This document formulates the basic notation and constraints for the Shift Scheduling Game.

1 Inputs: Sets and other Parameters

1.1 Sets

- $R$: set of residents
- $I \subseteq R$: set of interns
- $S$: set of shifts \{1,2,\ldots,7\}
- $N \subseteq S$: set of night shifts
- $D$: set of days
- $D_r \subseteq D \ \forall r \in R$: set of days that intern $r$ cannot work
- $L_{sd} \subseteq (S \times D)$: the set of shifts incompatible with $(s,d)$

1.2 Parameters

- $u_t = 16$: upper bound for number of total shifts
- $l_t = 12$: lower bound for number of total shifts
- $u_n = 5$: upper bound for number of night shifts
- $l_n = 2$: lower bound for number of night shifts

2 Decision Variables

$$x_{rsd} \ \ \text{do we assign resident } r \ \text{to shift } s \ \text{on day } d?$$

3 Objective Function

This simplified version of the shift scheduling problem does not have an objective function as the goal is simply to create a feasible schedule rather than one that is optimal based on certain metrics.

4 Variable Restrictions

$$x_{rsd} \in \{0,1\} \ \ \forall r \in R, s \in S, d \in D$$

5 Constraints

1. Every shift needs a resident
   For every shift $s$ on every day $d$, exactly one resident $r$ must be assigned to work.
   \[ \sum_{r \in R} x_{rsd} = 1, \ \ \forall s \in S, d \in D \]  \hspace{1cm} (1)

2. Every resident needs between 12 and 16 shifts
   For every resident $r$, we must assign between 12 and 16 shifts across all shifts $s$ over all days $d$. 

\[
\sum_{s \in S, d \in D} x_{rsd} \geq lt \quad \forall r \in R
\] (2)

\[
\sum_{s \in S, d \in D} x_{rsd} \leq ut \quad \forall r \in R
\] (3)

3. **Every resident needs between 2 and 5 night shifts**
   For every resident \( r \), we must assign between 2 and 5 shifts across all **night** shifts \( s \) in \( N \) over all days \( d \).

\[
\sum_{s \in N, d \in D} x_{rsd} \geq ln, \quad \forall r \in R
\] (4)

\[
\sum_{s \in N, d \in D} x_{rsd} \leq un, \quad \forall r \in R
\] (5)

4. **Every resident needs adequate rest between shifts**
   For every resident \( r \), day \( d \), and shift \( s \) the sum of \( x_{rsd} \) and its set of incompatible shifts in \((s', d')\) must be equal to at most 1.

\[
x_{rsd} + \sum_{(s', d') \in L_{sd}} x_{rs'd'} \leq 1 \quad \forall r \in R, s \in S, d \in D
\] (6)

5. **Interns cannot work the first or last shift of the day**
   Interns \( r \) in \( I \) cannot work the first (shift 1) or the last shift (shift 7) \( s \) of the day \( d \).

\[
\sum_{s \in \{1\}} \sum_{d \in D} x_{rsd} = 0 \quad \forall r \in I
\] (7)

\[
\sum_{s \in \{7\}} \sum_{d \in D} x_{rsd} = 0 \quad \forall r \in I
\] (8)

6. **Each resident has a specific day of week he or she cannot work**
   Resident \( r \) has a specific day of week that he or she cannot work. \( D_r \) is the set of corresponding days.

\[
x_{rsd} = 0 \quad \forall r \in R, s \in S, d \in D_r
\] (9)