Pareto Optimality in Pediatric Residency Shift Scheduling

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Collaborators

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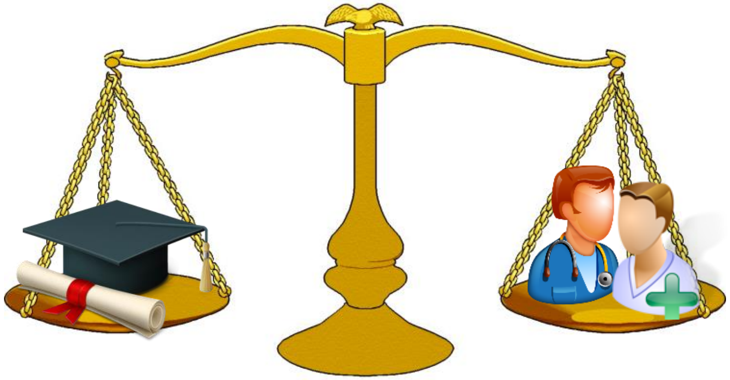
Content

- Background
- Motivation
- Formulations
  - Weighted sum method
  - Metric constraints method
- Result
- Ongoing Research
  - Pareto method

Resident Responsibilities in the U-M Pediatric Emergency Department

- 3-7 year medical training program
  - Responsibilities differ by residency year
- Balancing patient care and educational requirements
  - In hospital
    - Caring for patients
    - Teaching medical students
    - Learning from attending physicians
  - Out of hospital
    - Community clinics
    - Conferences
    - Other educational requirements

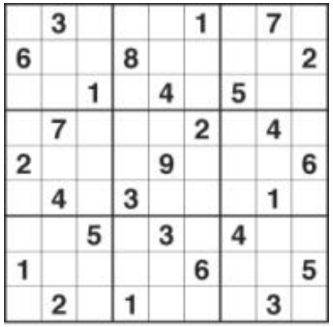


Pediatric ED: Scheduling Considerations

- All shifts assigned to a resident
- Appropriate coverage
  - e.g. certain shifts require a senior resident
- ACGME rules (similar to ABET for engineering)
  - e.g. 10 hour break rule
- Several different residency programs
  - Pediatrics (PED)
  - Family practice (FP)
  - Emergency medicine (EM)
- And others

Motivation

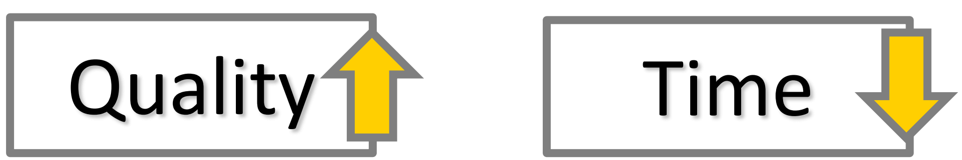
- **Scheduling residents**
  - **Complicated requirements**
    - **25 governing rules and preferences**
      - **Educational goals**
      - **Patient care**
      - **Regulations / Safety**
  - **Chief resident built monthly schedule by hand**
    - **Time consuming process: 20 - 25 hours / month**
    - **Transfer every year: no scheduling experience in July**
    - **Guess and check: errors / tedious correction process**



Motivation

- **Practical Significance**
  - Poor-quality schedule
    - Residents: decreased interest in learning
    - Patients: adverse health events

    (Smith-Coggins R, et. al. (1994) : "Relationship of day versus night sleep to physician performance and mood." Annals of Emergency Medicine)
- **Goals**
  - Solve for feasible schedule quickly
  - Create a good quality schedule with no violations



Objectives: Shift Fairness

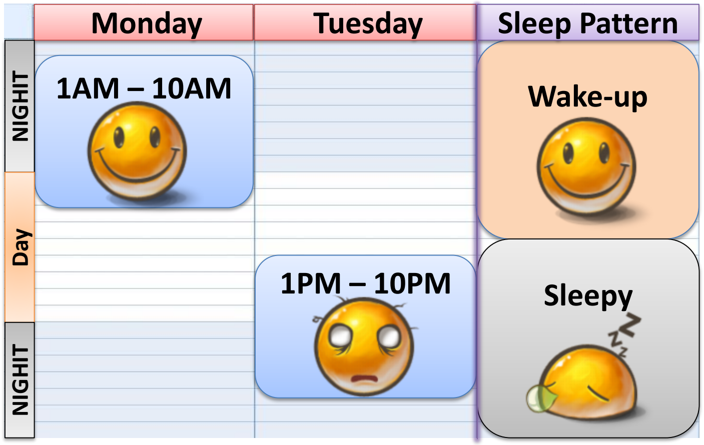
- **Total / night shift equity**
  - **Equal opportunities for training**
  - **Improved morale and learning ability**

Resident Name	Smith	Jones	Chen	Joe
Night Shifts / Total Shifts	0 / 7	1 / 7	1 / 7	5 / 7
Fairness				

  - **Total shift equity (TSE):** $(\sum t_{ij}\ , t_{ij} = |D_i - D_j|,\ i > j)$
  - **Night shift equity (NSE):** $(\sum n_{ij}\ ,\ n_{ij} = |N_i - N_j|,\ i > j)$

Objectives: Undesired Shift

- **Bad sleep patterns and post-clinic shifts**
  - **Reduces resident quality of life**
  - **Decreases patient safety**



Bad sleep pattern

Post-Clinic shift

  - **Minimum bad sleep patterns (BSP):** $(\sum count)$

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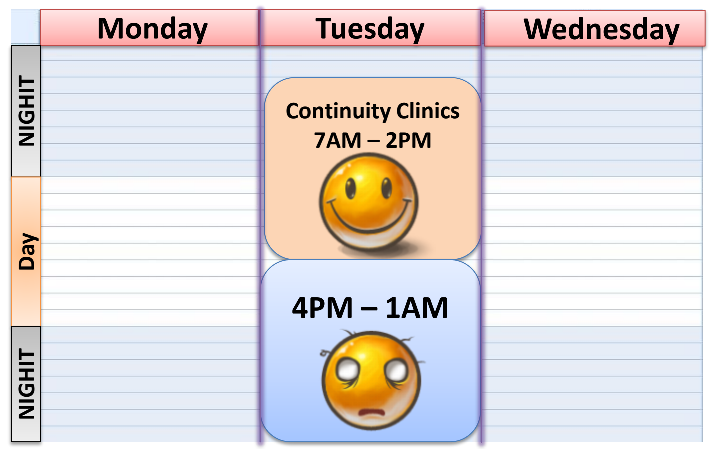
Bad sleep pattern



Post-Clinic shift

  - **Minimum post-clinic shifts (PCC):** $(\sum count)$

Formulation: Decision Variables

- **Sets**
  - R: set of residents
  - D: set of days in the schedule
  - S: set of shifts

Residents Name				
Smith	Sanchez	Chen	Shah	…

	27th	…	1st	…	31st
7a-4p	Shah	…		…	
9a-6p	Joe	…		…	Shah
12p-9p	Chen	…		…	Chen
4p-1a	Smith	…	Sanchez	…	
5p-2a		…		…	Sanchez
8p-5a	Sanchez	…	Smith	…	Smith
11p-8a		…	Chen	…	Joe

- **Decision Variables**
  - Binary: $x_{rds} \in \{0, 1\}$
    - 1 if resident r works shift s on day d
    - 0 otherwise

Formulation: Constraints

- Constraints (rules/requirements)
  - One resident assigned to each shift in the month
    - $\sum_{r \in \{\text{all}\}} x_{rds} = 1, \quad \forall d, \forall s$
  - Meets shift requests
    - $x_{rds} = 0, \quad \forall r, \forall d, s \in \{\text{day off, conferences, continuity clinic}\}$
  - Ensure resident type appropriate for shift
    - $\sum_{r \in \{\text{PED}\}} \sum_{s \in P} x_{rsd} \geq 1, \forall d, P = \{\{7a,9a\}, \{4p,5p\}, \{8p,11p\}\}$
  - Intern-forbidden shifts
    - $\sum_{r \in \{\text{interns}\}} \sum_{d} x_{rsd} = 0, \forall s \in \{7a, 11p\}$
  - And others

Multi-Criteria Problem

- Multi-Criteria schedule
  - Metrics for UM Pediatric Emergency Department
    - Total shift equity (TSE)
    - Night shift equity (NSE)
    - Minimum bad sleep patterns (BSP)
    - Minimum post-clinic shifts (PCC)
    - ⋮

Weights?
Preferences?
Trade-off?

Multi-objective Mathematical Programming

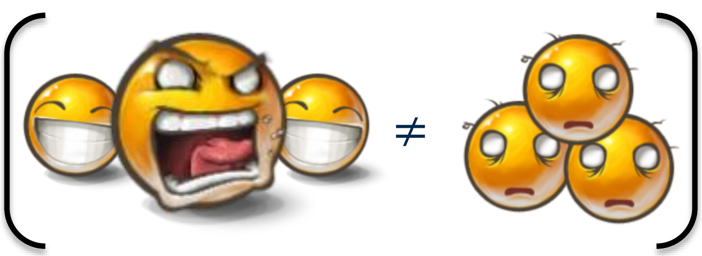
Weighted Sum Method

$$\textbf{Min } \color{red}{\boldsymbol{w_1}}(TSE) + \color{red}{\boldsymbol{w_2}}(NSE) + \color{red}{\boldsymbol{w_3}}(BSP) + \color{red}{\boldsymbol{w_4}}(PCC)$$

$$\textbf{s.t.} \qquad \text{"rules/requirements"}$$

$$x_{rds} \in \{0,1\}$$

- **Quantifying preferences (w_i) is difficult**
  - **Weights are subjective and difficult to quantify**
    - **Resulting schedule does not match their intentions**
  - **Various measurement units**
    - **Equity (σ , $\mathrm{Max}|\mathrm{diff}_{ij}|$, $\sum|\mathrm{diff}_{ij}|$, …)**
  - **Non-linearity**
    - **10 BSPs ≠ 10 x 1 BSP**



Metrics Formulation

- **Feasibility problem**
  - Constraint on metrics

$$\begin{aligned} &\textbf{Min } \boldsymbol{w_1}(TSE) + \boldsymbol{w_2}(NSE) + \boldsymbol{w_3}(BSP) + \boldsymbol{w_4}(PCC) \\ &\textbf{s.t.} \qquad \text{"rules/requirements"} \\ &\qquad\qquad x_{rds} \in \{0,1\} \end{aligned}$$

- **Benefits of a feasibility problem**
  - More flexible
  - Faster to solve: < 2 sec.
    - Given: 35 days / 20 PEDs / 7 shifts

Metrics Formulation

- **Feasibility problem**
  - Constraint on metrics

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&\qquad x_{rds} \in \{0,1\} \\
&\qquad \boldsymbol{lb_{TSE}} \le (\boldsymbol{TSE}) \le \boldsymbol{ub_{TSE}} \\
&\qquad \boldsymbol{lb_{NSE}} \le (\boldsymbol{NSE}) \le \boldsymbol{ub_{NSE}} \\
&\qquad \boldsymbol{lb_{BSP}} \le (\boldsymbol{BSP}) \le \boldsymbol{ub_{BSP}} \\
&\qquad \boldsymbol{lb_{PCC}} \le (\boldsymbol{PCC}) \le \boldsymbol{ub_{PCC}}
\end{aligned}
$$

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Interactive Improvement

- Example output of metrics
  - Value (Lower bound, Upper bound)

Resident Name	Number of Shifts	Number of Night Shifts	Number of Post CC	Number of Bad Sleep Templates
Smith	8 (7,9)	2 (0,10)	0 (0,1)	1 (0,1)
Sanchez	8 (7,10)	1 (0,10)	0 (0,1)	1 (0,1)
Chen	8 (7,9)	5 (0,10)	1 (0,1)	1 (0,1)
Shah	14 (13,15)	3 (0,10)	1 (0,1)	1 (0,1)
⋮	⋮	⋮	⋮	⋮

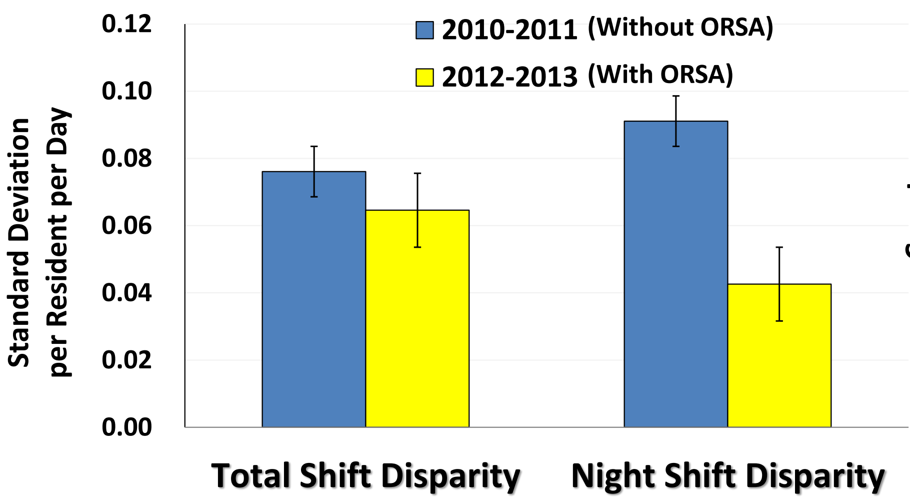
- Interactive approach engaging chief resident
  - Iteratively adjust bounds on metric constraints
  - Quickly build high quality schedule

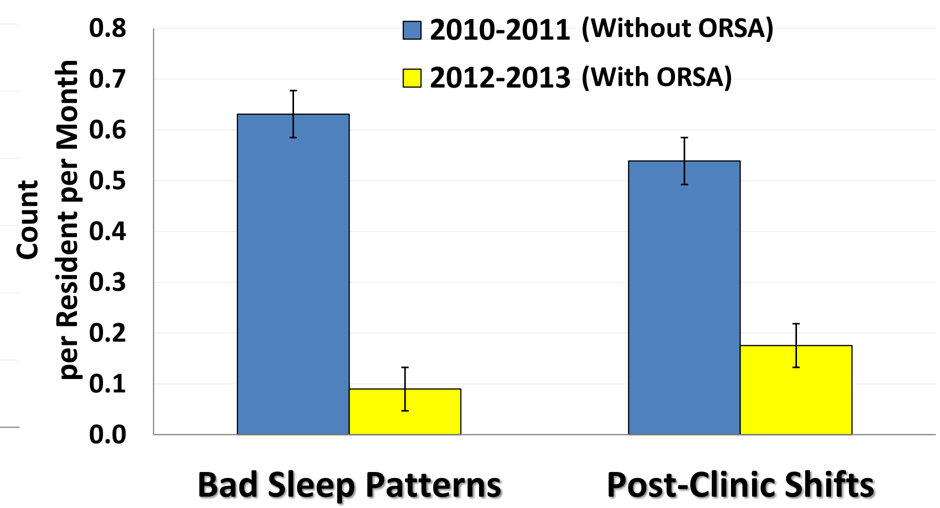
Results

- **Our metrics-based scheduling tool:**
  - **Reduces time to create schedules**

20 hours /month → 1 hour /month

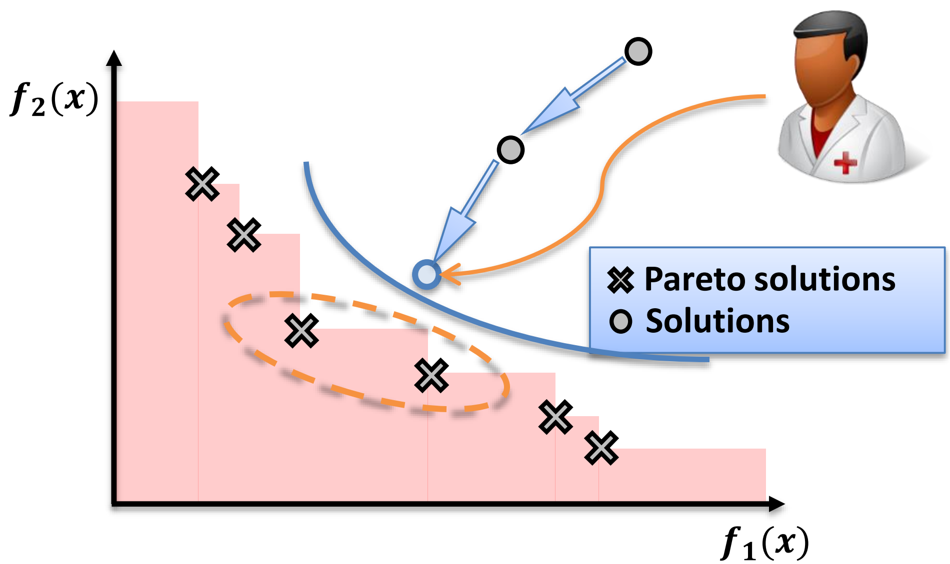
  - **Solves a multi-criteria scheduling problem**





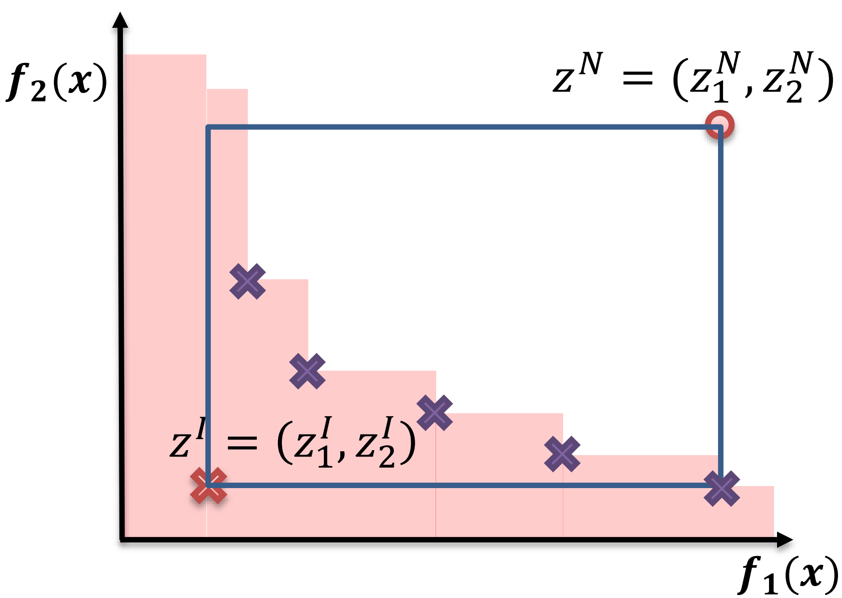
Limitations

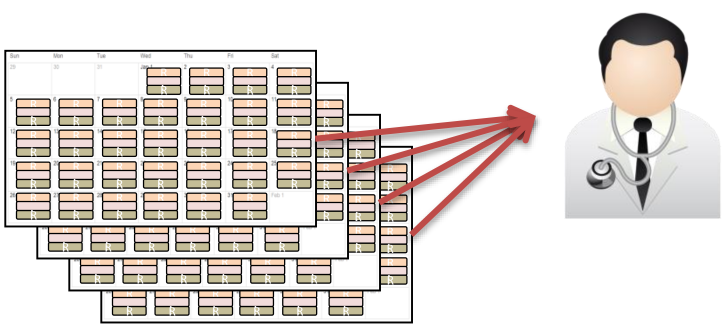
- Myopic Solution
  - Non-Pareto solution could be selected by chief residents
    - Never see the whole picture (the set of Pareto solutions)
    - The most preferred solution is "most preferred" with respect to their satisfaction



Next Step

- **Pareto Front**
  - Generate the Pareto solutions of the problem (all of them or a sufficient representation)
    - Select the most preferred one among them





Efficient Schedules

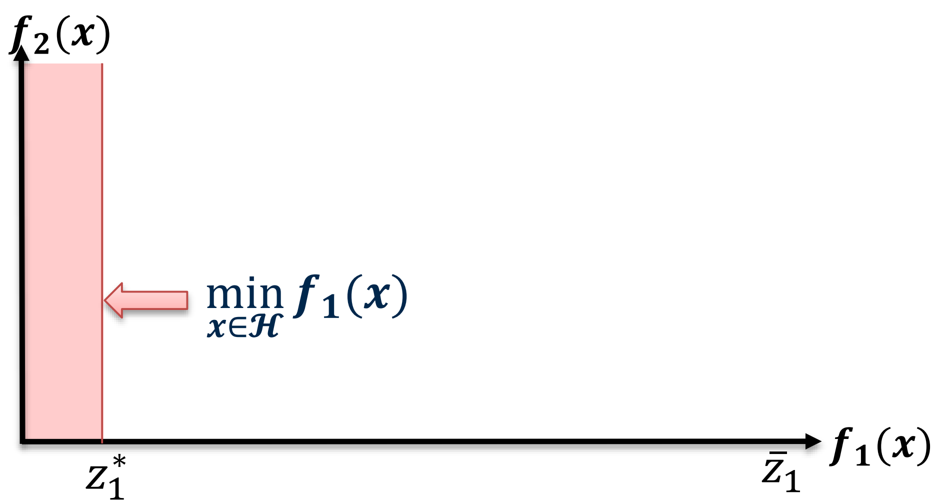
Pareto: Bi-Objective Problem

- **Notation**
  - $\mathcal{H}$: Solution Space, the set of feasible solutions
  - $\mathcal{P}$: Pareto Front, the set of solutions in objective space
  - $z_i = f_i(x)$: ith integer objective function, $\in \mathbb{Z}$
  - Dominance ($\prec$) : $x \prec x'$ if and only if $z_i \leq z_i'$ where at least one inequality is strict
- **Bi-Objective Problem**

$$\min f(x) = (f_1(x), f_2(x))$$
$$s.t.\ x \in \mathcal{H}$$

Pareto: Bi-Objective Problem

- **Pareto Square Region**
  - **Ideal Point:**
    - $\mathbf{z}_1^* = \min_{x \in \mathcal{H}} f_1(x)$ and $\mathbf{z}_2^* = \min_{x \in \mathcal{H}} f_2(x)$
  - **Nadir Point:**
    - $\bar{z}_1 = \min_{x \in \mathcal{H} \cap f_2(x) = \mathbf{z}_2^*} f_1(x)$ and $\bar{z}_2 = \min_{x \in \mathcal{H} \cap f_1(x) = \mathbf{z}_1^*} f_2(x)$



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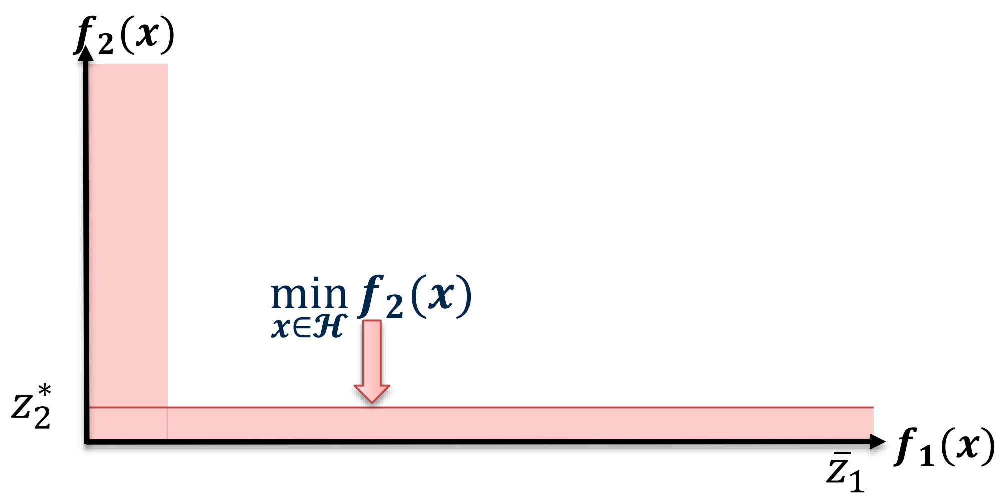

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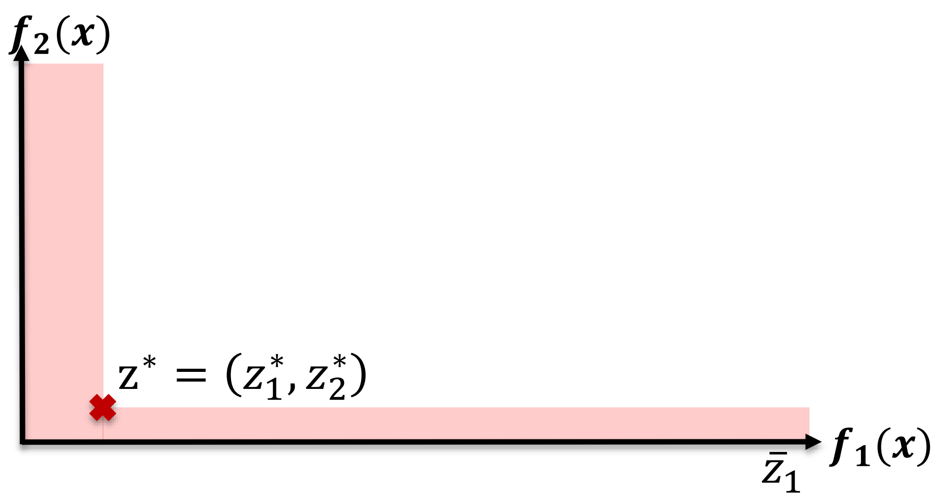

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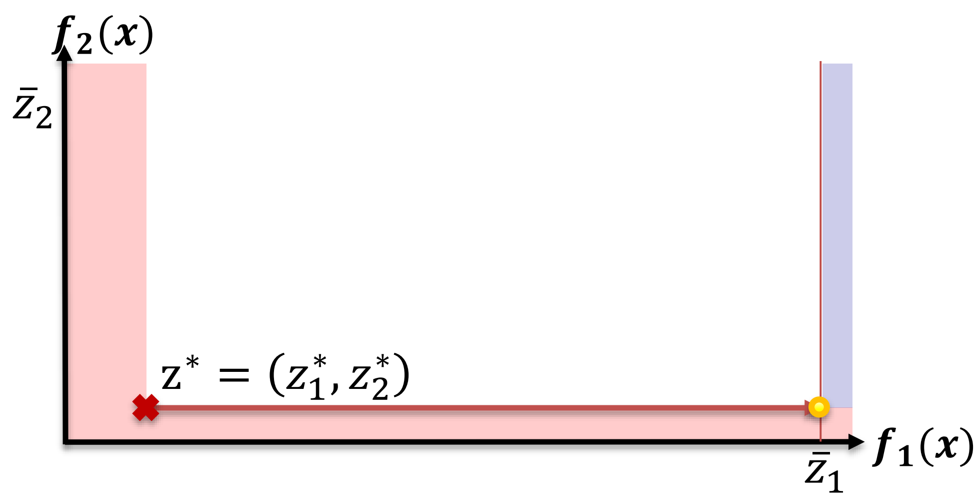

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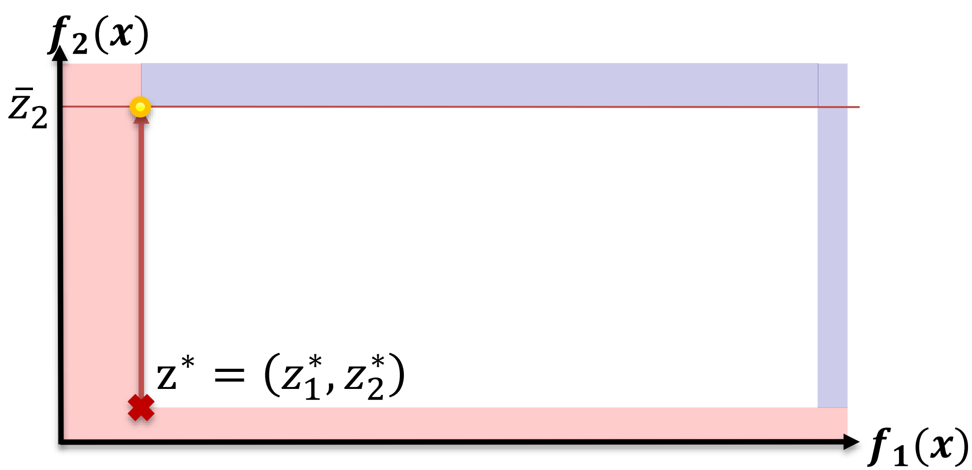

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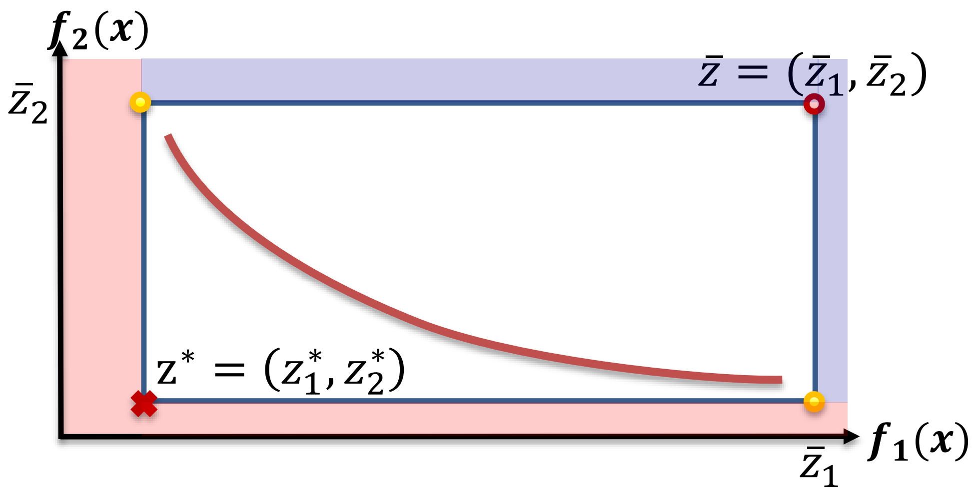


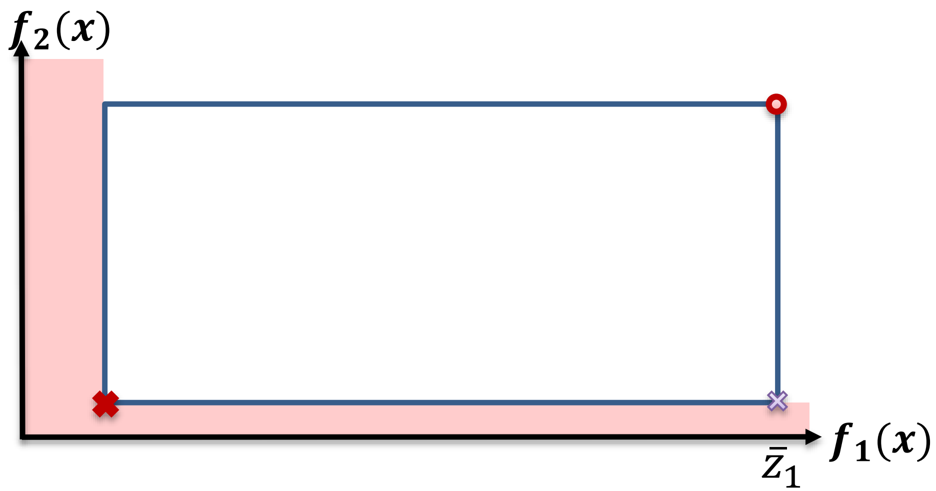
Pareto: Bi-Objective Problem

- ## Pareto Squeezing Algorithm

Algorithm 1 Exact squeezing algorithm for bi-objective problems

Let P is set of pareto solutions we've found;

Compute the ideal (z_1^*, z_2^*) and Nadir $(\bar{z}_1, \bar{z}_2)$ points;
Set $P := \{(\bar{z}_1, z_2^*)\}$ and $\delta := \bar{z}_1 - 1$;
WHILE $\delta \geq z_1^*$
  Solve $P_2(\delta)$ and get optimal solution (z_1^i, z_2^i) to $P_2(\delta)$;
  //Given (z_1^i, z_2^i), Find a left-botton corner $(\hat{z}_1^i)$ in the Pareto set;
  Solve $SQZ_1(z_2^i)$ and get optimal solution $(\hat{z}_1^i, z_2^i)$ to $SQZ_1(z_2^i)$;
END WHILE
Set $P := P + (\hat{z}_1^i, z_2^i)$ and $\delta = \hat{z}_1^i - 1$;
GO Step 2 UNTIL $z_1 = z_1^*$;



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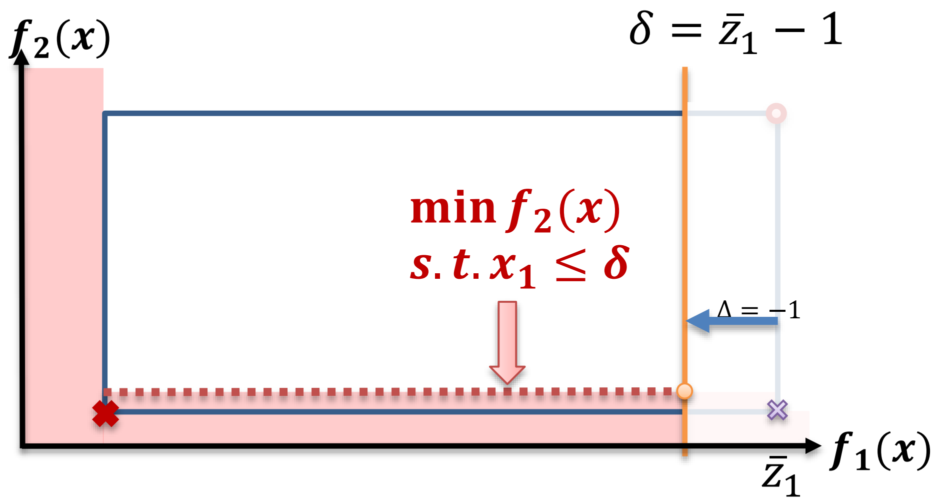

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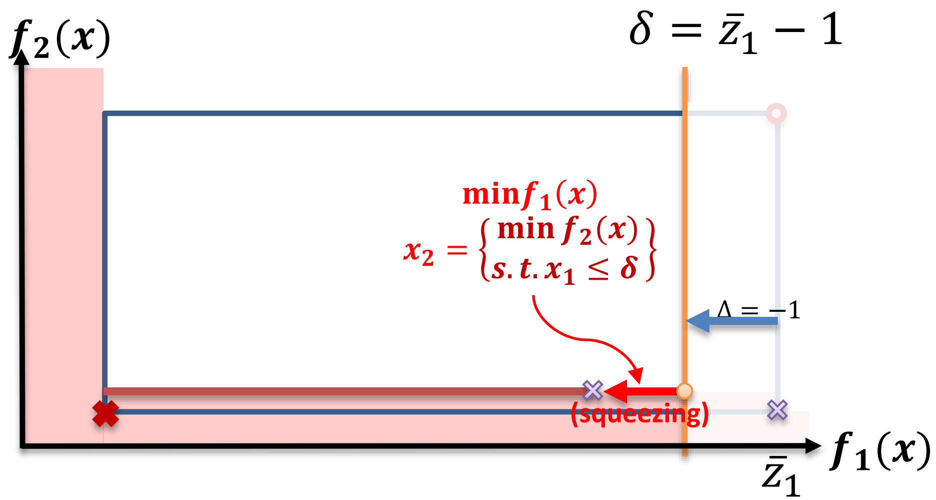
```



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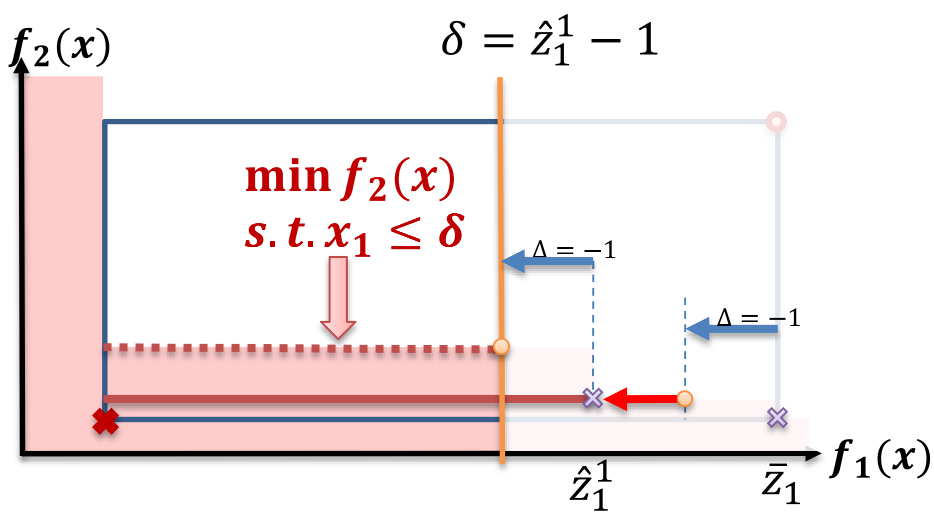

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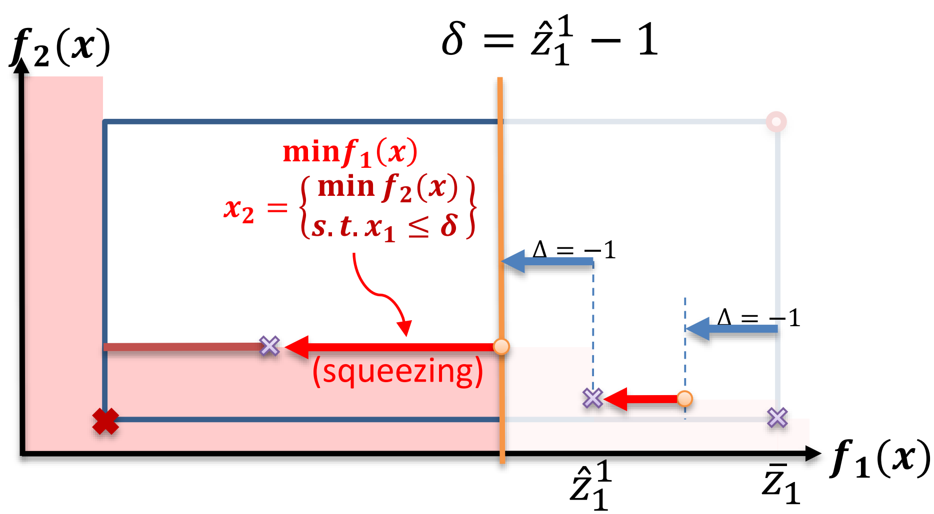

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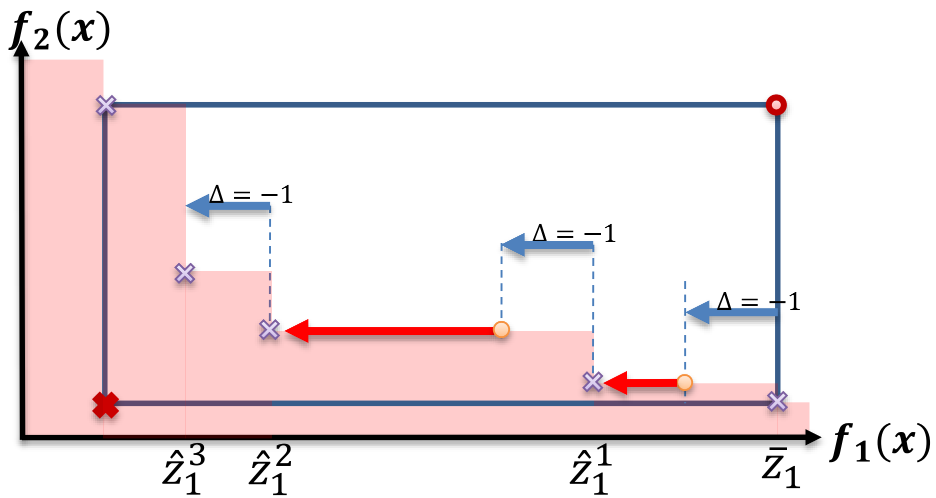


Ongoing Research

- **Tri-Objective Problem**

$$\min \boldsymbol{f}(\boldsymbol{x}) = \left(\boldsymbol{f}_1(\boldsymbol{x}), \boldsymbol{f}_2(\boldsymbol{x}), \boldsymbol{f}_3(\boldsymbol{x})\right)$$
$$\boldsymbol{s.t.}\ \boldsymbol{x} \in \mathcal{H}$$

- **$\boldsymbol{n}$-Objective Problem**

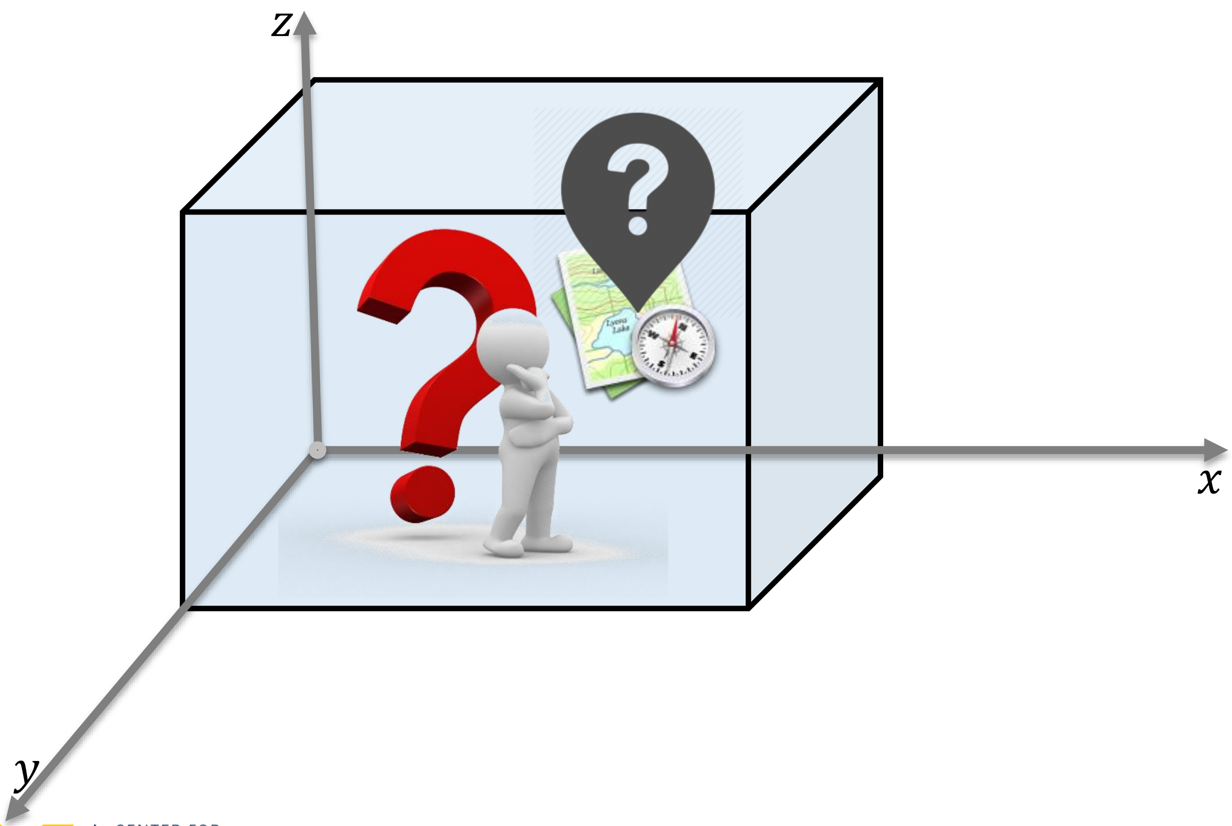
$$\min \boldsymbol{f}(\boldsymbol{x}) = \left(\boldsymbol{f}_1(\boldsymbol{x}), \boldsymbol{f}_2(\boldsymbol{x}), \dots, \boldsymbol{f}_n(\boldsymbol{x})\right)$$
$$\boldsymbol{s.t.}\ \boldsymbol{x} \in \mathcal{H}$$

- **“Three is more than two plus one.”**

(Gandibleux, Xavier, ed. Multiple criteria optimization: state of the art annotated bibliographic surveys. Vol. 52. Springer Science & Business Media, 2002)

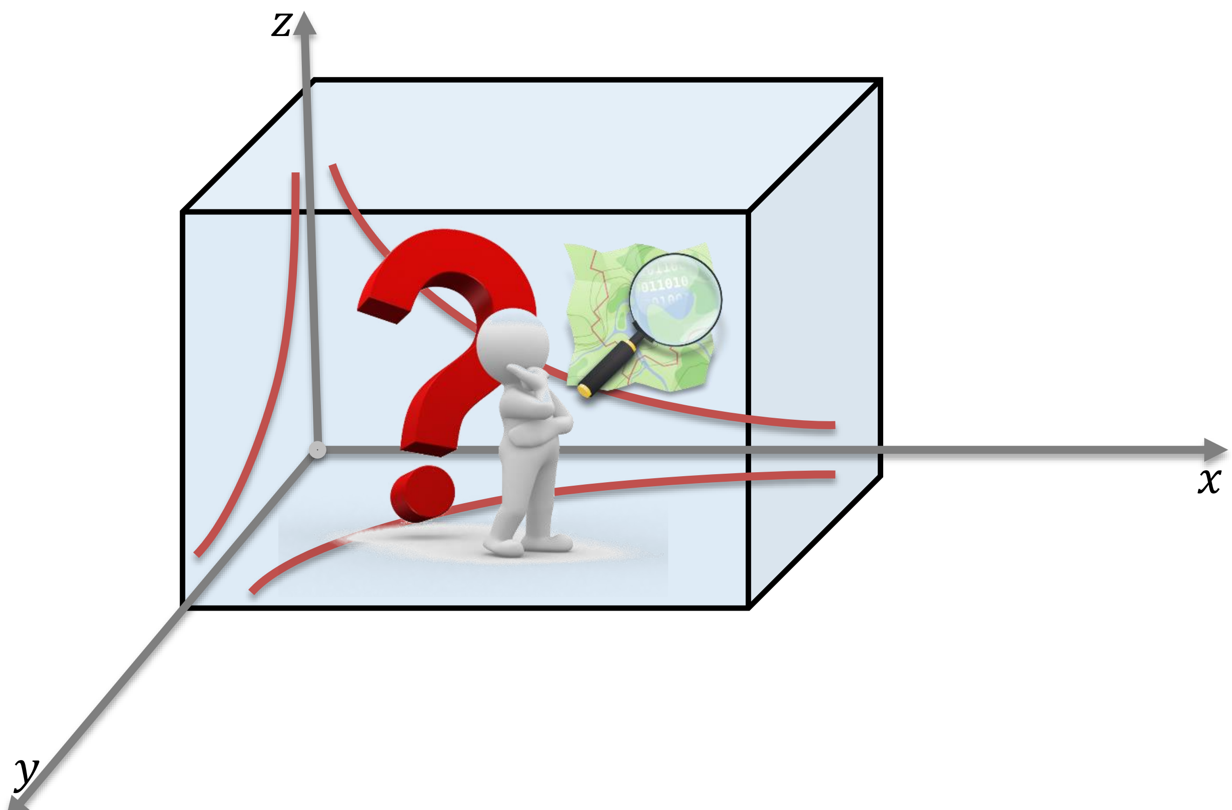
Pareto: Tri-Objective Problem

- **Tri-Objective Problem**
  - **Where is Pareto Front?**



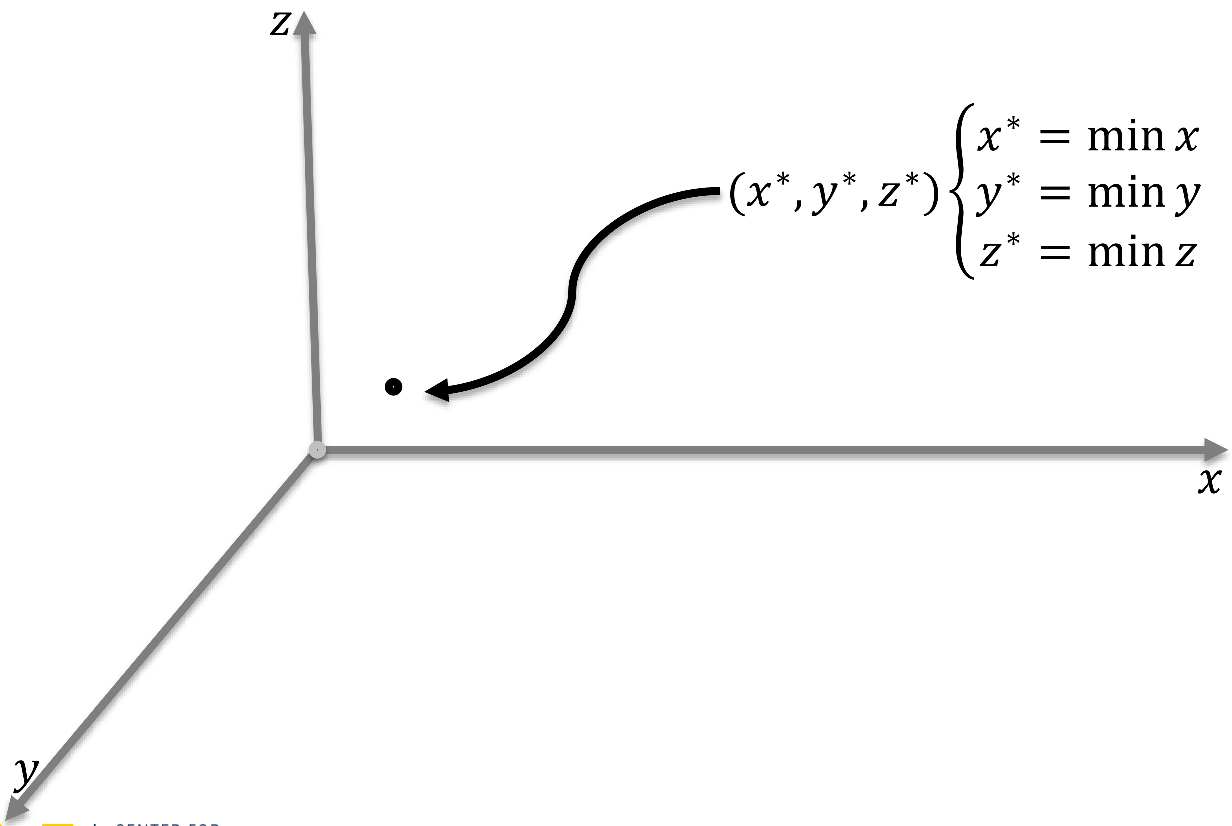
Pareto: Tri-Objective Problem

- **Tri-Objective Problem**
  - **How to find Pareto Front?**



Pareto: Tri-Objective Problem

- **Pareto Surface**
  - **Ideal Point and Nadir Points**



Pareto: Tri-Objective Problem

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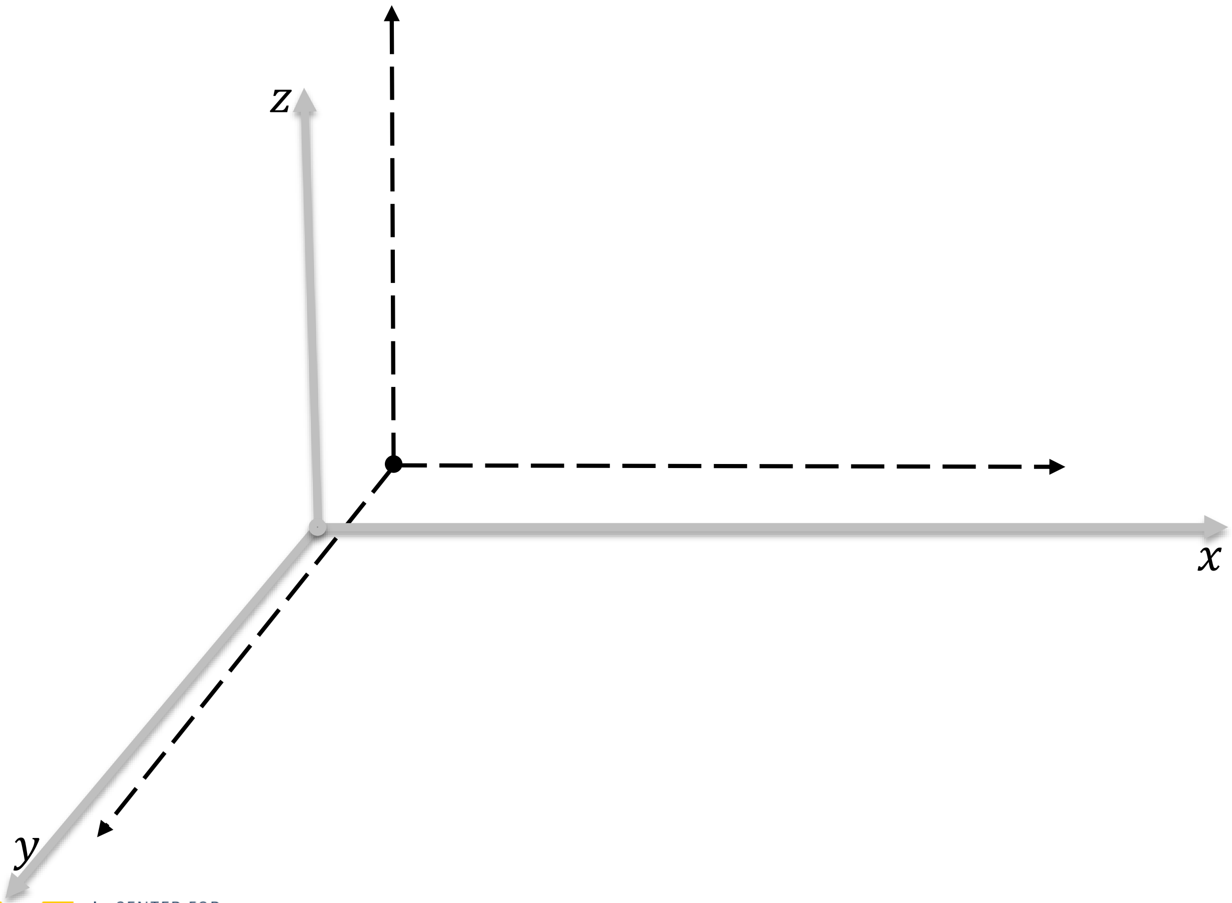

Pareto: Tri-Objective Problem

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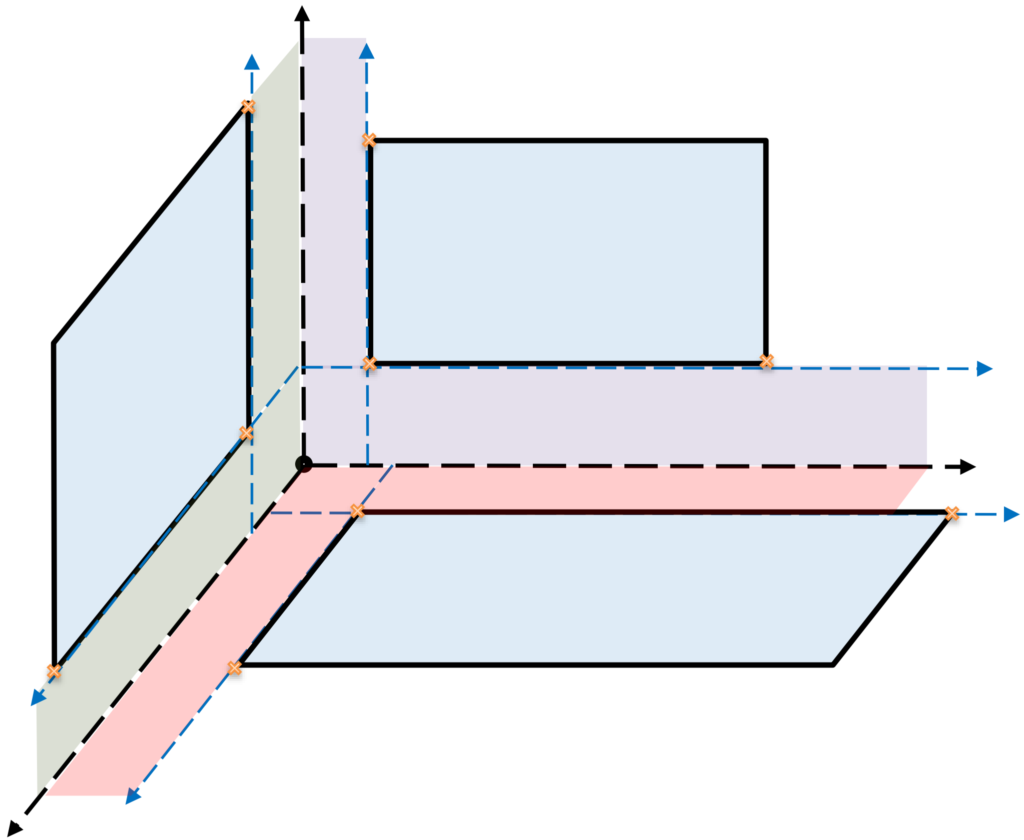

Pareto: Tri-Objective Problem

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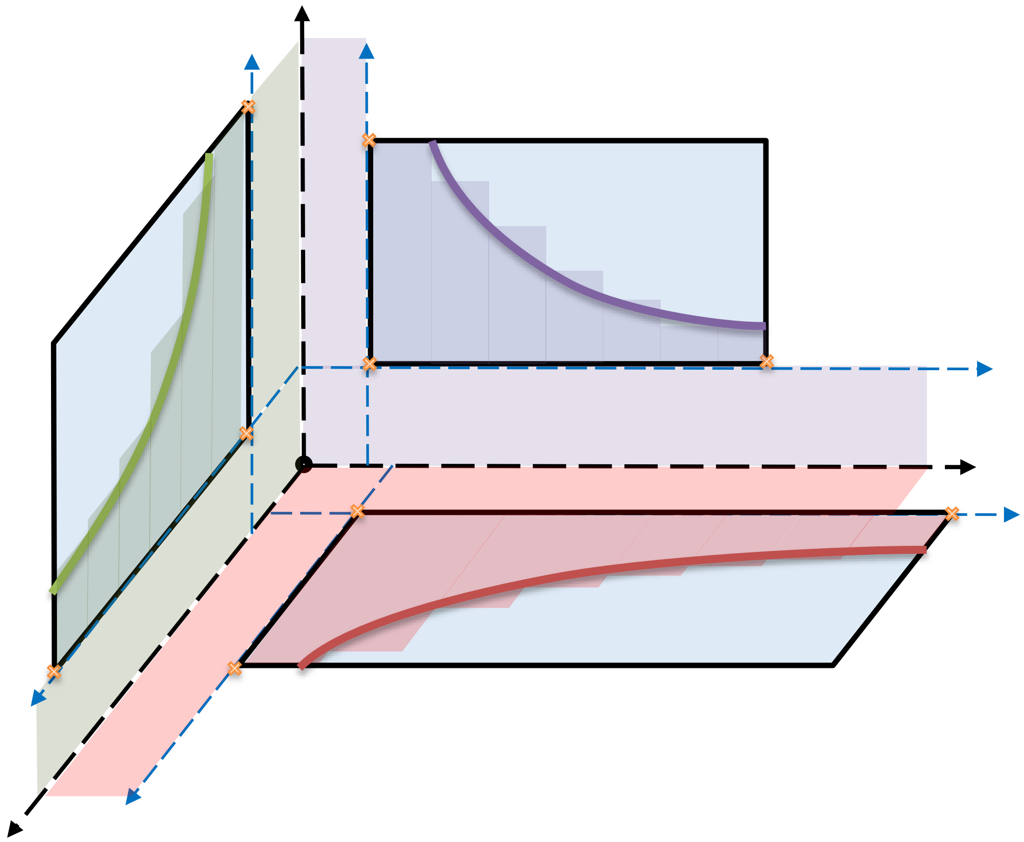
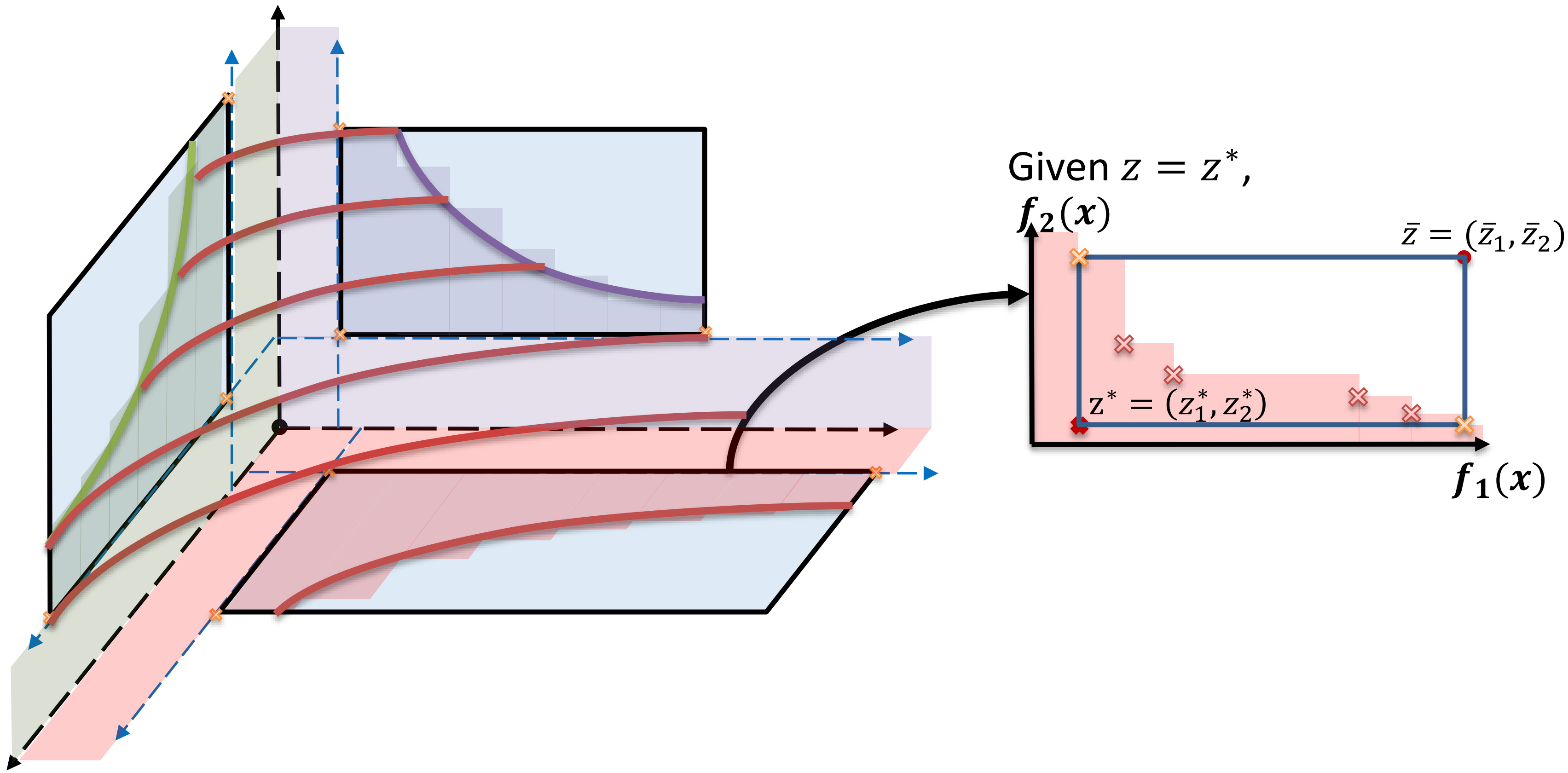


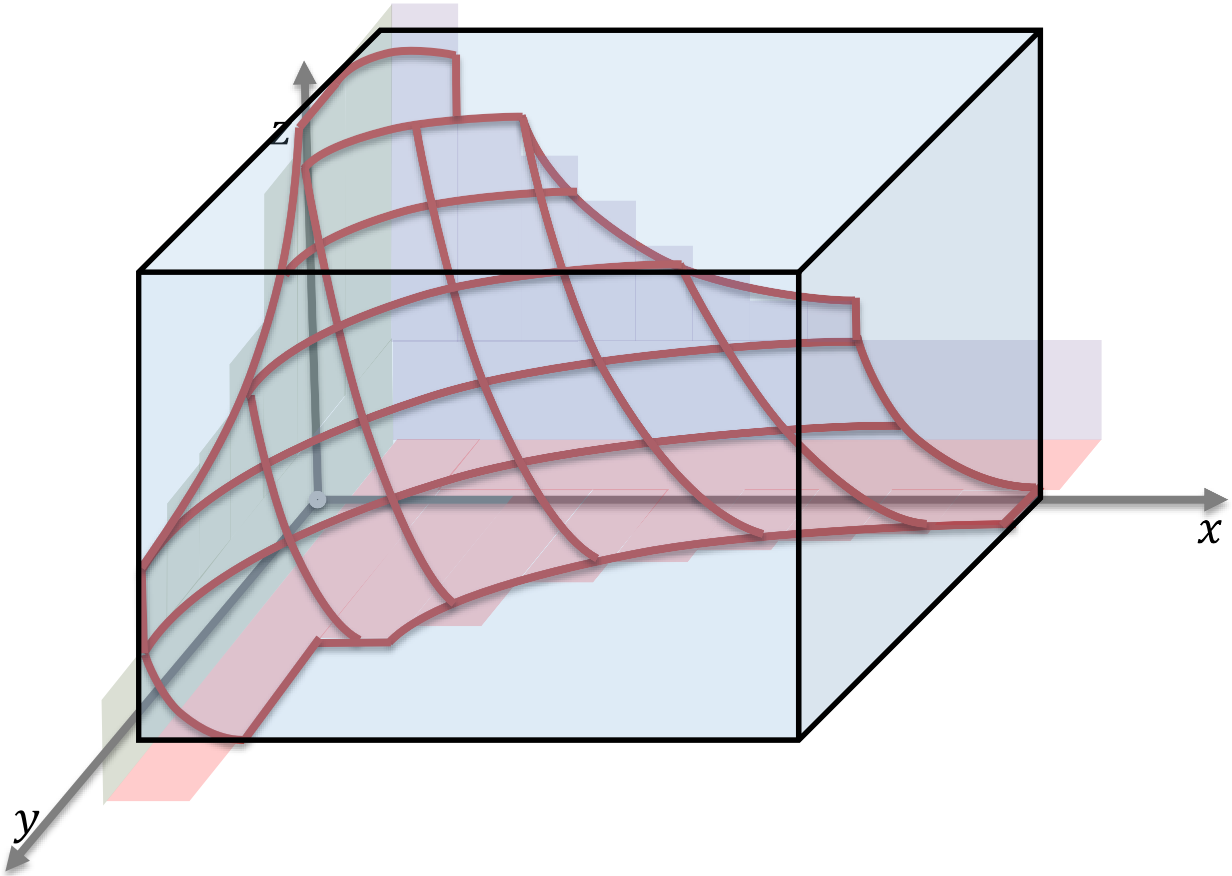
Pareto: Tri-Objective Problem

- **Pareto Surface**
  - **Given fixed z level, apply bi-objective approach**



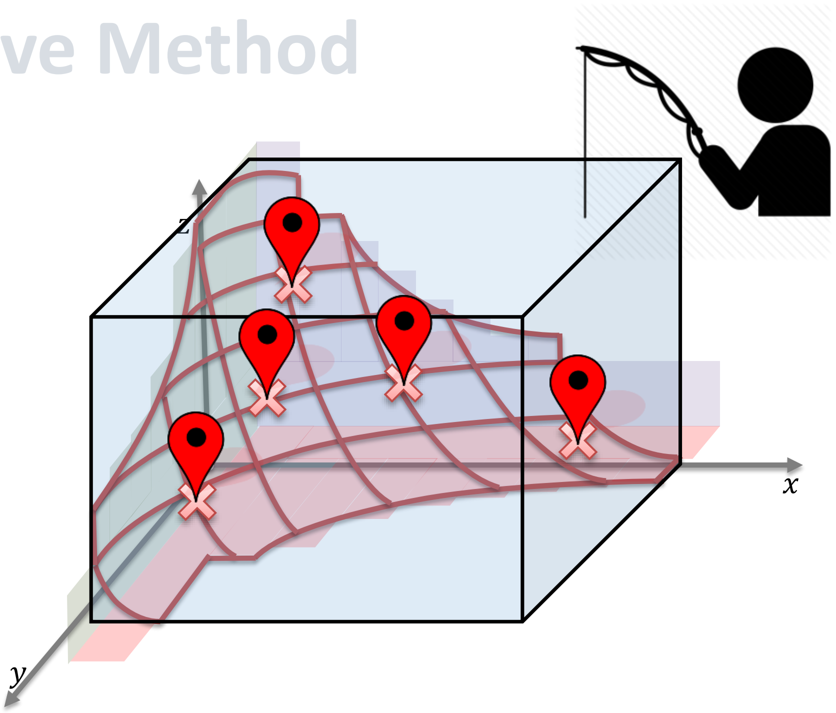
Pareto: Tri-Objective Problem

- **Pareto Surface**
  - **Given fixed z level, apply bi-objective approach**



Future Research

- **Approximation**
  - Sampling
  - Speed up
- **Integration with Interactive Method**
  - Interactive feedback
  - Decision on Pareto Front



Future Research

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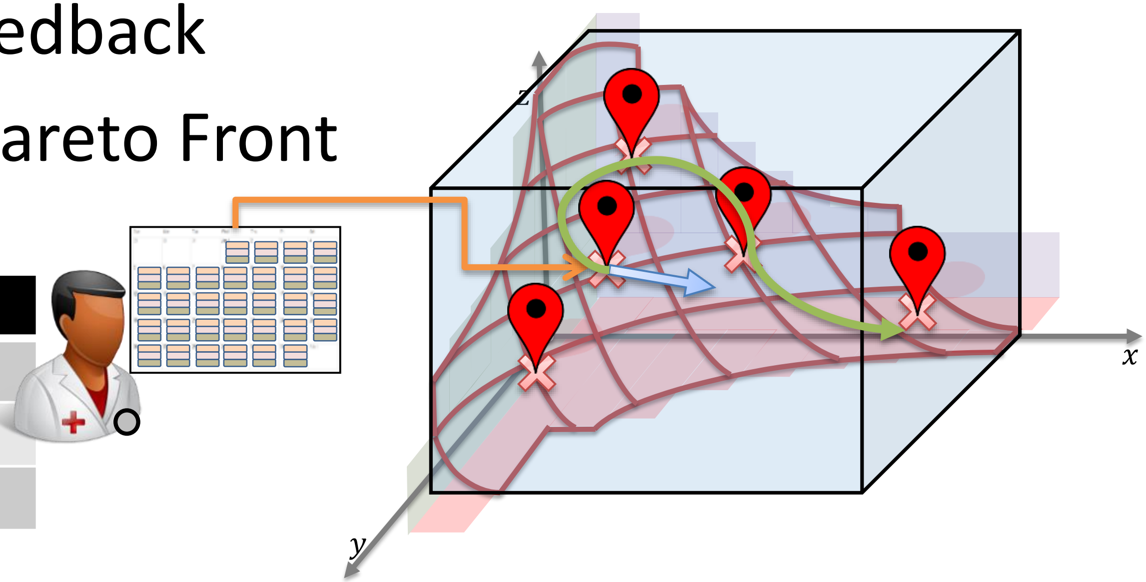
Metrics	$f_1(x)$	$f_2(x)$	$f_3(x)$	…
ΔValue	↑	↓	↑	…
ΔValue	↓	↑	↓	…
ΔValue	↓	↑	↓	…



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Feedback and Questions

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