Using Stochastic Programming to Improve Service Quality in an Outpatient Infusion Center

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Spyros Potiris

Center for Healthcare Engineering & Patient Safety

Collaborators

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Background

- Cancer Statistics
  - Second leading cause of death in the US
  - 1,638,910 new cases of cancer in 2012
- Outpatient Oncology
  - 29.2 million visits in the US with a primary diagnosis of cancer
  - 23 million adult patient visits for chemotherapy
- University of Michigan Comprehensive Cancer Center
  - 93,319 Outpatient Visits
  - 51,884 Infusion Treatments

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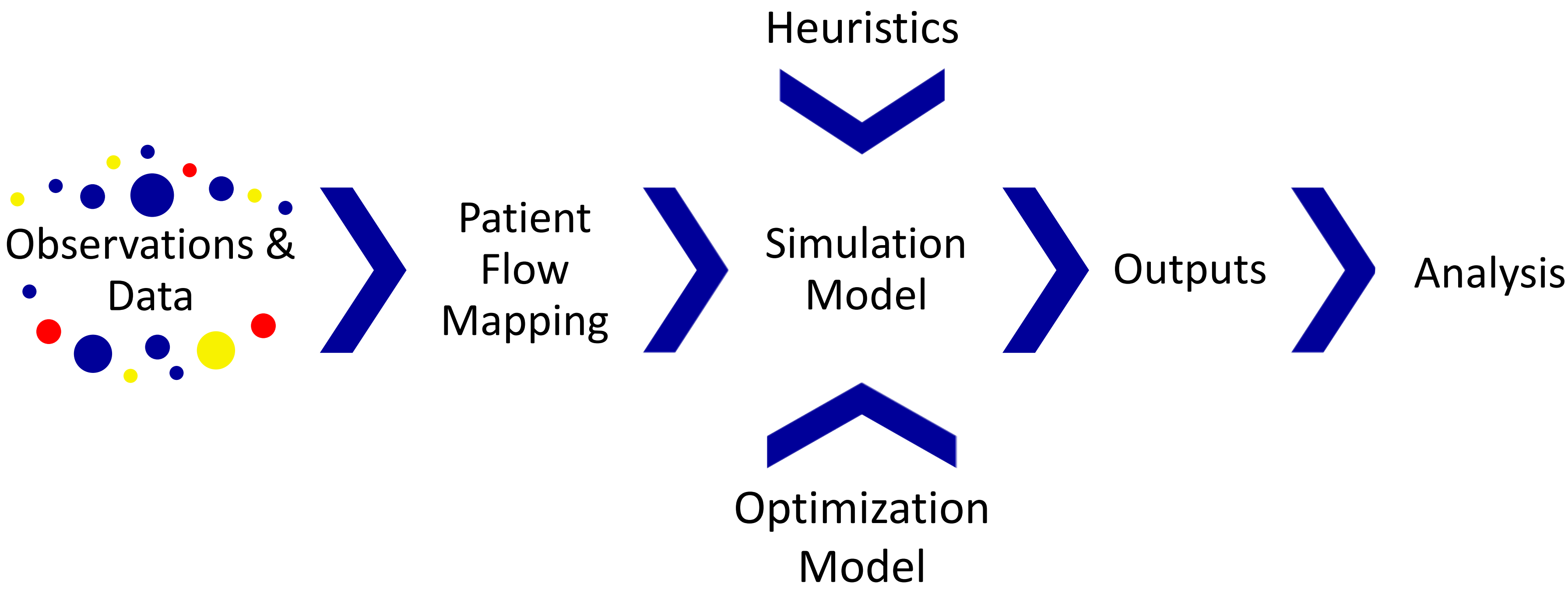
Project Goals

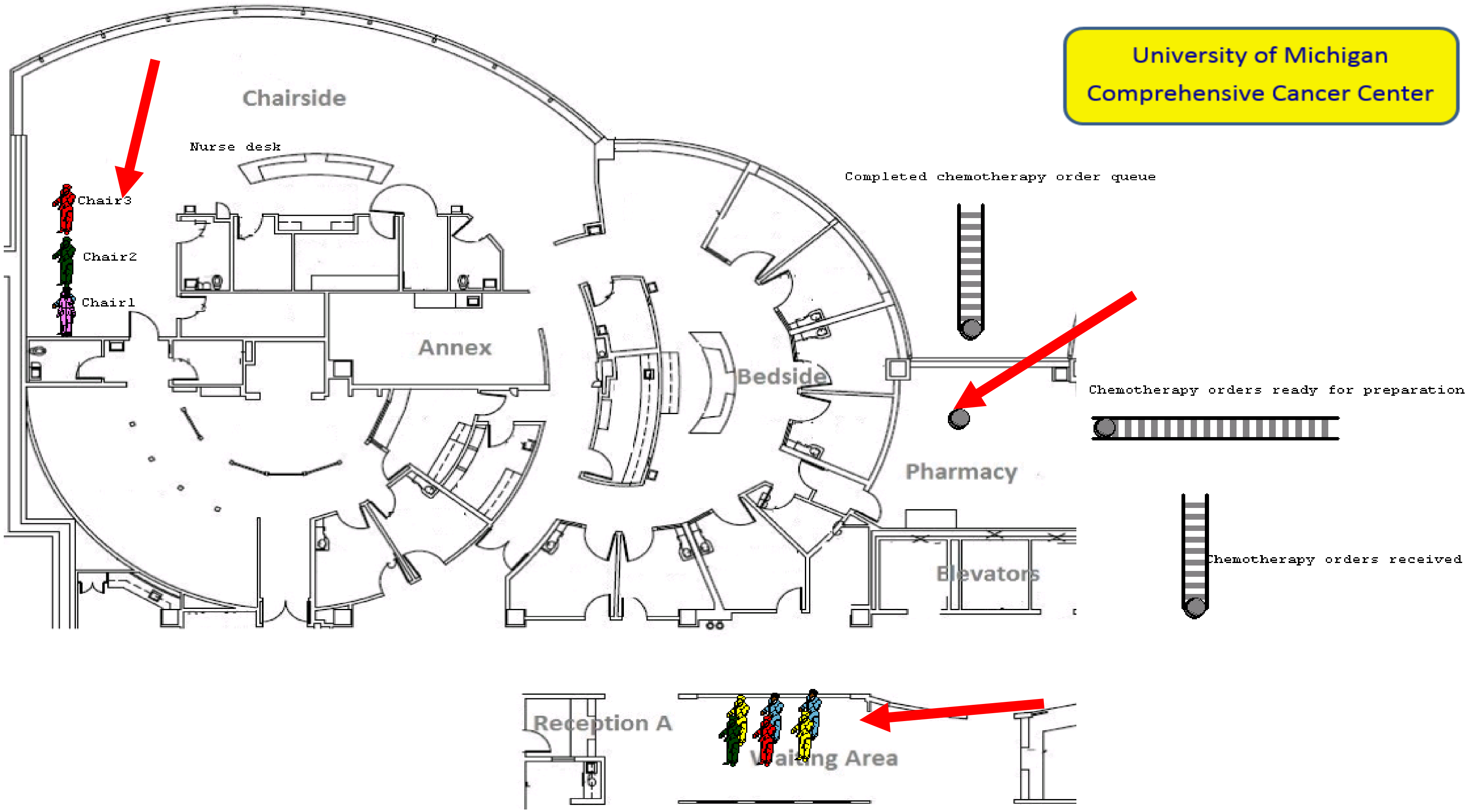
- Improve quality of cancer care delivery in the infusion center
  - Reduce patient waiting times
  - Reduce total length of day of operations
  - Improve patient and nurse safety

Approach


Heuristics
Observations & Data
Patient Flow Mapping
Simulation Model
Outputs
Analysis
Optimization Model

Computer Simulation Model

University of Michigan
Comprehensive Cancer Center



Computer Simulation Model
Inputs - Outputs

Inputs

Patient types

Nurse preparation time

Nurse discharge time

Pharmacy preparation time

Patient appointment schedules

- Baseline
- LPT heuristic
- SPT heuristic
- Stochastic optimization model

Computer Simulation

Outputs

Average patient waiting times

Hours of operation

Chair utilization

Average time in system

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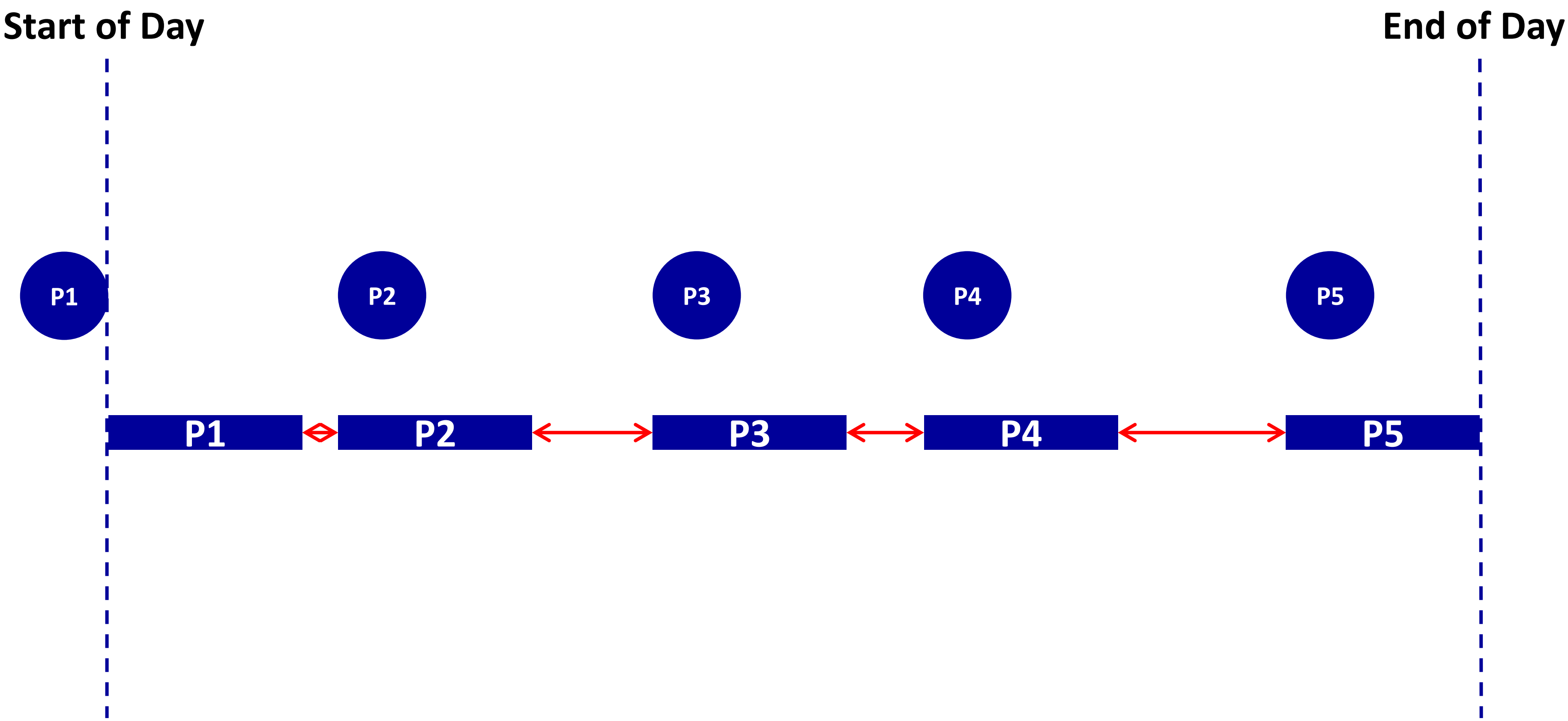
Patient wait times vs. Length of day



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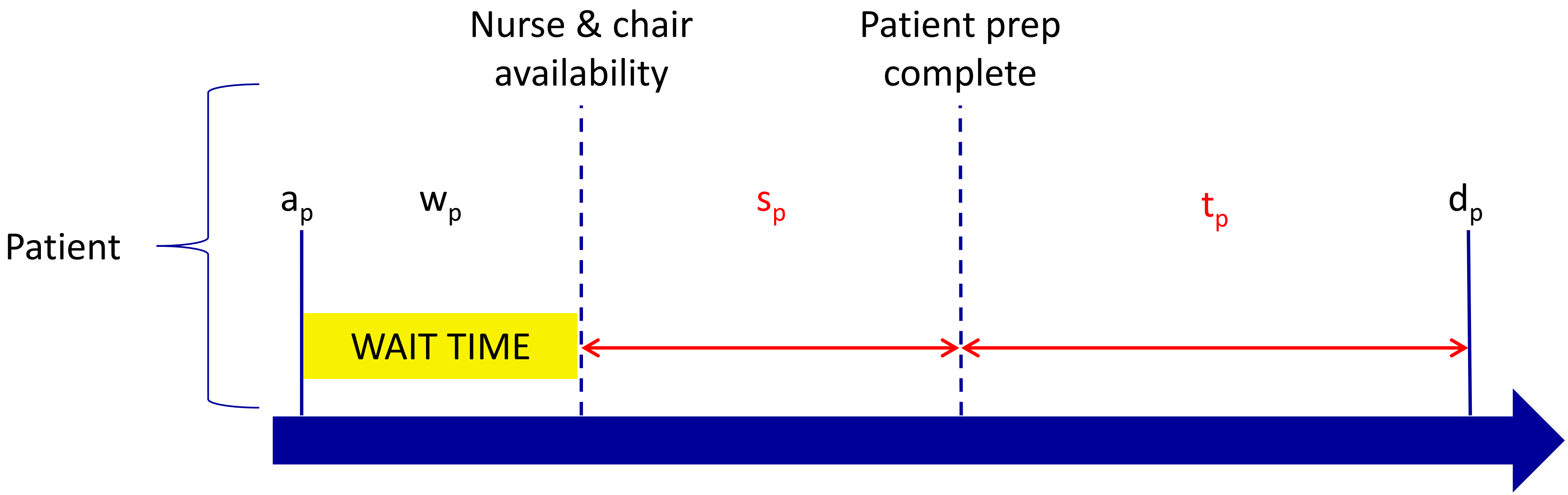
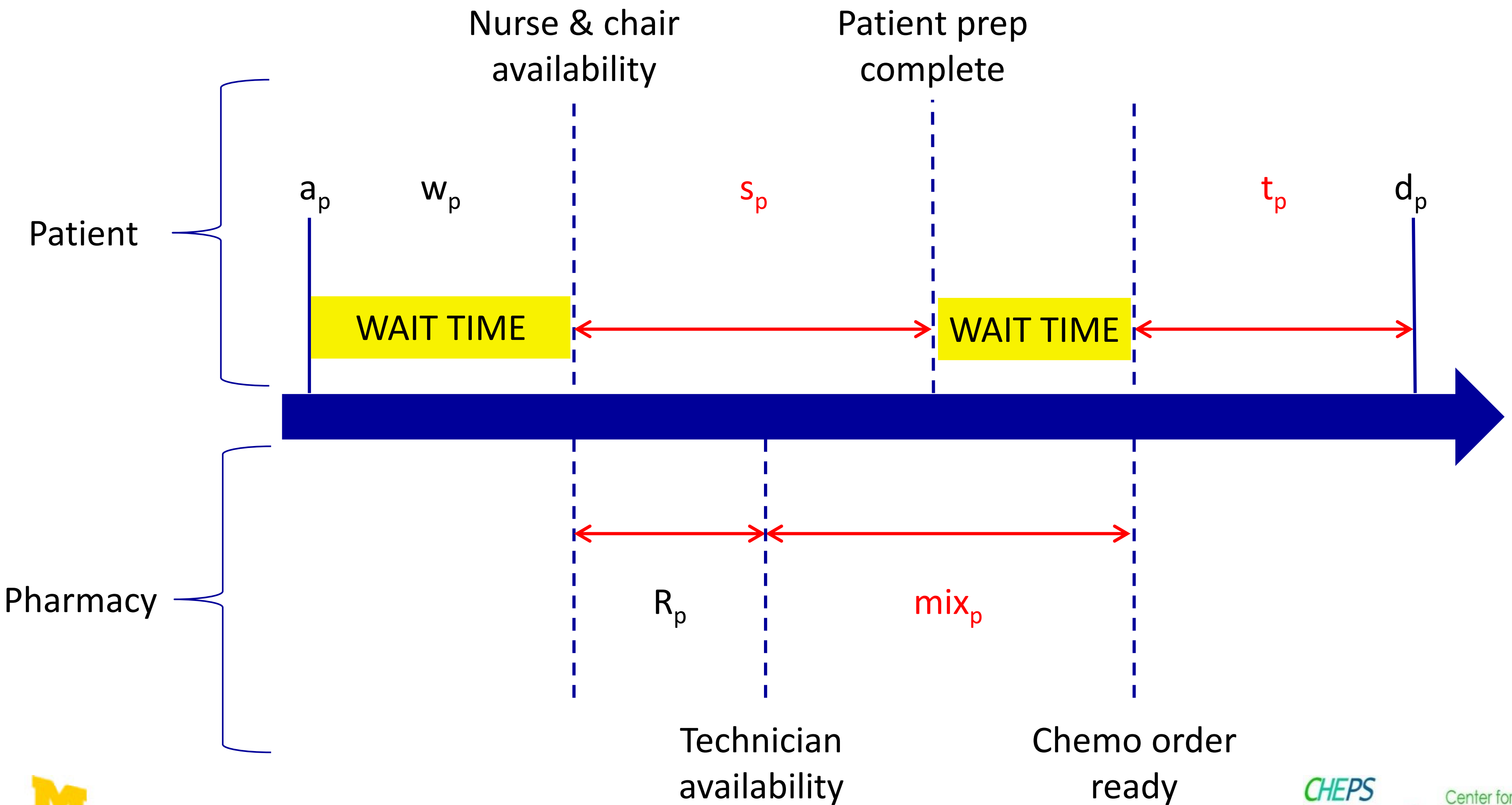
Stochastic Programming Model

- Generates appointment schedule that reduces patient waiting times and total length of day of operations
- Decisions
  - Appointment times
  - Patient sequence
  - Patient-Chair assignment
- Stochasticity
  - Scenarios represent variability
  - Scenarios sample infusion, preparation, discharge, and drug mix times from distributions

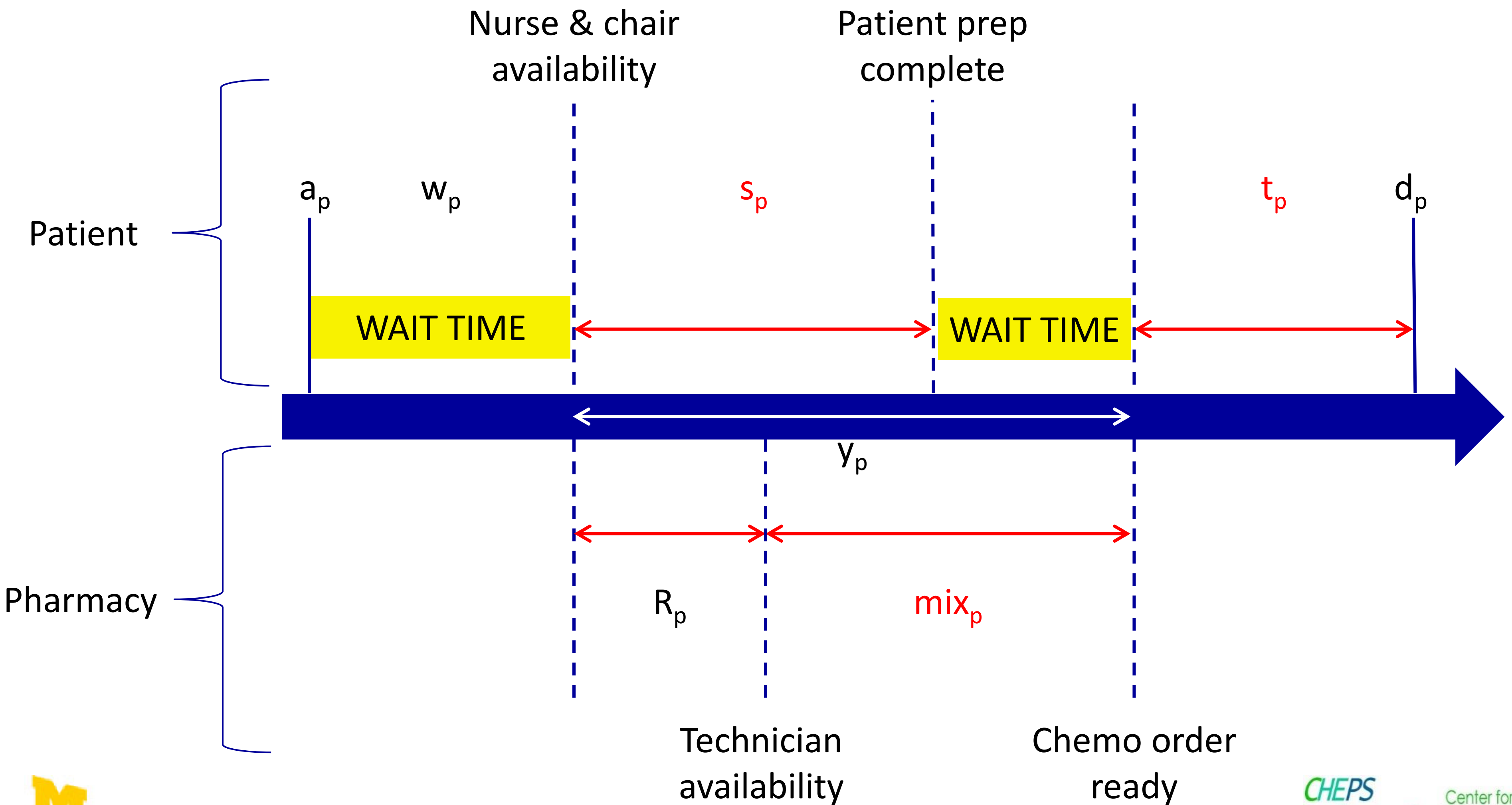
Stochastic Programming Model Variables & Parameters Explanation



Stochastic Programming Model Variables & Parameters Explanation



Stochastic Programming Model Variables & Parameters Explanation



Stochastic Programming Model Sample Path Approximation

Decision Variables

a_p : appointment time of patient p
w_p^ω : waiting time of patient p in scenario ω
R_p^ω : waiting time for available technician to mix patient's p chemo drug at the pharmacy in scenario ω
y_p^ω : time patient p sits in the chair before infusion starts in scenario ω
d_p^ω : exit time of patient p in scenario ω
x_{pc}^ω : binary, 1 if patient p is assigned to chair c in scenario ω ; 0 otherwise
$z_{p'p}$: binary, 1 if patient p' is scheduled before patient p ; 0 otherwise
L^ω : end of day in scenario ω

Parameters

s_p^ω : preparation time of patient p in scenario ω
t_p^ω : infusion time plus discharge time of patient p in scenario ω
mix_p^ω : chemo drug mix time for patient p in scenario ω
m : number of scenarios
λ : weight in the objective function
M : large number

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Stochastic Programming Model
Sample Path Approximation

$$\min \quad \lambda \sum_{p\in P} \sum_{\omega\in\Omega} \frac{1}{m}[w_p^\omega + (y_p^\omega - s_p^\omega)] + (1-\lambda) \sum_{\omega\in\Omega} \frac{1}{m} L^\omega$$

Objective Function: Trade-off between the total expected waiting time and the expected length of day

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Stochastic Programming Model
Sample Path Approximation

$$\begin{aligned}
\min \quad & \lambda \sum_{p \in P} \sum_{\omega \in \Omega} \frac{1}{m}[w_p^\omega + (y_p^\omega - s_p^\omega)] + (1-\lambda) \sum_{\omega \in \Omega} \frac{1}{m} L^\omega \\
\text{subject to} \quad & \sum_{c \in C} x_{pc}^\omega = 1 \qquad \forall p \in P,\ \forall \omega \in \Omega
\end{aligned}$$

Each patient should be assigned to exactly one infusion chair

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Stochastic Programming Model
Sample Path Approximation

$$
\begin{aligned}
\min \quad & \lambda \sum_{p\in P}\sum_{\omega\in\Omega}\frac{1}{m}[w_p^\omega + (y_p^\omega - s_p^\omega)] + (1-\lambda)\sum_{\omega\in\Omega}\frac{1}{m}L^\omega \\
\text{subject to} \quad & \sum_{c\in C} x_{pc}^\omega = 1 && \forall p\in P,\ \forall\omega\in\Omega \\
& a_p + w_p^\omega + y_p^\omega + t_p^\omega = d_p^\omega && \forall p\in P,\ \forall\omega\in\Omega
\end{aligned}
$$

Value of exit time of patient p in each scenario

Stochastic Programming Model
Sample Path Approximation

$$
\begin{aligned}
\min \quad & \lambda \sum_{p\in P}\sum_{\omega\in\Omega} \tfrac{1}{m}[w_p^\omega + (y_p^\omega - s_p^\omega)] + (1-\lambda)\sum_{\omega\in\Omega}\tfrac{1}{m}L^\omega \\
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& a_p + w_p^\omega + y_p^\omega + t_p^\omega = d_p^\omega && \forall p\in P,\ \forall \omega\in\Omega \\
& a_p + w_p^\omega + M(3 - x_{pc}^\omega - x_{p'c}^\omega - z_{p'p}) \geq d_{p'}^\omega && \forall c\in C,\ \forall p,p'\in P,\ \forall\omega\in\Omega
\end{aligned}
$$

Free chair constraint: A patient can sit in a chair only if all previously sequenced patients assigned to the same chair have been discharged

Stochastic Programming Model Sample Path Approximation

$$
\begin{aligned}
\min \quad & \lambda \sum_{p \in P} \sum_{\omega \in \Omega} \frac{1}{m}[w_p^\omega + (y_p^\omega - s_p^\omega)] + (1-\lambda) \sum_{\omega \in \Omega} \frac{1}{m} L^\omega \\
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& a_p + w_p^\omega + M(1 - z_{p'p}) \geq a_{p'} + w_{p'}^\omega + s_{p'}^\omega && \forall p, p' \in P,\ \forall \omega \in \Omega
\end{aligned}
$$

Available nurse constraint: A patient can sit in a chair if the nurse has finished preparing all previously sequenced patients

Stochastic Programming Model Sample Path Approximation

$$
\begin{aligned}
\min \quad & \lambda \sum_{p\in P} \sum_{\omega\in\Omega} \frac{1}{m}[w_p^\omega + (y_p^\omega - s_p^\omega)] + (1-\lambda) \sum_{\omega\in\Omega} \frac{1}{m} L^\omega \\
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& a_p + w_p^\omega + M(1 - z_{p'p}) \geq a_{p'} + w_{p'}^\omega && \forall p, p' \in P,\ \forall \omega \in \Omega
\end{aligned}
$$

FIFO rule: A patient that arrives before another patient has to sit in an infusion chair first

Stochastic Programming Model
Sample Path Approximation

$$
\begin{aligned}
\min \quad & \lambda \sum_{p\in P} \sum_{\omega\in\Omega} \frac{1}{m}[w_p^\omega + (y_p^\omega - s_p^\omega)] + (1-\lambda) \sum_{\omega\in\Omega} \frac{1}{m} L^\omega \\
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& a_p + w_p^\omega + R_p^\omega + M(1 - z_{p'p}) \geq a_{p'} + w_{p'}^\omega + R_{p'}^\omega + mix_{p'}^\omega && \forall p, p' \in P,\ \forall \omega \in \Omega
\end{aligned}
$$

Available pharmacy technician constraint: A pharmacy technician can prepare only one chemotherapy order at a time

Stochastic Programming Model
Sample Path Approximation

$$
\begin{array}{lll}
\min & \lambda \sum_{p\in P} \sum_{\omega\in\Omega} \frac{1}{m}[w_p^\omega + (y_p^\omega - s_p^\omega)] + (1-\lambda) \sum_{\omega\in\Omega} \frac{1}{m} L^\omega & \\
\text{subject to} & \sum_{c\in C} x_{pc}^\omega = 1 & \forall p \in P,\ \forall \omega \in \Omega \\
& a_p + w_p^\omega + y_p^\omega + t_p^\omega = d_p^\omega & \forall p \in P,\ \forall \omega \in \Omega \\
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& a_p + w_p^\omega + M(1 - z_{p'p}) \geq a_{p'} + w_{p'}^\omega + s_{p'}^\omega & \forall p, p' \in P,\ \forall \omega \in \Omega \\
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& a_p + w_p^\omega + R_p^\omega + M(1 - z_{p'p}) \geq a_{p'} + w_{p'}^\omega + R_{p'}^\omega + mix_{p'}^\omega & \forall p, p' \in P,\ \forall \omega \in \Omega \\
& L^\omega \geq d_p^\omega & \forall p \in P,\ \forall \omega \in \Omega
\end{array}
$$

All patients should be discharged to end the day

Stochastic Programming Model
Sample Path Approximation

$$
\begin{array}{lll}
\min & \lambda \sum_{p\in P}\sum_{\omega\in\Omega} \frac{1}{m}[w_p^\omega + (y_p^\omega - s_p^\omega)] + (1-\lambda)\sum_{\omega\in\Omega}\frac{1}{m}L^\omega & \\
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& L^\omega \geq d_p^\omega & \forall p\in P,\ \forall\omega\in\Omega \\
& (a_{p'} - a_p) + M z_{p'p} \geq 0 & \forall p,p'\in P \\
& z_{pp'} = 1 - z_{p'p} & \forall p,p'\in P
\end{array}
$$

Definition of variable $z_{p'p}$

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Stochastic Programming Model
Sample Path Approximation

$$
\begin{array}{lll}
\min & \lambda \sum_{p\in P} \sum_{\omega\in\Omega} \frac{1}{m}[w_p^\omega + (y_p^\omega - s_p^\omega)] + (1-\lambda) \sum_{\omega\in\Omega} \frac{1}{m} L^\omega & \\
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& z_{pp'} = 1 - z_{p'p} & \forall p, p' \in P \\
& y_p^\omega \geq s_p^\omega & \forall p \in P,\ \forall \omega \in \Omega \\
& y_p^\omega \geq R_p^\omega + mix_p^\omega & \forall p \in P,\ \forall \omega \in \Omega
\end{array}
$$

Definition of variable y^ω_p

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Sample Path Approximation

$$
\begin{array}{lll}
\min & \lambda \sum_{p\in P} \sum_{\omega\in\Omega} \frac{1}{m}[w_p^\omega + (y_p^\omega - s_p^\omega)] + (1-\lambda) \sum_{\omega\in\Omega} \frac{1}{m} L^\omega & \\
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& z_{pp'} = 1 - z_{p'p} & \forall p, p' \in P \\
& y_p^\omega \geq s_p^\omega & \forall p \in P,\ \forall \omega \in \Omega \\
& y_p^\omega \geq R_p^\omega + mix_p^\omega & \forall p \in P,\ \forall \omega \in \Omega \\
& x_{pc}^\omega \in \{0, 1\} & \forall c \in C,\ \forall p \in P,\ \forall \omega \in \Omega \\
& z_{p'p} \in \{0, 1\} & \forall p, p' \in P \\
& a_p \geq 0 & \forall p \in P \\
& w_p^\omega, d_p^\omega, R_p^\omega \geq 0 & \forall p \in P,\ \forall \omega \in \Omega
\end{array}
$$

Binary and non-negativity constraints

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Stochastic Programming Model Tractability

Optimized patient sequence

Optimized appointment times

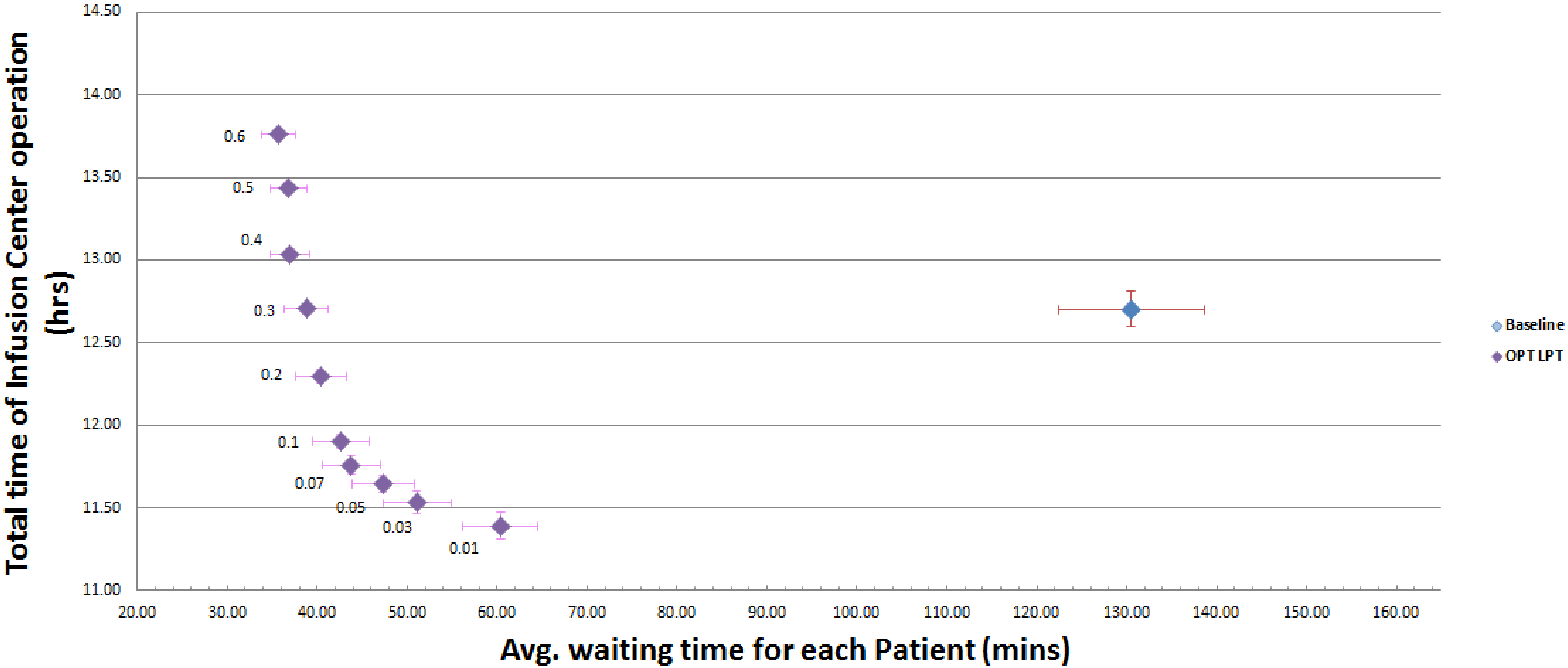
→

Fixed patient sequence

Optimized appointment times

- Stochastic MIP model
  - Large number of constraints
  - Using big M in constraints results in weak relaxations
- Stochastic LP model assuming a fixed sequence of patients (e.g. LPT)
  - Eliminate $z_{p'p}$ and x^{ω}_{pc} decision variables

Results



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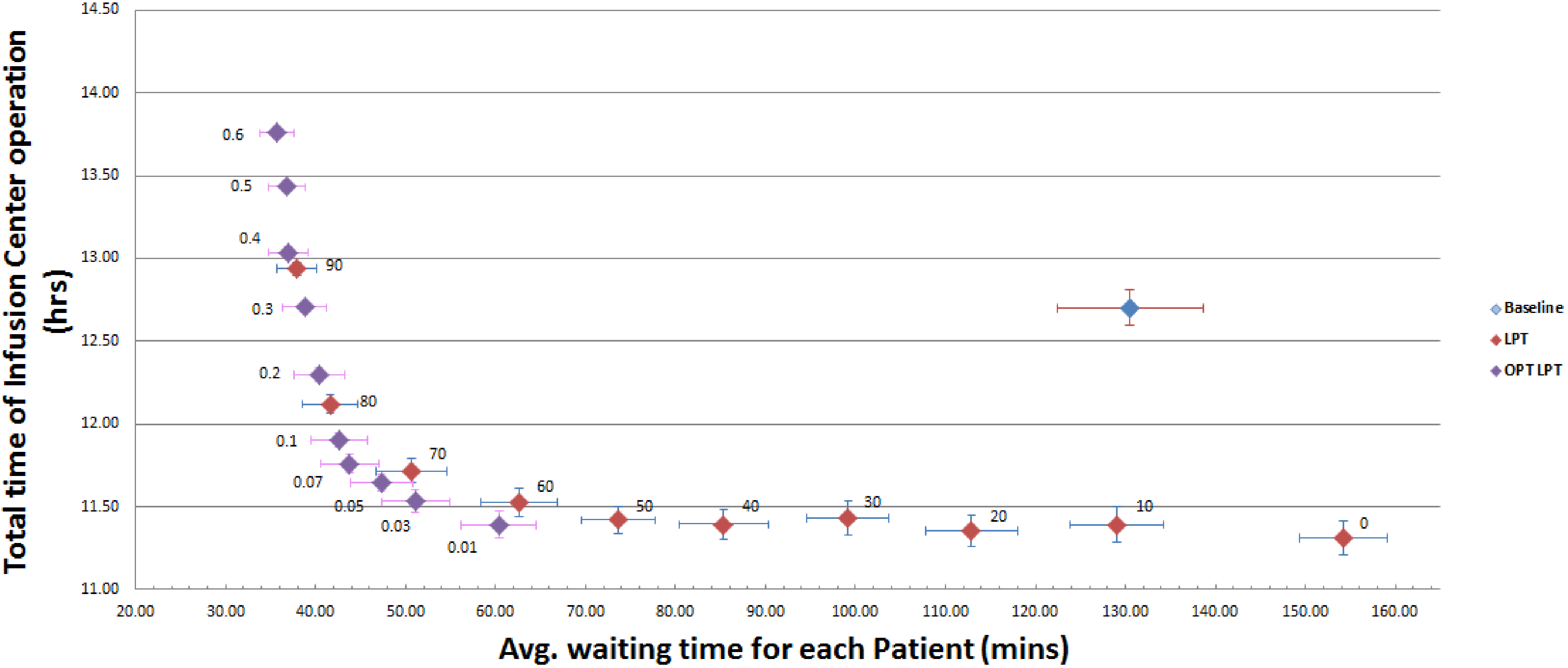
Results



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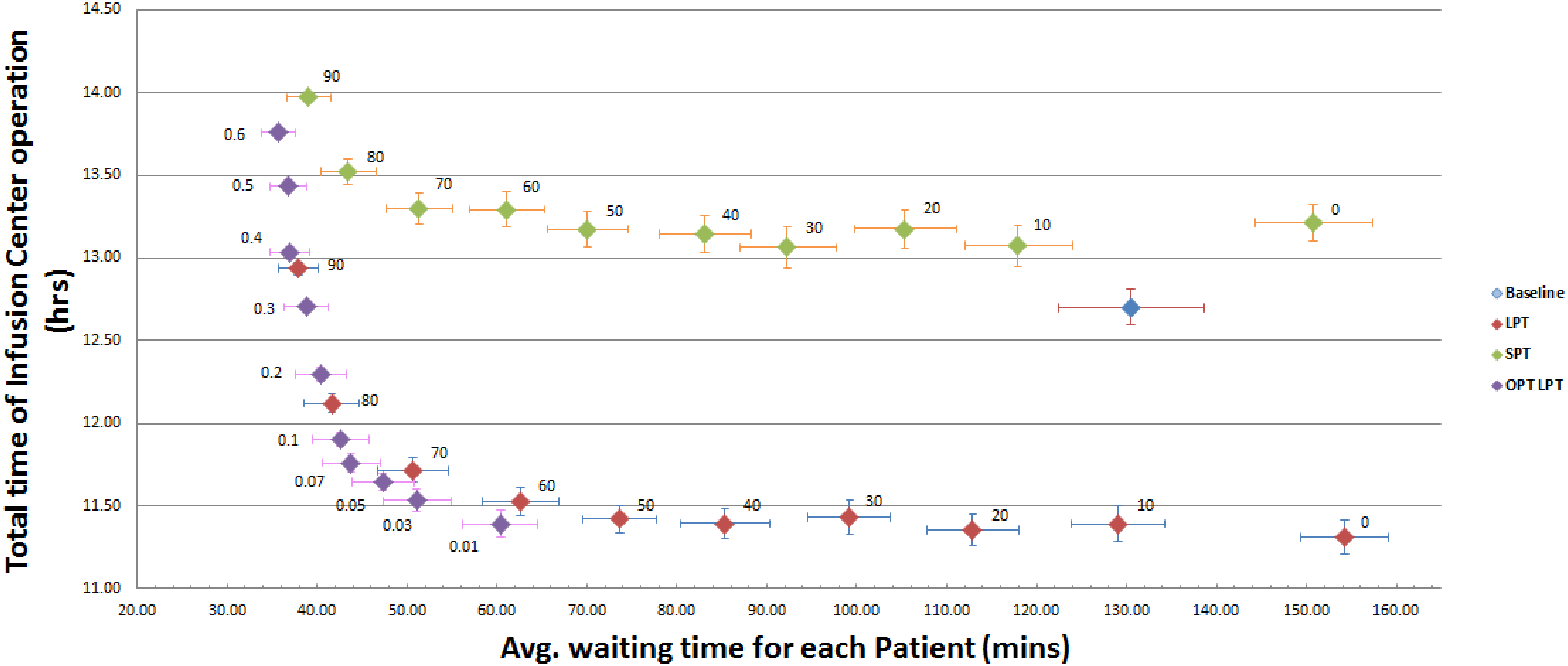
Results



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Conclusions

- Key aspects of the infusion center scheduling process can be approximated as a two stage stochastic integer program
- Fixing sequencing decisions based on LPT results in an easy to solve continuous stochastic program that performs well compared to current practice and other heuristics
- The LPT sequence is near optimal when compared to solvable instances of the MIP model for a small number of scenarios

Future Steps

- Improve stochastic MIP model
- Development of a heuristic that can be easily implemented by schedulers
- Enhancing simulation model
  - Addition of oncology clinic

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  - The TDC Foundation
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  - Center for Healthcare Engineering and Patient Safety (CHEPS)

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