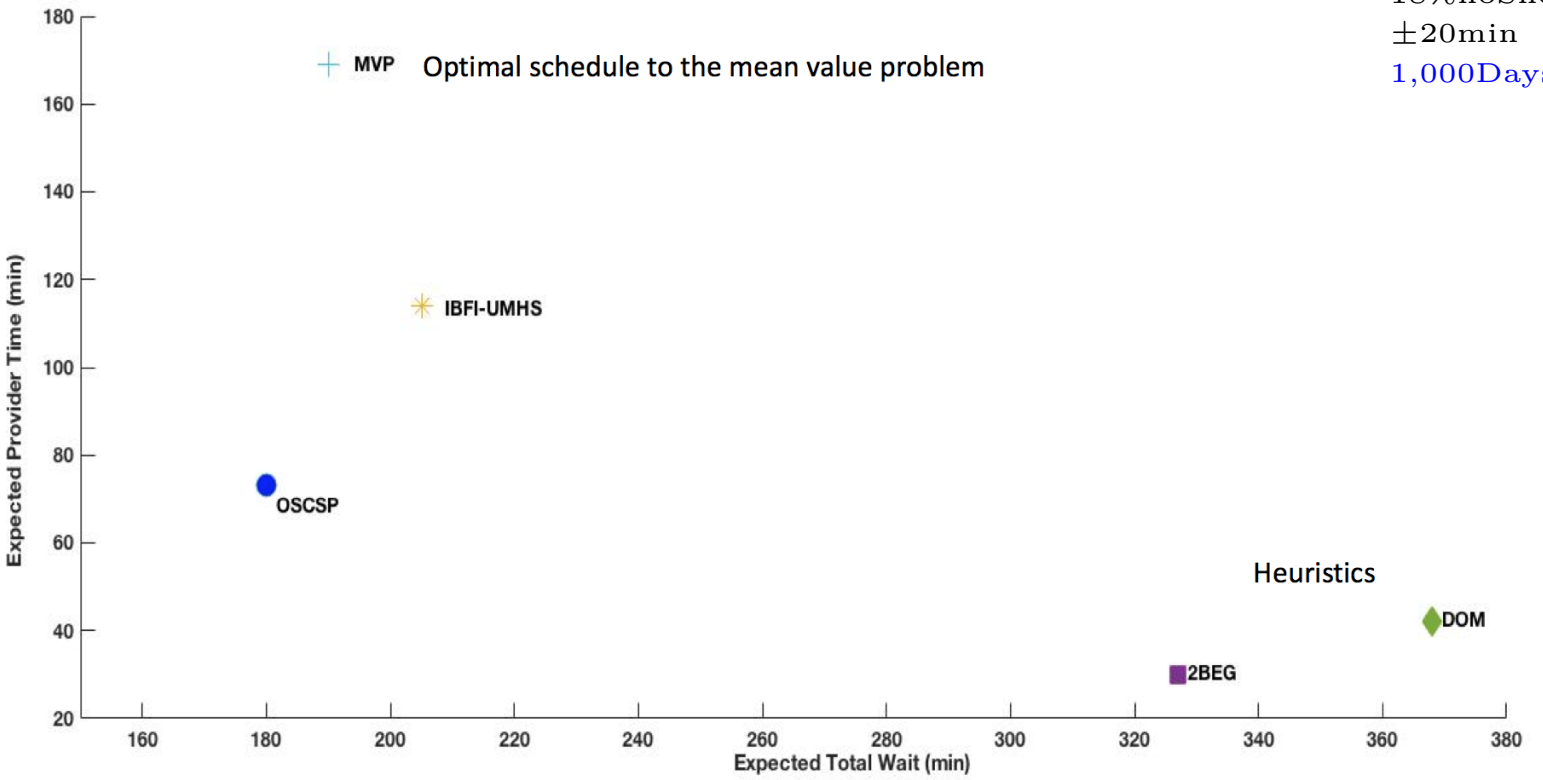


Figure 4: Expected performance form 1,000 simulation runs of OSCSP schedule and some traditional scheduling rules.The case of 18% no-show rate and 20 min early/late

Presentation Overview

Conclusion

- Colonoscopy scheduling have unique characteristics that are different in nature and potentially different from other OPC
- Better scheduling policies can be developed by approximating the uncertainty within these characteristics
- Properties of the "approximated" schedules can be exploited in designing a fast and easy to use schedule optimization tool

Acknowledgment

- My Star Professor Amy Cohn
- The Center for Healthcare Engineering and Patient Safety
- The Seth Bonder Foundation
- The IOE Family at the University of Michigan

Thank You!

"***No one can whistle a symphony. It takes a whole orchestra to play it***"

Q&A

Karmel Shehadeh
Ksheha@umich.edu

Professor. Amy Cohn
amycohn@umich.edu