Linear Programming Tools for Scheduling Trainees in Healthcare

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Presentation Outline

• Background
• Motivation
• Model Formulation
• Model Implementation
• Conclusions and Future Work
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Healthcare Training at Michigan

- 1,199 trainees
- 105 training programs
- 25 residencies
- 80 fellowships
Importance of Scheduling
Who does the Scheduling?

- Program dependent
  - Chief Resident
  - Faculty (Program Director)
  - Senior Administrative Staff
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Challenges in Scheduling

- Time-intensive process
- Numerous stakeholders
- Complex rules and legal requirements
- Conflicting goals
- Varying strategies and interdependencies
- “Good enough” mentality
Resident Education Requirements

- Each program has unique educational requirements (operative and disease exposure)

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General Plastic Surgery (dermatology fellowship level per year)
Service Coverage Requirements

- Each service requires a resident complement comprised of varying skillsets and disciplines
Traditional Scheduling Approach

1. Build rotation templates
2. Adjust for coverage and educational needs
3. Renegotiate after reaching a dead-end

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Design a linear program which automates creation of a block schedule that satisfies the needs of the residents and services.
Presentation Outline

- Background
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- **Model Formulation**
- Model Implementation
- Conclusions and Future Work
A technique to solve complicated story problems

Four basic parts
- Sets and parameters
- Decision variables
- Objective function
- Constraints

\[
\begin{align*}
\text{min} & \quad 2x_1 + x_2 \\
\text{subject to} & \quad x_1 + x_2 \geq 5 \\
& \quad 2x_1 + 3x_2 \leq 11 \\
& \quad x_1, x_2 \geq 0
\end{align*}
\]

Optimal Solution: (1, 4)
Objective Value = 6
Sets

$R$: residents

$C$: resident categories

$S$: services

$M$: months
Parameters

\( a_{rc} \in \{0, 1\} \): indicates if resident \( r \) fits category \( c \)

\( \mathcal{L}_{csm} \): lower bound on number of residents fitting category \( c \) in service \( s \) during month \( m \)

\( \mathcal{U}_{csm} \): upper bound on number of residents fitting category \( c \) in service \( s \) during month \( m \)

\( \lambda_{rs} \): lower bound on number of months resident \( r \) must spend on service \( s \)

\( \mu_{rs} \): upper bound on number of months resident \( r \) must spend on service \( s \)
Decision Variables

\[ x_{rsm} \in \{0, 1\}: \text{whether resident } r \text{ is assigned to service } s \text{ in month } m \]
\[ \forall r \in R, s \in S, m \in M \]

*The base model does not have an objective function.*
Constraints

1. Every resident gets assigned to one service every month
   \[ \sum_{s \in S} x_{rsm} = 1, \quad \forall r \in R, m \in M \]

2. Every resident satisfies their educational requirements
   \[ \lambda_{rs} \leq \sum_{m \in M} x_{rsm} \leq \mu_{rs}, \quad \forall r \in R, s \in S \]

3. Every service satisfies their service coverage needs
   \[ L_{csm} \leq \sum_{r \in R} a_{rc} x_{rsm} \leq U_{csm}, \quad \forall c \in C, s \in S, m \in M \]

\vdots
Constraints

1. Every resident gets assigned to one service every month

\[ x_{Smith, Maize, July} \]
Is Dr. Smith assigned to the Maize service in July?

If yes, \( x_{Smith, Maize, July} = 1 \). If no, \( x_{Smith, Maize, July} = 0 \).

\[ x_{Smith, Blue, July} \]
Is Dr. Smith assigned to the Blue service in July?

\[ x_{Smith, White, July} \]
Is Dr. Smith assigned to the White service in July?

\[ x_{Smith, Maize, July} + x_{Smith, Blue, July} + x_{Smith, White, July} = 1 \]
Constraints

1. Every resident gets assigned to one service every month

\[ x_{Smith, Maize, July} + x_{Smith, Blue, July} + x_{Smith, White, July} = 1 \]
\[ x_{Smith, Maize, Aug} + x_{Smith, Blue, Aug} + x_{Smith, White, Aug} = 1 \]
\[ \vdots \]
\[ x_{Smith, Maize, June} + x_{Smith, Blue, June} + x_{Smith, White, June} = 1 \]

\[ x_{Jones, Maize, July} + x_{Jones, Blue, July} + x_{Jones, White, July} = 1 \]
\[ \vdots \]
\[ x_{Jones, Maize, June} + x_{Jones, Blue, June} + x_{Jones, White, June} = 1 \]

\[ \sum_{s\in S} x_{rsm} = 1, \quad \forall \ r \in R, \ m \in M \]
2. Every resident satisfies their educational requirements

\[ x_{Smith, Maize, July} \]

Is Dr. Smith assigned to the Maize service in July?

If yes, \( x_{Smith, Maize, July} = 1 \). If no, \( x_{Smith, Maize, July} = 0 \).

\[ x_{Smith, Maize, Aug} \]

Is Dr. Smith assigned to the Maize service in August?

\[ x_{Smith, Maize, June} \]

Is Dr. Smith assigned to the Maize service in June?

\[ 1 \leq x_{Smith, Maize, July} + x_{Smith, Maize, Aug} + \ldots + x_{Smith, Maize, June} \leq 2 \]
Constraints

2. Every resident satisfies their educational requirements

\[ 1 \leq x_{Smith, Maize, July} + x_{Smith, Maize, Aug} + \ldots + x_{Smith, Maize, June} \leq 2 \]
\[ 1 \leq x_{Smith, Blue, July} + x_{Smith, Blue, Aug} + \ldots + x_{Smith, Blue, June} \leq 2 \]
\[ 1 \leq x_{Smith, White, July} + x_{Smith, White, Aug} + \ldots + x_{Smith, White, June} \leq 2 \]

\[ 1 \leq x_{Jones, Maize, July} + x_{Jones, Maize, Aug} + \ldots + x_{Jones, Maize, June} \leq 2 \]
\[ \vdots \]
\[ 1 \leq x_{Jones, Blue, July} + x_{Jones, Blue, Aug} + \ldots + x_{Jones, Blue, June} \leq 2 \]

\[ \lambda_{rs} \leq \sum_{m \in M} x_{rsm} \leq \mu_{rs}, \quad \forall \ r \in R, s \in S \]
3. Every service satisfies their service coverage needs

\[ x_{\text{Smith, Maize, July}} \]
Is Dr. Smith assigned to the Maize service in July?
If yes, \( x_{\text{Smith, Maize, July}} = 1 \). If no, \( x_{\text{Smith, Maize, July}} = 0 \).

\[ a_{\text{Smith, GS}} \]
Is Dr. Smith a General Surgery resident?
If yes, \( a_{\text{Smith, GS}} = 1 \). If no, \( a_{\text{Smith, GS}} = 0 \).

\[ a_{\text{Smith, PGY1}} \]
Is Dr. Smith a PGY1 resident?
If yes, \( a_{\text{Smith, PGY1}} = 1 \). If no, \( a_{\text{Smith, PGY1}} = 0 \).

\[ a_{\text{Smith, GS_PGY1}} \]
Is Dr. Smith a General Surgery PGY1 resident?
If yes, \( a_{\text{Smith, GS_PGY1}} = 1 \). If no, \( a_{\text{Smith, GS_PGY1}} = 0 \).
3. Every service satisfies their service coverage needs

\[
3 \leq a_{\text{Smith,GS}} \times \text{Smith, Maize, July} + a_{\text{Jones,GS}} \times \text{Jones, Maize, July} + a_{\text{Chan,GS}} \times \text{Chan, Maize, July} + \cdots + a_{\text{Gupta,GS}} \times \text{Gupta, Maize, July} \\
\leq 4
\]

\[
1 \leq a_{\text{Smith,PGY1}} \times \text{Smith, Maize, July} + a_{\text{Jones,PGY1}} \times \text{Jones, Maize, July} + a_{\text{Chan,PGY1}} \times \text{Chan, Maize, July} + \cdots + a_{\text{Gupta,PGY1}} \times \text{Gupta, Maize, July} \leq 2
\]

\[
\mathcal{L}_{\text{csm}} \leq \sum_{r \in R} a_{rc} x_{rsm} \leq \mathcal{U}_{\text{csm}}, \quad \forall \ c \in C, \ s \in S, \ m \in M
\]
Expanded Model

- Distributed Educational Requirements
- Distributed Coverage Needs
- Extended Rotations
- Service Sequencing
- Service Spacing
- Resident Pairing
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Implementation Process

Sets
- R: residents
- C: resident categories
- S: services
- M: months

Parameters
- $\alpha_r \in (0, 1)$: whether resident $r$ fits category $c$
- $\ell_{c,m}, \bar{u}_{c,m}$: lower, upper bounds on staffing of residents fitting category $c$ in service during month $m$
- $\lambda_{r,s}, \mu_{r,s}$: lower, upper bounds on months resident $r$ must spend on service $s$

Decision Variables
- $x_{r, s, m} \in \{0,1\}$: whether resident $r$ is assigned to service $s$ in month $m$

Objective Function
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Recap

• Scheduling issues affect hospital workflow, training quality, and patient safety
• Scheduling residency programs at UMHS is highly interdependent, complex, and poorly executed
• We can address these scheduling needs using a linear programming formulation
Future Work

• Define metrics for schedule optimality
  – Minimize deviation from desired resident complement by service
  – Maximize satisfied requests for educational customization

• Apply model to improve scheduling for other training programs
Related Applications

- Pediatric Medicine rotation schedule
- C.S. Mott Emergency Department shift schedule
- Chemotherapy infusion patient schedule
- Physician clinic/OR schedule
- Master surgical schedule problem
- Nurse staff scheduling
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Questions [ ? ] and Comments [ ! ]

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